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Unsharp Finite Element Analysis Based on Random Set Theory

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Unsharp physical quantities

Unsharp physical quantities:

Unsharp physical quantities are a serious problem often faced especially in geotechnical engineering

Geometrical and structural configuration, material parameters, boundary conditions, initial stress state, contact behaviour, load history and other thinkable modelling information remains often unsharp

Unsharp physical quantities:

Limited and spatially restricted geotechnical site investigation

Dominant diversity in structure, in mechanical, hydraulic and thermal behaviour of geotechnical materials

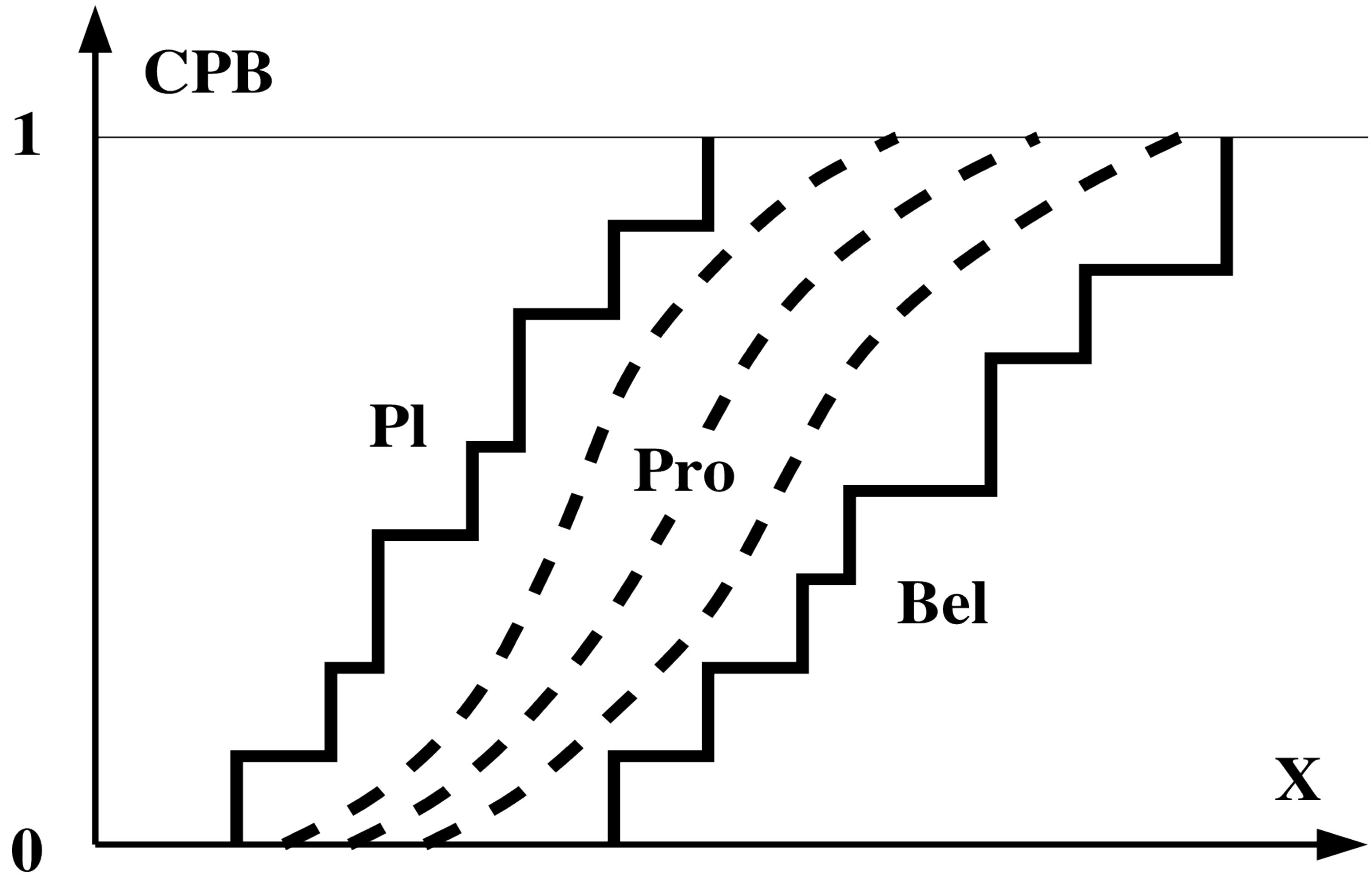
Uncertainties in stress history, loading conditions and laboratory testing results

