An Educational Code for Topological Optimization in Gibiane

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I. Optimization operators/procedures in Cast3M

A brief reminder

Find the minimum/maximum of a linear/nonlinear function with several variables, with/without constraints (linear/nonlinear)

● Simplex method

SIMP max. of a linear function, with linear equalities/inequalities constraints

● Least Squares method

MOCA min. of the sum of squares distances (observation – prediction)^2 for a linear function

AJUSTE idem but on a nonlinear function (the iterative scheme is included)

LEVM Levenberg Marquardt method ("damped" least squares)

● Method of Moving Asymptotes (MMA) [1]

Min. of a nonlinear function with nonlinear inequalities

EXCE a variation of the MMA

MMA new "generic" Method of Moving Asymptotes [2]

Approximate the original nonlinear problem with a succession of (simpler) convex problems

Must be included in a iterative scheme to deal with nonlinear functions

I. Optimization operators/procedures in Cast3M

Method of Moving Asymptotes (MMA)

- **New MMA operator:** “generic” Method of Moving Asymptotes [2]

$$\min_{\mathbf{x},y,z} \quad f_0(\mathbf{x}) + a_0 z + \sum_{i=1}^{m} [c_i y_i + 0.5 d_i {y_i}^2] \qquad (f_0 \text{ objective function})$$

$$\begin{aligned}
\text{subject to: } & f_i(\mathbf{x}) - a_i z - y_i \leq 0 && \text{for } i = 1, \dots, m && (f_i : \text{constraints functions}) \\
& x_j^{\min} \leq x_j \leq x_j^{\max} && \text{for } i = 1, \dots, n && (x_j : \text{design variables}) \\
& y_j \geq 0 && \text{for } j = 1, \dots, m && (y_i : \text{slack variables for constraints}) \\
& z \geq 0 && && (z : \text{regularization variable})
\end{aligned}$$

new **MMA** operator:
- uses LISTREELs (instead of TABLEs) which improves performances
- more generic framework

Can handle several “classical” optimization problems:

Constrained “standard” min. problem:
$$\min_{\mathbf{x}} \quad f_0(\mathbf{x})$$
$$\text{s.t.:} \quad f_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, m$$

Most used for topological optimization

Constrained “min-max” problem:
$$\min_{\mathbf{x}} \quad \max\{h_1(\mathbf{x}), h_2(\mathbf{x}), \dots, h_p(\mathbf{x})\}$$
$$\text{s.t.:} \quad g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, q$$

Constrained “least-squares” problem:
$$\min_{\mathbf{x}} \quad 0.5 \sum_{i=1}^{p} {h_i}^2(\mathbf{x})$$
$$\text{s.t.:} \quad g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, q$$

- **KKT_MMA procedure:** residuals for the Karush-Kuhn-Tucker conditions

test cases: mma_00.dgibi mma_01.dgibi mma_02.dgibi mma_03.dgibi mma_04.dgibi

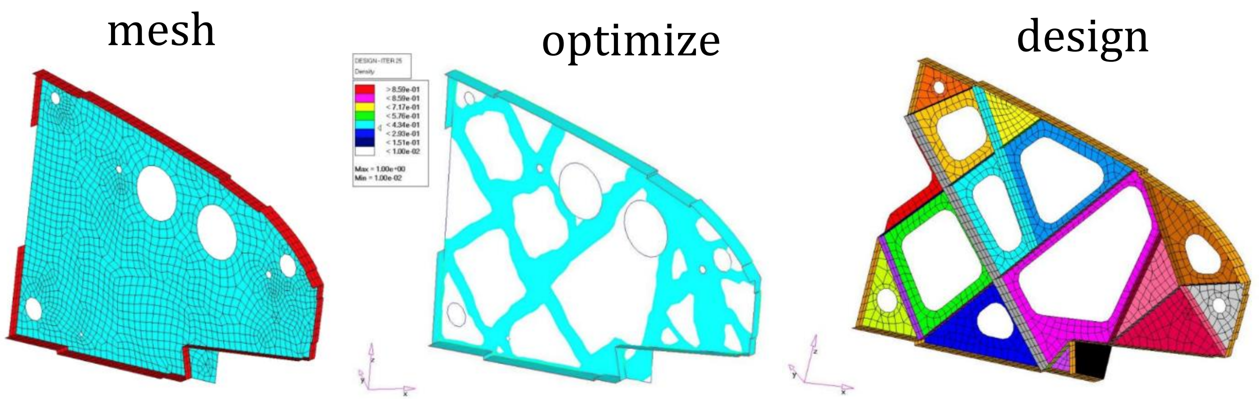
II. Topological Optimization

Introduction to density-based TO

- **Find the optimal matter distribution in a meshed space to minimize/maximize some properties**

optimization variable: continuous element volume fraction (density) $x_e \in [0,1]$

- **Identify holes for shape/parametric optimization**



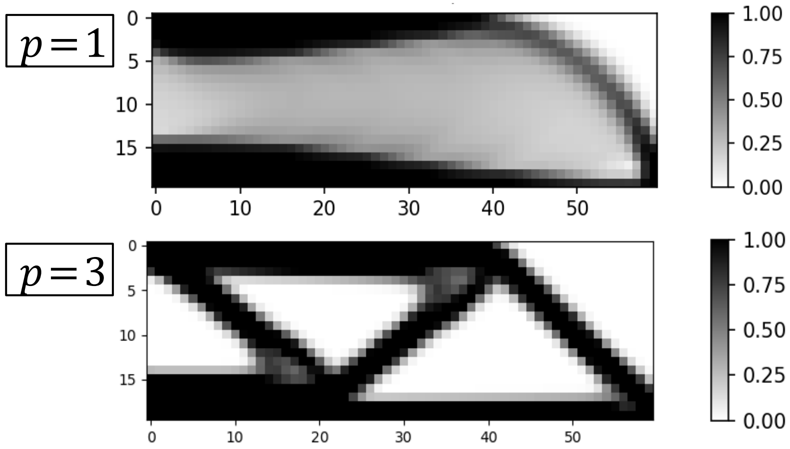
Airplane structure TO application [3]