The Random Set Theory

The Random Set Theory:

Unsharp quantities are enclosed in a lower and an upper bound interval range using discrete Pl(X) (plausibility) and Bel(X) (belief) functions bracketing possible continuous probability distribution functions Pro(X)

Stochastic property of unsharp physical quantities is represented by a cumulative probability CPB


CPB
1
0
Pl
Pro
Bel
X

The Random Set Theory:

Unsharp quantities are represented with a set of focal elements defining intervals combined with the probability of inclusion

$$P\left(m_i \middle| MIN_i \leq m_i \leq MAX_i\right)$$

The Random Set Theory:

In computations, all combinations of focal element limiting values are systematically considered and new limiting values are derived in the result

The probability corresponding to the resulting focal elements is the product of the input focal element probabilities assuming probabilistic independence

The Random Set Theory:

The Random Set Theory combines interval logic with a probabilistic modelling

The number of required computations is significantly lower than in comparable methods such as Monte Carlo simulations

The Random Set Theory is best suited for analytical or numerical analyses with limited number of computations

Random Set Theory implementation in GIBIANE with object orientation

The Random Set Procedure Collection:

#@RSTH.procedur
#@RSTH.notice

Object oriented GIBIANE library implementing the Random Set Theory in Cast3M

(Possible) inclusion into Cast3M after final validation and verification

The Random Set Procedure Collection:

’#’ symbol marks a container of multiple methods in a single file (as METHods are restricted to OBJEcts)

’@’ symbol marks external contribution

Methods create, initialize, operate and evaluate a (repeated or simultaneous) simulation with a given Random Set parametrized in a TABLE variable

The Random Set Procedure Collection:

Creation of a Random Set Object:

RS = OBJET @RSTH ;

Initialisation of a Random Set Object with the operator %’RST’ (reset) with Random Set data in the TABLE R:

RS%’RST’ R ;

The Random Set Procedure Collection:

Operation of a Random Set Object with the operators RS%’RSV’ (value) and RS%’RSR’ (result)

Evaluation of a Random Set Object with the auxiliary operators RS%’SCV’ and RS%’SCS’ generating evolution components for visualisation and output

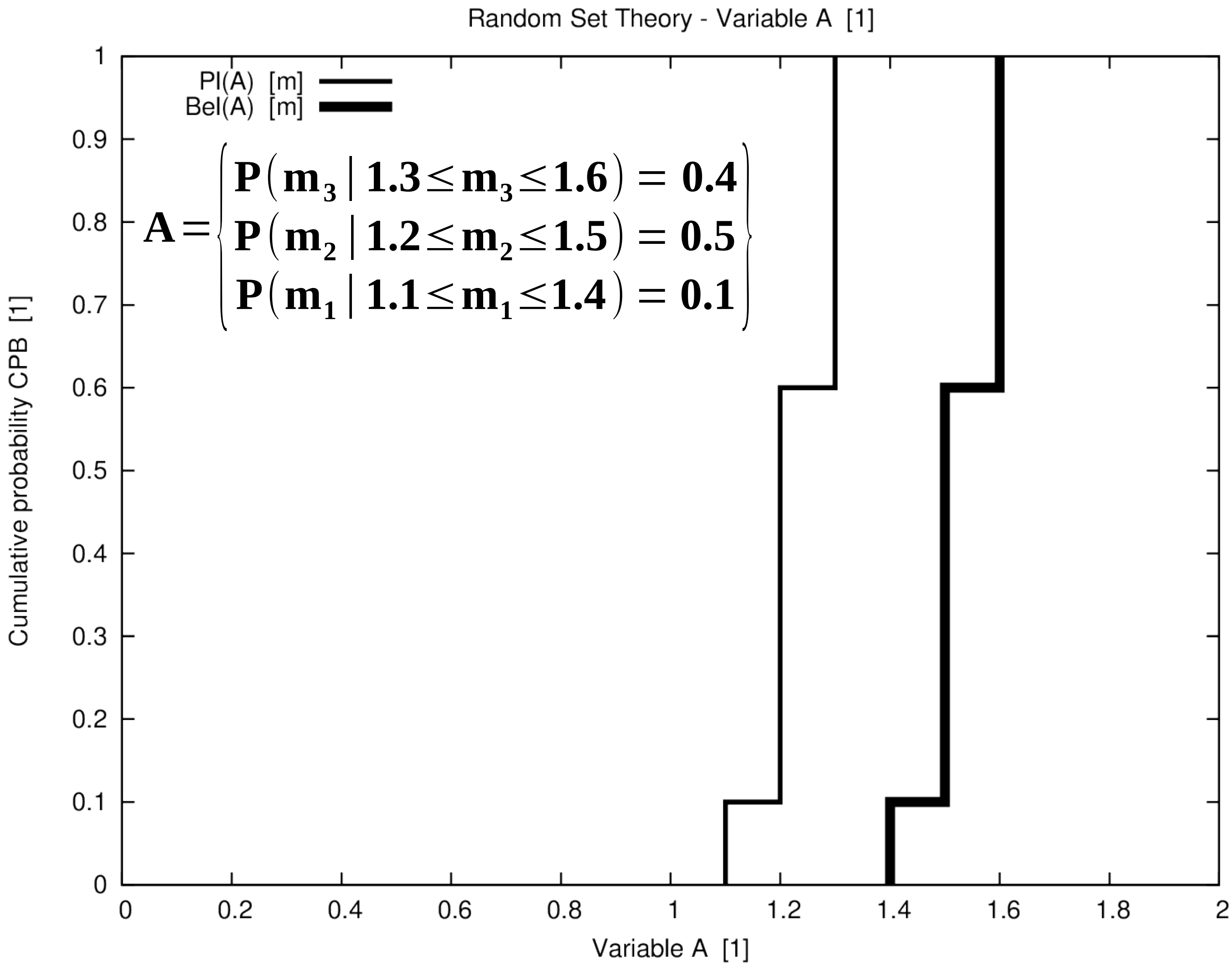
Trivial analytical example

Trivial analytical Random Set analysis example for three functions:

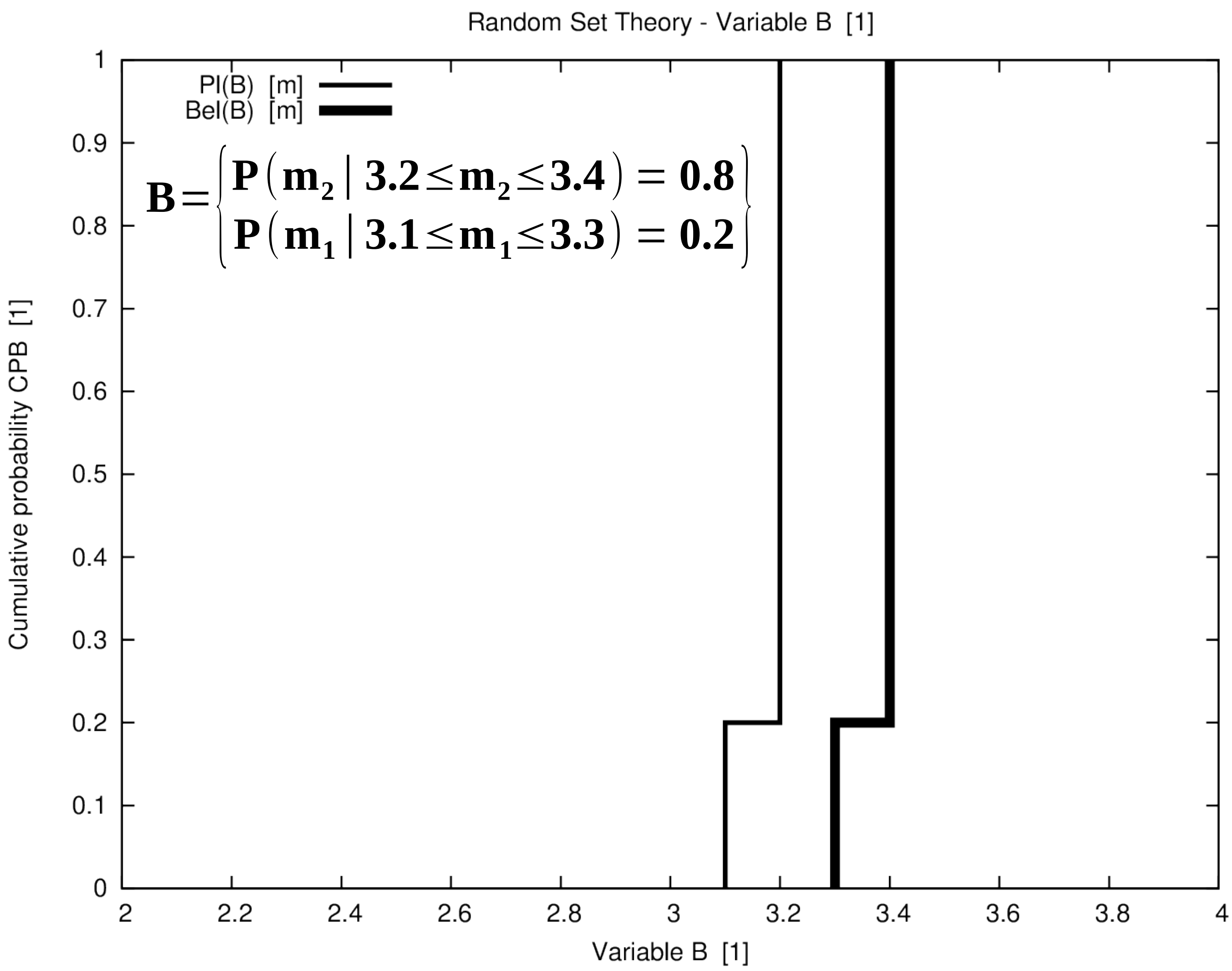
$$PA(a,b,c,d)=a+(b*c*d)$$

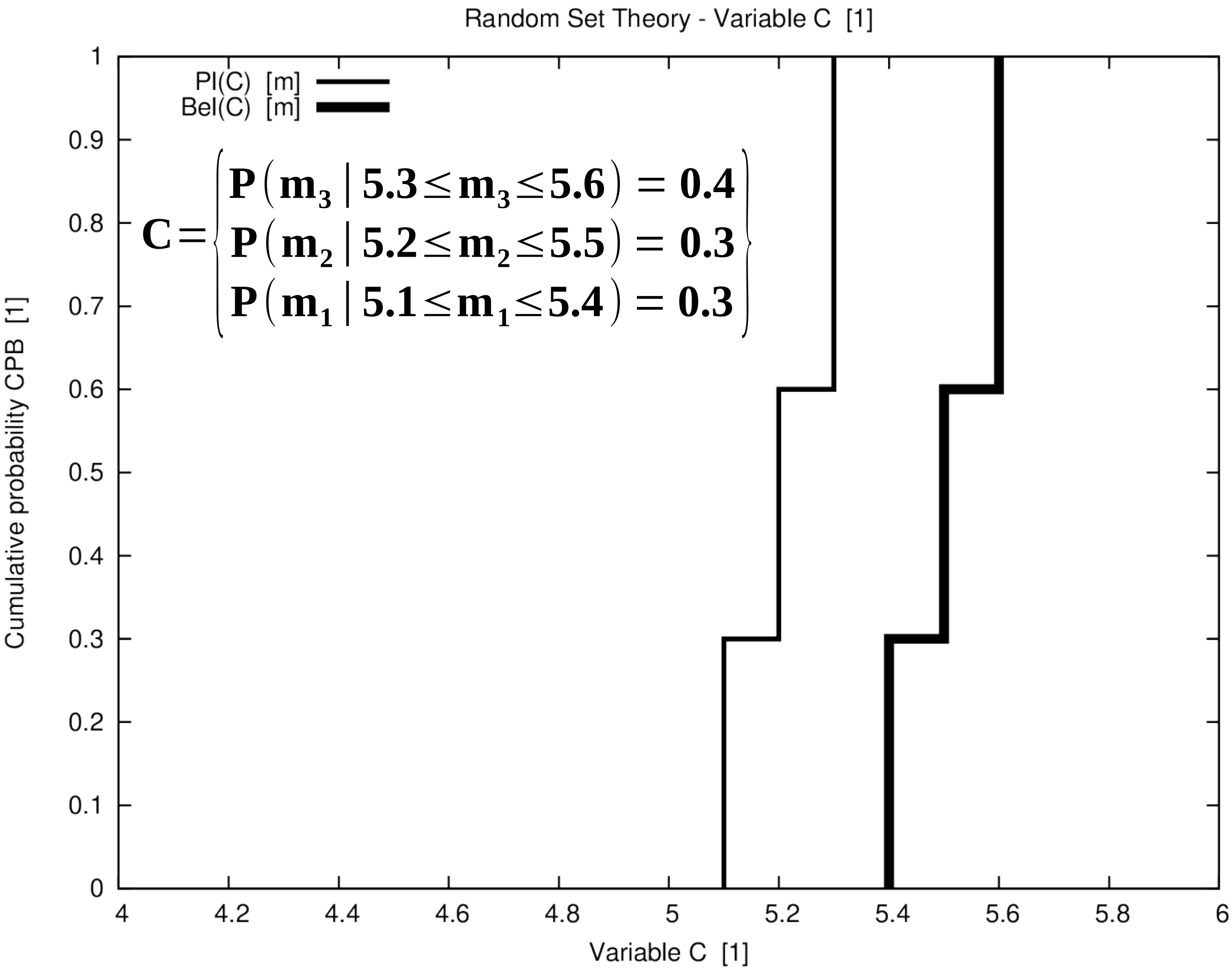
$$PB(a,b,c,d)=(a*b)+(c*d)$$

$$PC(a,b,c,d)=(a*b*c)+d$$



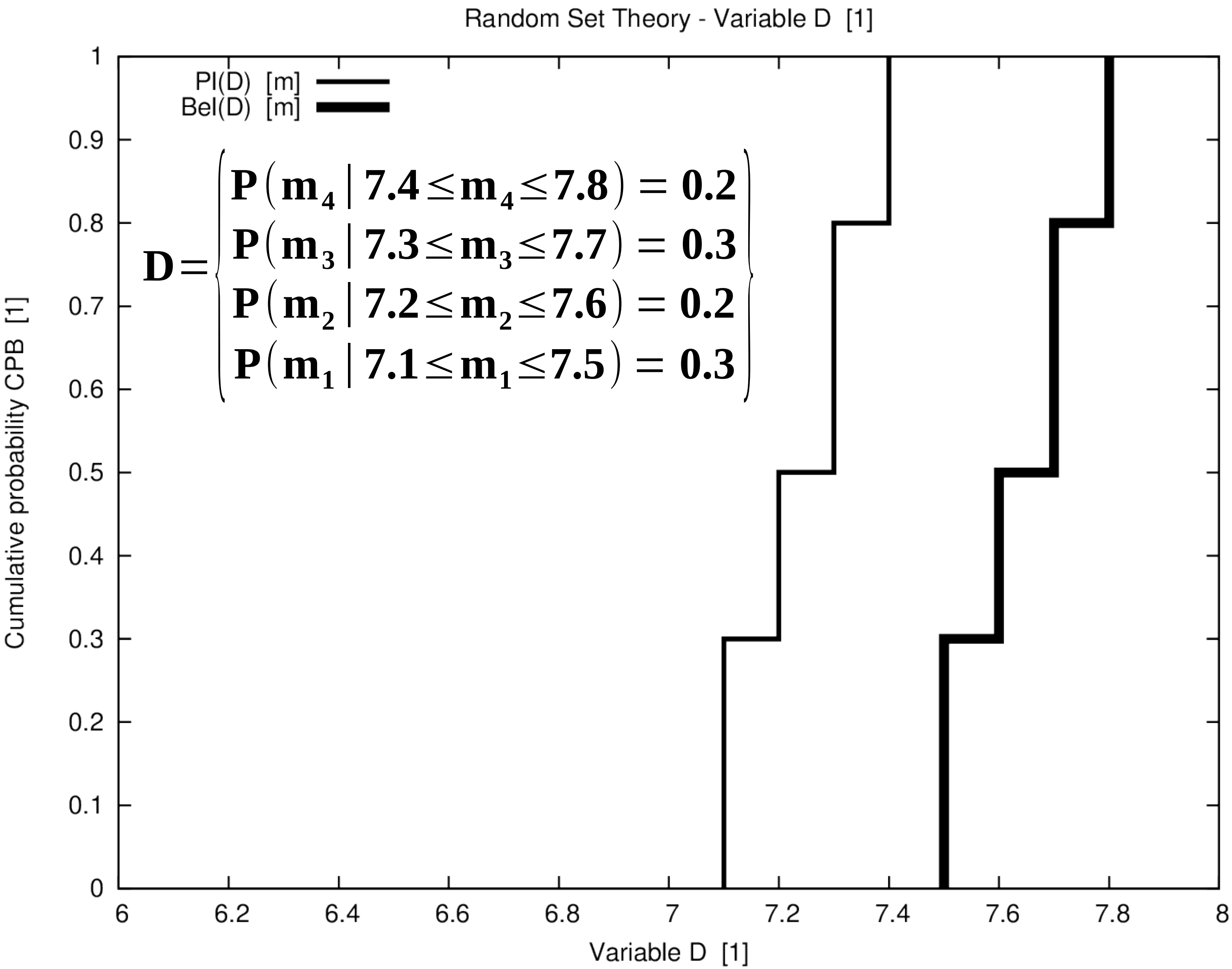
$$\mathbf{A}=\begin{Bmatrix} \mathbf{P}\left(\mathbf{m}_3 \mid 1.3 \le \mathbf{m}_3 \le 1.6\right) = 0.4 \\ \mathbf{P}\left(\mathbf{m}_2 \mid 1.2 \le \mathbf{m}_2 \le 1.5\right) = 0.5 \\ \mathbf{P}\left(\mathbf{m}_1 \mid 1.1 \le \mathbf{m}_1 \le 1.4\right) = 0.1 \end{Bmatrix}$$


Random Set Theory - Variable B [1]
Pl(B) [m]
Bel(B) [m]
$$\mathbf{B}=\left\{\begin{array}{l}\mathbf{P}\left(\mathbf{m}_2 \mid 3.2\leq\mathbf{m}_2\leq3.4\right)=0.8\\ \mathbf{P}\left(\mathbf{m}_1 \mid 3.1\leq\mathbf{m}_1\leq3.3\right)=0.2\end{array}\right\}$$
Cumulative probability CPB [1]
Variable B [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4



$$\mathbf{C} = \begin{Bmatrix} \mathbf{P}\left(\mathbf{m}_3 \mid 5.3 \le \mathbf{m}_3 \le 5.6\right) = 0.4 \\ \mathbf{P}\left(\mathbf{m}_2 \mid 5.2 \le \mathbf{m}_2 \le 5.5\right) = 0.3 \\ \mathbf{P}\left(\mathbf{m}_1 \mid 5.1 \le \mathbf{m}_1 \le 5.4\right) = 0.3 \end{Bmatrix}$$