- **Solid Isotropic Material Penalization (SIMP)**

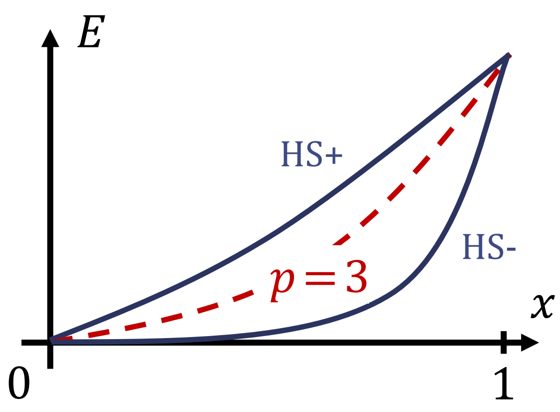
Penalize intermediate density contribution to objective or constraint functions

- Mechanical application: Young Modulus E

$$E_e(x_e) = x_e^p\ E$$



Hashin-Strickmann criteria



II. Topological Optimization

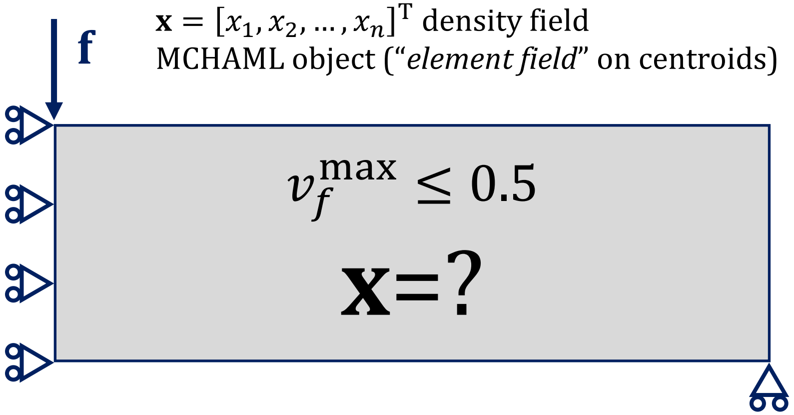
In Cast3M: the TOPOPTIM procedure [4]

- **Easy to use procedure**

Solve the **"classical" compliance** minimization problem:

$$\min_{\mathbf{x}} \quad c(\mathbf{x}) = \mathbf{u}^{\mathrm{T}}.\mathbf{f} = \mathbf{u}^{\mathrm{T}}.\mathbf{K}.\mathbf{u} \quad \text{eq. to minimize strain energy}$$

$$\text{subject to:} \quad v_f(\mathbf{x}) = \frac{V(\mathbf{x})}{V_0} \leq v_f^{\max} \qquad \text{max. global volume fraction}$$



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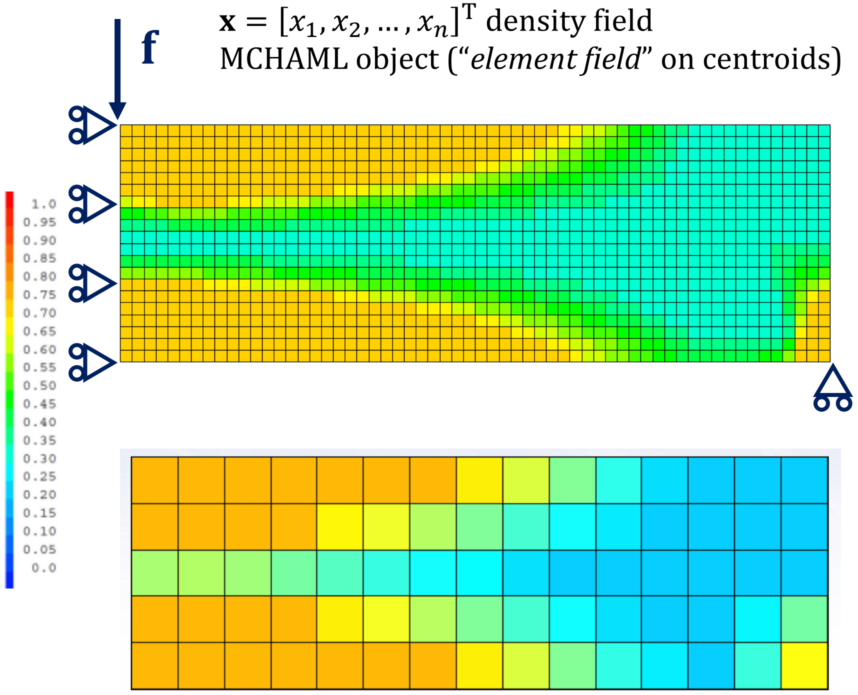
- **Additional features:**
  - unstructured meshes
  - 2D (plane/axis) 3D models
  - fixed full/empty zones
  - multi-case loading
  - automatic element removal
  - thermal analysis
  - nonlinear behavior (plasticity, contact,…)
  - can be stopped/restarted (remeshing…)
  - geometric restrictions

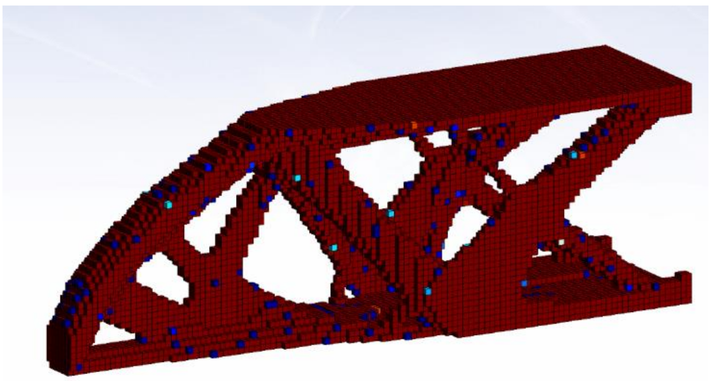
- **Additional smoothing procedure TOPOSURF**

A lot of **examples**

topoptim_01.dgibi … topoptim_11.dgibi

toposurf_01.dgibi toposurf_03.dgibi





III. The educational codes top_oc.dgibi & top_mma.dgibi

Presentation

- **Transcription in Gibiane of the "88 lines code in Matlab" [5]**
- **For educational and personal purposes**
    - \+ Short, simple, detailed (~280 lines)
    - \+ Easy to modify
    - \- TOPOPTIM functionalities not available

- **Description**

Solve the **"classical" compliance** minimization on MBB:

$$\min_{\mathbf{x}} \; c(\mathbf{x}) = \mathbf{u}^{\mathrm{T}}.\mathbf{f} = \mathbf{u}^{\mathrm{T}}.\mathbf{K}.\mathbf{u}$$

subject to: $v_f(\mathbf{x}) = \dfrac{V(\mathbf{x})}{V_0} \leq v_f^{\max}$

With: $\mathbf{K}(\mathbf{x}).\mathbf{u} = \mathbf{f}$

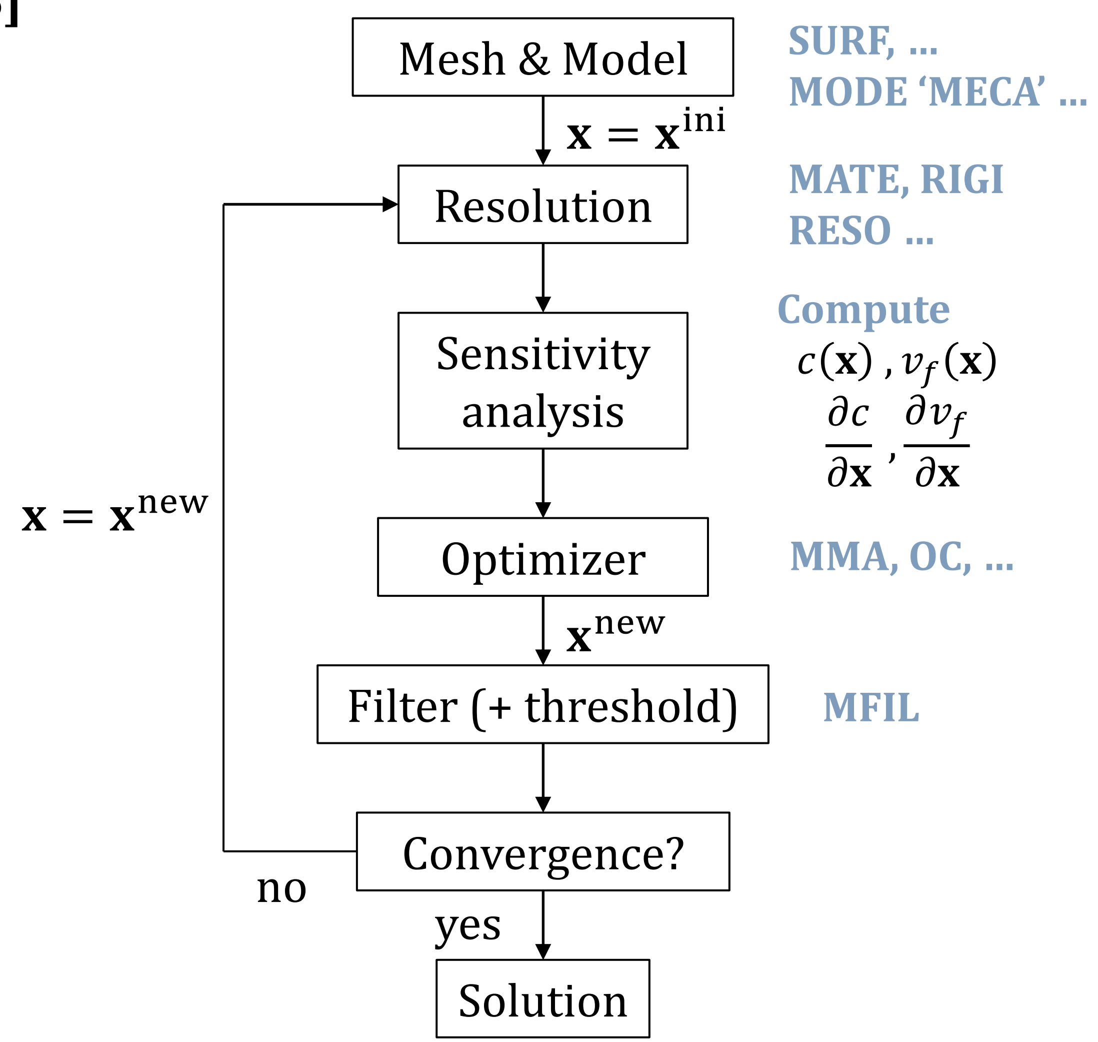
$$E_e(x_e) = E_{\min} + x_e^p (E_{\max} - E_{\min})$$

- **Sensitivity analysis**

$$\frac{\partial c}{\partial x_e} = -\mathbf{u}_e^{\mathrm{T}}.\frac{\partial \mathbf{K}_e}{\partial x_e}.\mathbf{u}_e = \int_{v_e} \frac{\partial}{\partial x_e} \boldsymbol{\sigma}_e : \boldsymbol{\varepsilon}_e \, \mathrm{d}V$$

$$= \frac{\partial E_e}{\partial x_e} \times \textbf{INTG}\ \text{'ELEM'}\ \text{model}\ (\textbf{ENER}\ \text{model}\ \boldsymbol{\varepsilon}\ \boldsymbol{\sigma}_1)\ ;$$

Code flowchart



Some examples!

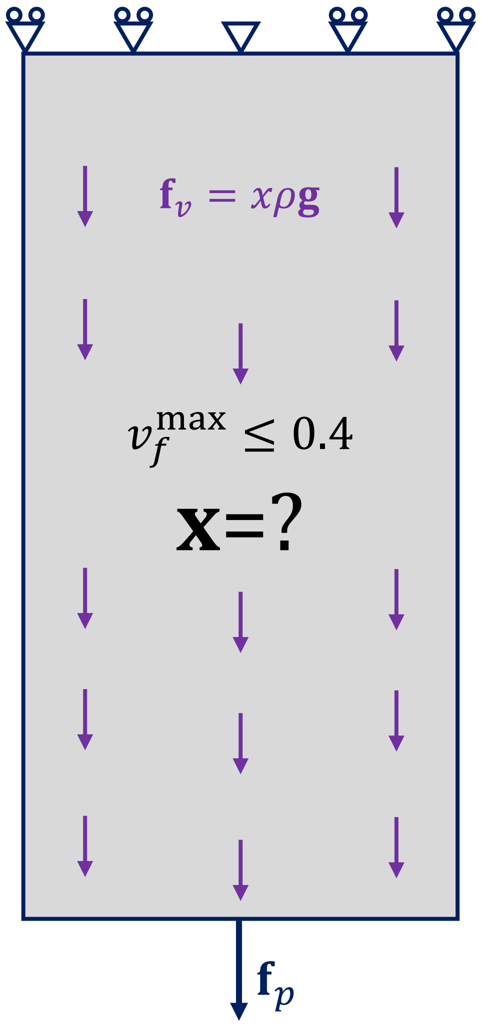
IV. Nonconvex problems

Example of design-dependent loads (gravity, centrifugal forces, Laplace forces,…)

- **Compliance minimization problem with a non convex object:**

Compliance gradient becomes: $\dfrac{\partial c}{\partial x_e} = -\mathbf{u}^{\mathrm{T}}.\dfrac{\partial \mathbf{K}}{\partial x_e}.\mathbf{u} + 2\mathbf{u}^{\mathrm{T}}.\dfrac{\partial \mathbf{f}}{\partial x_e}$

- **Test case: hanging bar (punctual force + gravity)**



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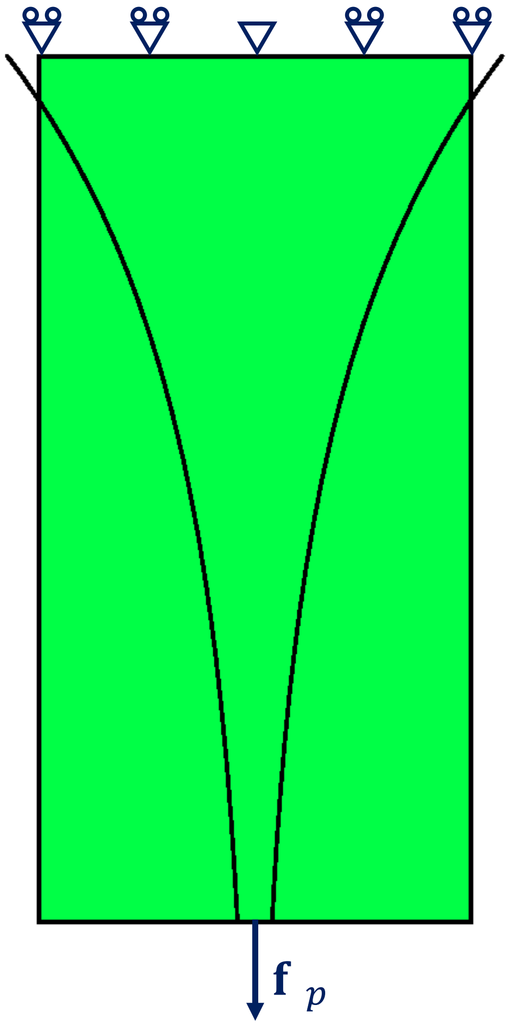
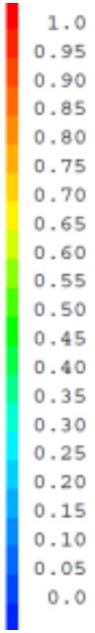


Good agreement with the reference analytical solution [6]



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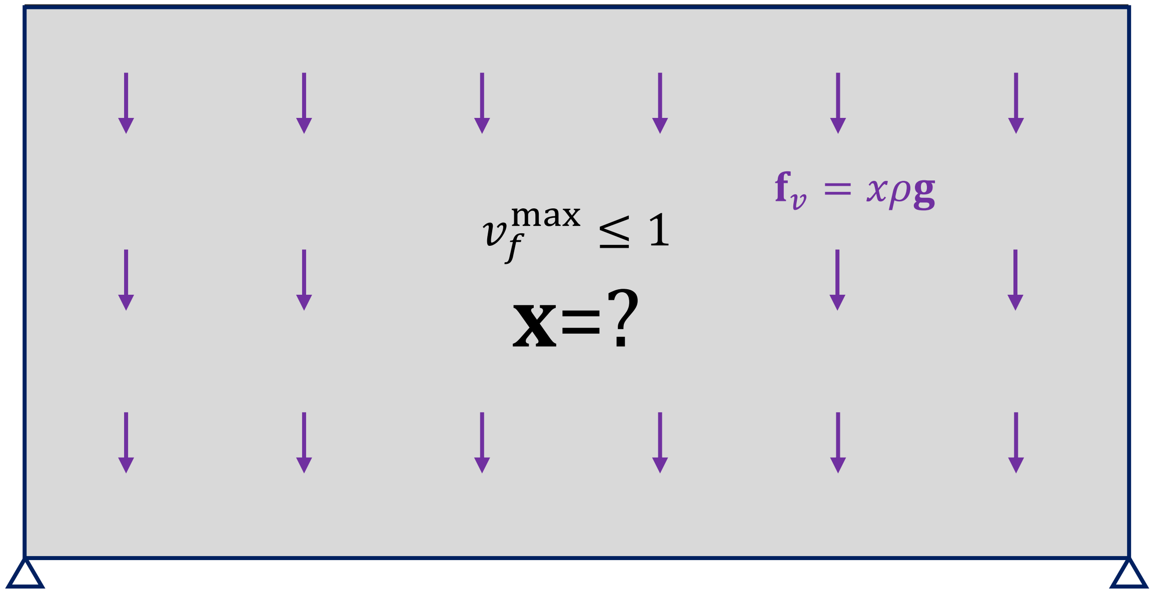
- **Test case: hanging bar (punctual force + gravity)**

- **Test case: arch bridge (gravity only!)**



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- **Test case: hanging bar (punctual force + gravity)**

- **Test case: arch bridge (gravity only!)**

Solution may depend on the starting point
and be only a local minimum!

IV. Multi-constrained problem

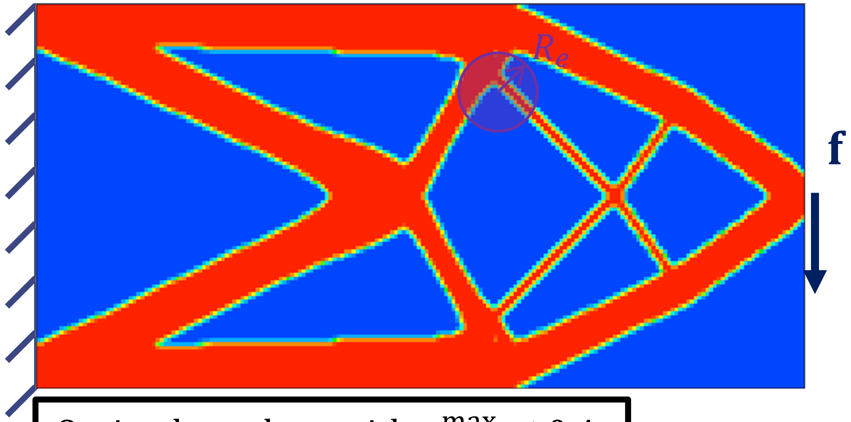
Example of "infill" optimization for additive manufacturing [7]

- **From the compliance minimization problem:**

$$\min_{\mathbf{x}} \quad c(\mathbf{x}) = \mathbf{u}^{\mathrm{T}}.\mathbf{f} = \mathbf{u}^{\mathrm{T}}.\mathbf{K}.\mathbf{u}$$

$$\text{subject to:} \quad v_f(\mathbf{x}) = \frac{V(\mathbf{x})}{V_0} \leq v_f^{\max} \qquad \text{max. global volume fraction}$$

$$\left(\frac{1}{n}\sum_e \bar{x}_e{}^P\right)^{1/P} \leq \alpha \qquad \text{max. local volume fraction}$$



Optimal topology with $v_f^{\max} \leq 0.4$

- **Measure of the "local" (neighboring) volume fraction**

$$\bar{x}_e = \frac{\sum_{i\in\mathbb{N}_e} x_i}{\sum_{i\in\mathbb{N}_e} 1} \qquad \text{where } \mathbb{N}_e = \left\{ i \,\middle|\, \underbrace{\text{dist}(i,e)}_{} \leq r_e \right\}$$

Linear filter
MFIL operator

centroid
distance $i \leftrightarrow e$

- **Local constraints → aggregated into a single differentiable constraint**

$$\bar{x}_e \leq \alpha \quad \forall e \in [1,n] \quad \rightarrow \max_{\forall e}(\bar{x}_e) \leq \alpha \quad \rightarrow \left(\frac{1}{n}\sum_e \bar{x}_e{}^P\right)^{1/P} \leq \alpha$$

P-mean function: soft max.
AGRE operator

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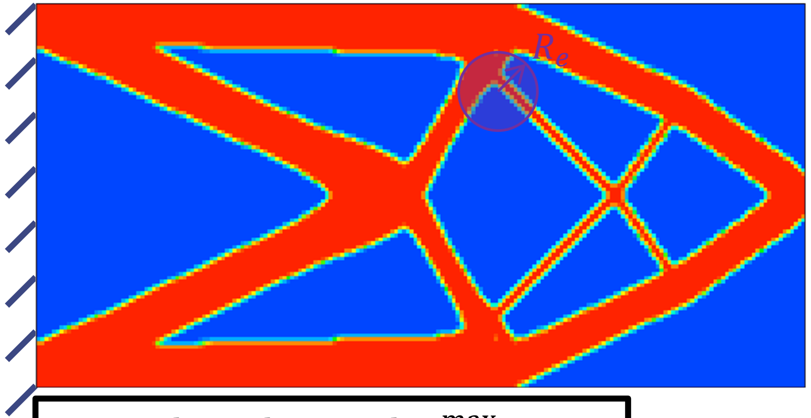

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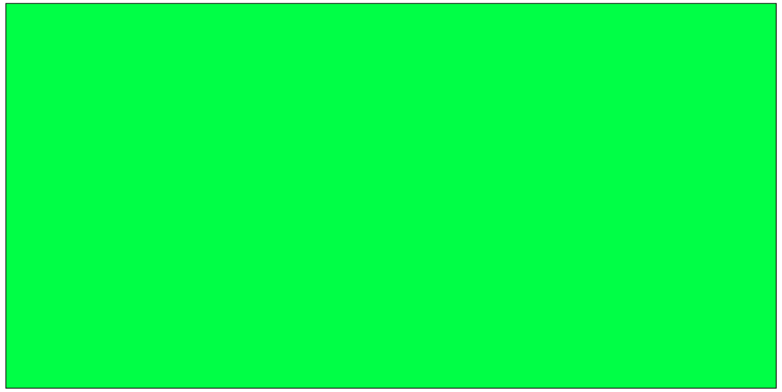


Optimal topology with $v_f^{\max} \leq 0.4$ and
$\alpha \leq 0.6$ $r_e = 6 \times$ element size $P = 16$

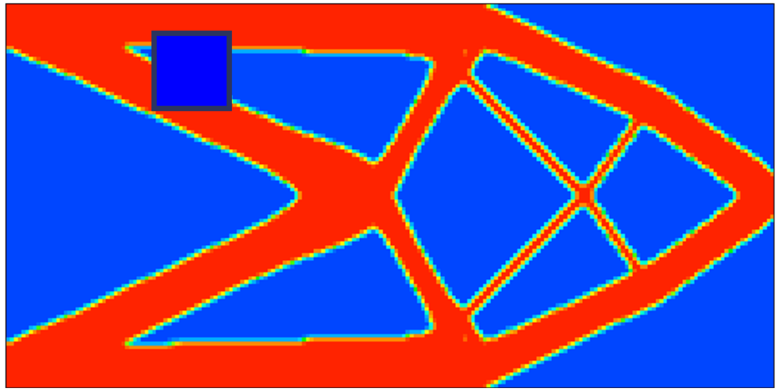
IV. Multi-constrained problem

Example of “infill” optimization for additive manufacturing [7]

- **Robustness** (to damage)

Let’s introduce a damaged zone in the structure

What’s the effect on the compliance (objective function)?



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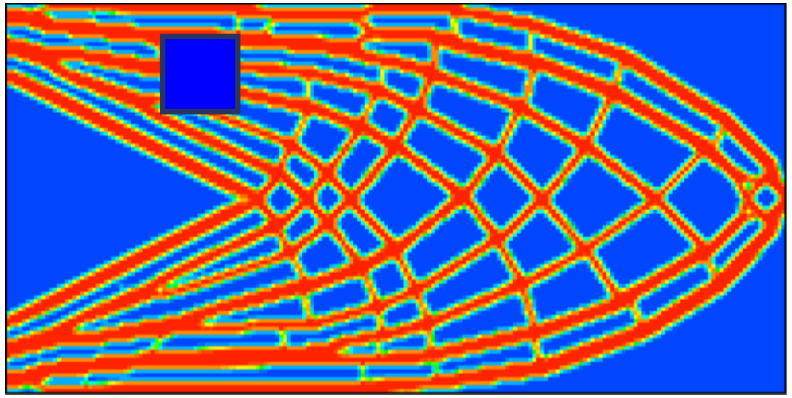

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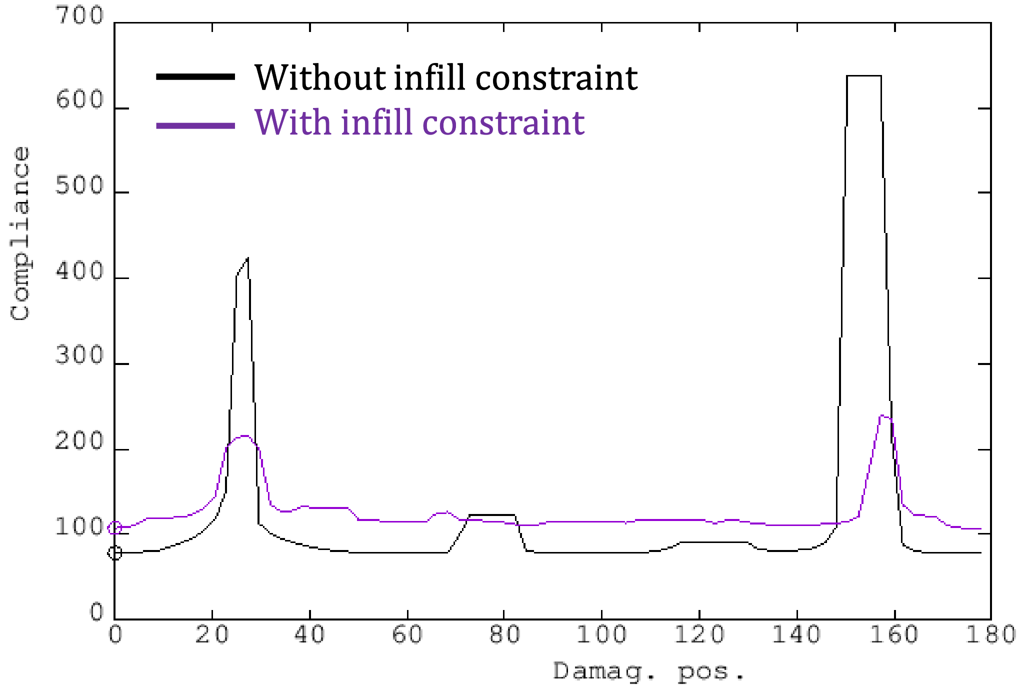

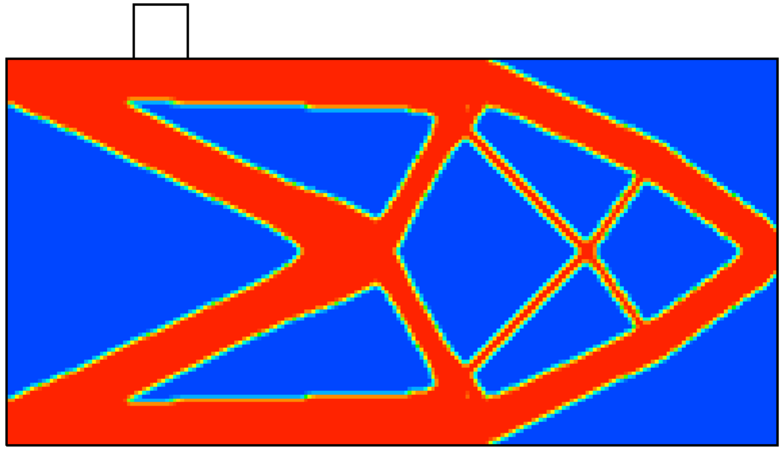

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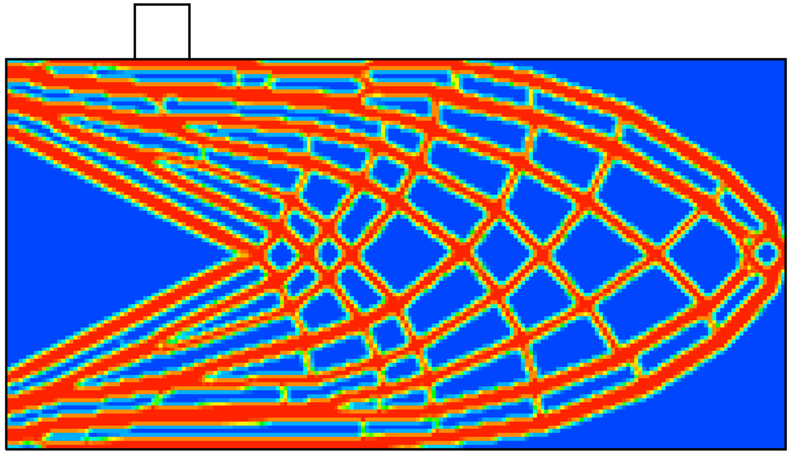

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- **Robustness** (to load uncertainty)

IV. Adjoint problem

Case of compliant mechanism [8]

- **Maximize structure displacement according to a load/displacement input**
- **Optimization problem:**

$$\min_{\mathbf{x}} \; u_{\text{out}}(\mathbf{x}) = \mathbf{e}_{\text{out}}^{\mathrm{T}}.\,\mathbf{u} \qquad \mathbf{e}_{\text{out}}^{\mathrm{T}} = -\langle 0 \ldots 0 \; 1 \; 0 \; \ldots 0 \rangle$$

$$\text{subject to: } v_f(\mathbf{x}) = \frac{V(\mathbf{x})}{V_0} \leq v_f^{\max}$$

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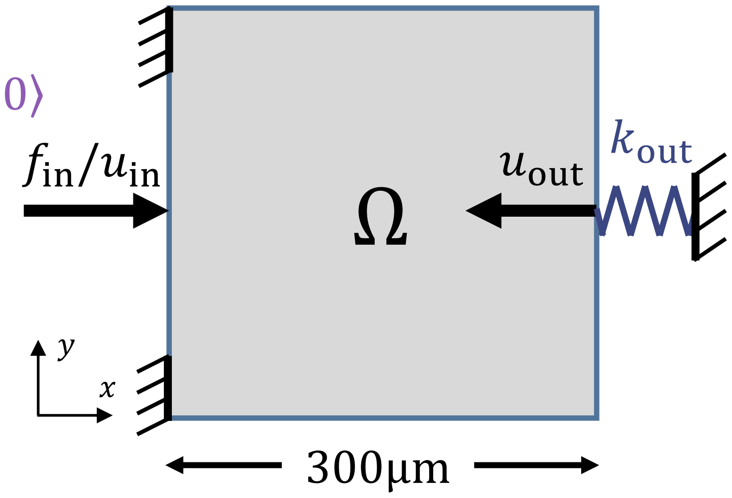

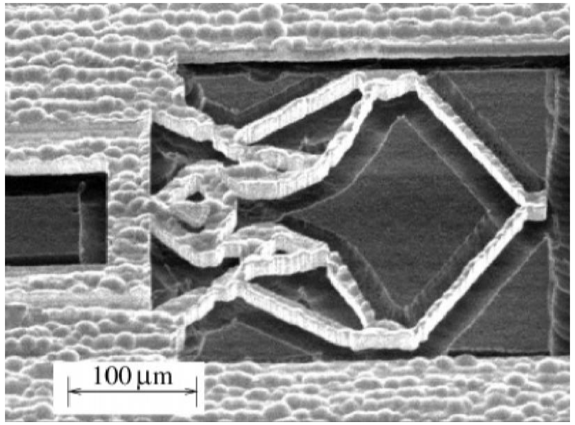

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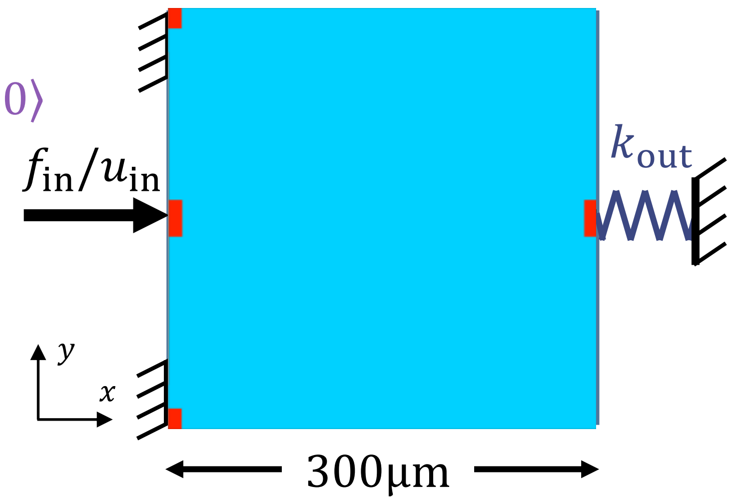

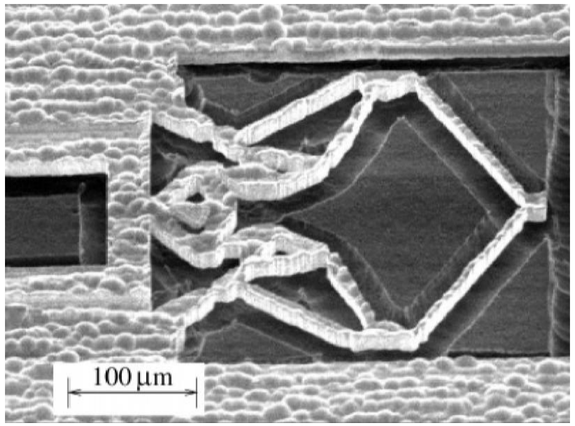

- **Adjoint problem:**

$$\frac{\partial u_{\text{out}}}{\partial x_e} = \mathbf{e}_{\text{out}}^{\text{T}} \frac{\partial \mathbf{u}}{\partial x_e} = \underbrace{\mathbf{e}_{\text{out}}^{\text{T}} \mathbf{K}^{-1}}_{= \boldsymbol{\lambda}^{\text{T}}} \frac{\partial \mathbf{K}}{\partial x_e}.\,\mathbf{u}$$

<u>Adjoint system:</u>

$$\mathbf{K}\,.\,\boldsymbol{\lambda} = \mathbf{e}_{\text{out}} \qquad \mathbf{u} \;\; \boldsymbol{\lambda} = \textbf{RESO} \;\; \mathbf{K} \;\; \mathbf{f} \;\; \mathbf{e}_{\text{out}}$$

$$\frac{\partial u_{\text{out}}}{\partial x_e} = -\boldsymbol{\lambda}_e^{\text{T}}.\frac{\partial \mathbf{K}}{\partial x_e}.\,\mathbf{u}_e$$

$$= \frac{\partial E_e}{\partial x_e} \times \textbf{INTG} \;\; \text{'ELEM'} \; \text{model} \left(\textbf{ENER} \; \text{model} \; \boldsymbol{\varepsilon}_{\mathbf{u}} \;\; \boldsymbol{\sigma}_{\lambda,1} \right);$$

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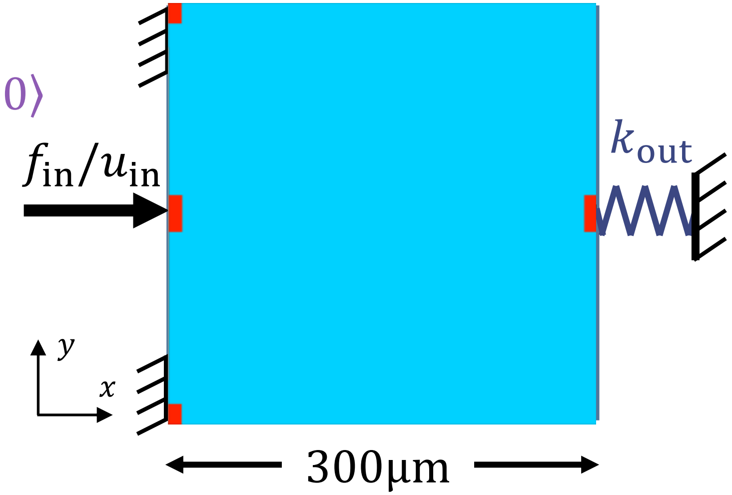

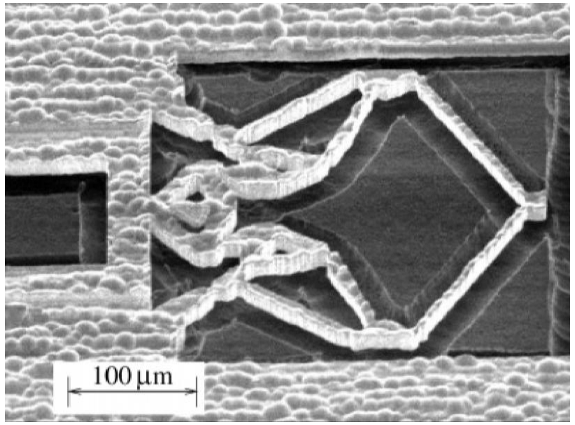

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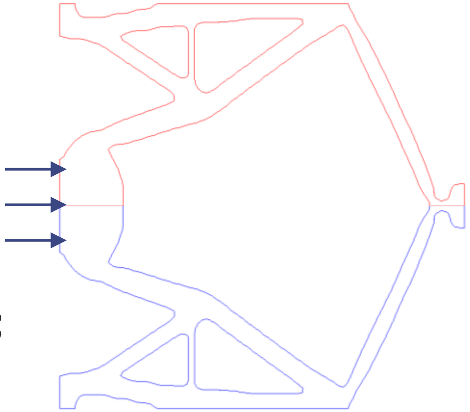
<u>Adjoint system:</u>

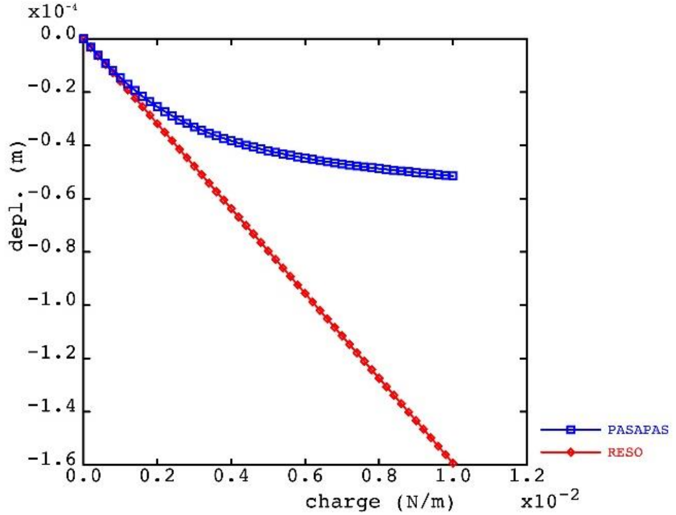
$\mathbf{K} \, . \, \boldsymbol{\lambda} = \mathbf{e}_{\text{out}}$ $\qquad$ $\mathbf{u}\ \ \boldsymbol{\lambda} =$ **RESO** $\mathbf{K}\ \ \mathbf{f}\ \ \mathbf{e}_{\text{out}}$

$$\frac{\partial u_{\text{out}}}{\partial x_e} = -\boldsymbol{\lambda}_e^{\mathrm{T}} . \frac{\partial \mathbf{K}}{\partial x_e} . \mathbf{u}_e$$

$$= \frac{\partial E_e}{\partial x_e} \times \textbf{INTG} \ \text{'ELEM'} \ \text{model} \ \left(\textbf{ENER} \ \text{model} \ \boldsymbol{\varepsilon}_{\mathbf{u}} \ \ \boldsymbol{\sigma}_{\lambda,1} \right) ;$$

⚠ small disp. ≢ large disp.





Conclusion and perspectives

- **Available here (for Cast3M version ≥2026):**

  top_oc.dgibi   top_mma.dgibi

- **New simple Gibiane test cases for topological optimization (T.O.)**

Easy to read and adapt to many T.O. problems
Examples of application to non-convex/multi-constrained T.O. problems
Extension to other objective/constraints functions
"Sandbox" for testing new formulations

- **Coming soon:**

GC MMA (Globally Convergent)
Other examples: stress-based T.O., multi-physics (thermo-mechanical, …) T.O.
Implementation in the **TOPOPTIM** procedure (for the most common problems)

References

[1] **K. Svanberg**
The method of moving asymptotes – a new method for structural optimization
International Journal for Numerical Methods in Engineering, 24, 359-373 (1987)

[2] **K. Svanberg**
MMA and GCMMA – two methods for nonlinear optimization (2017)
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