Unsharp Finite Element Analysis based on Random Set Theory

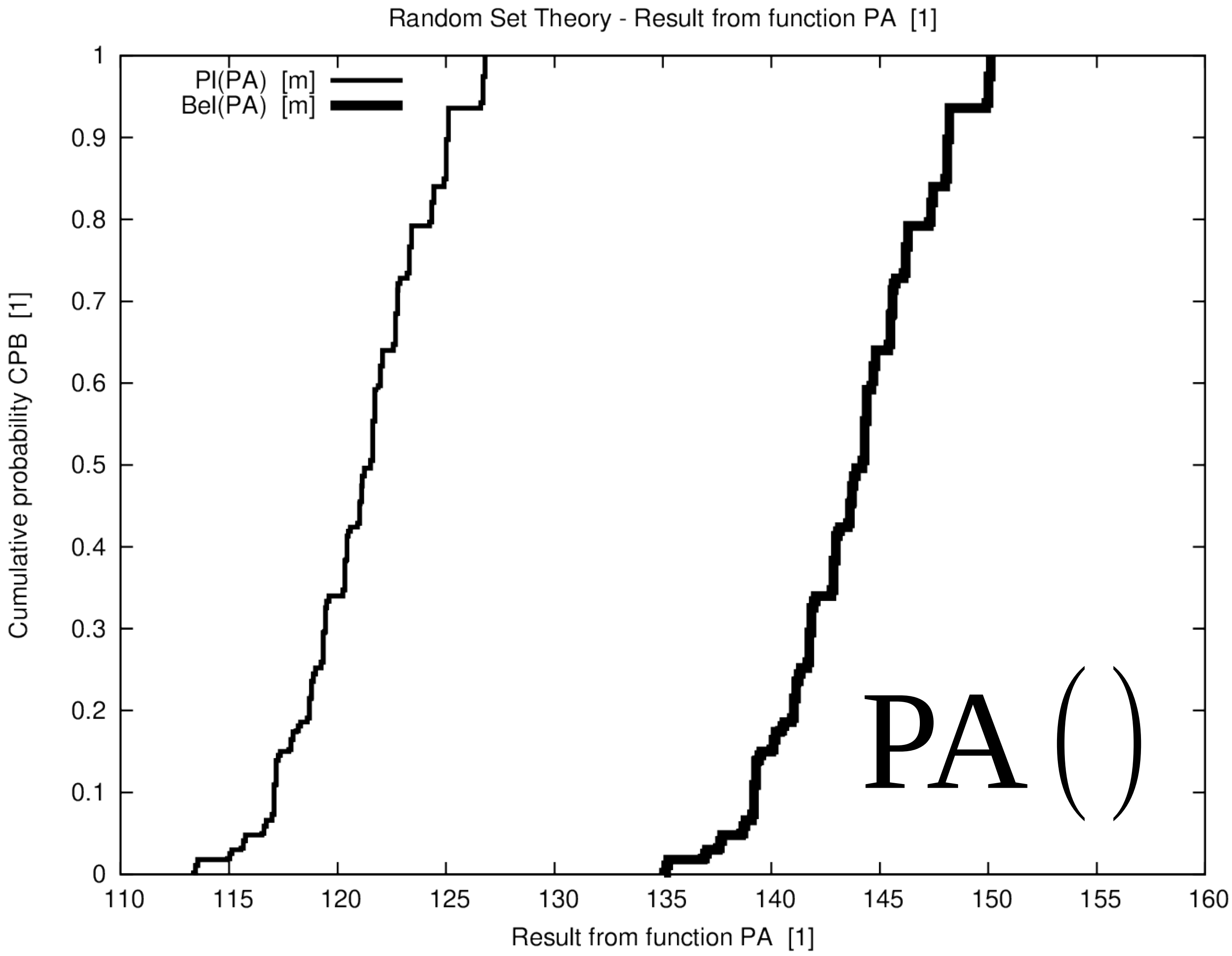


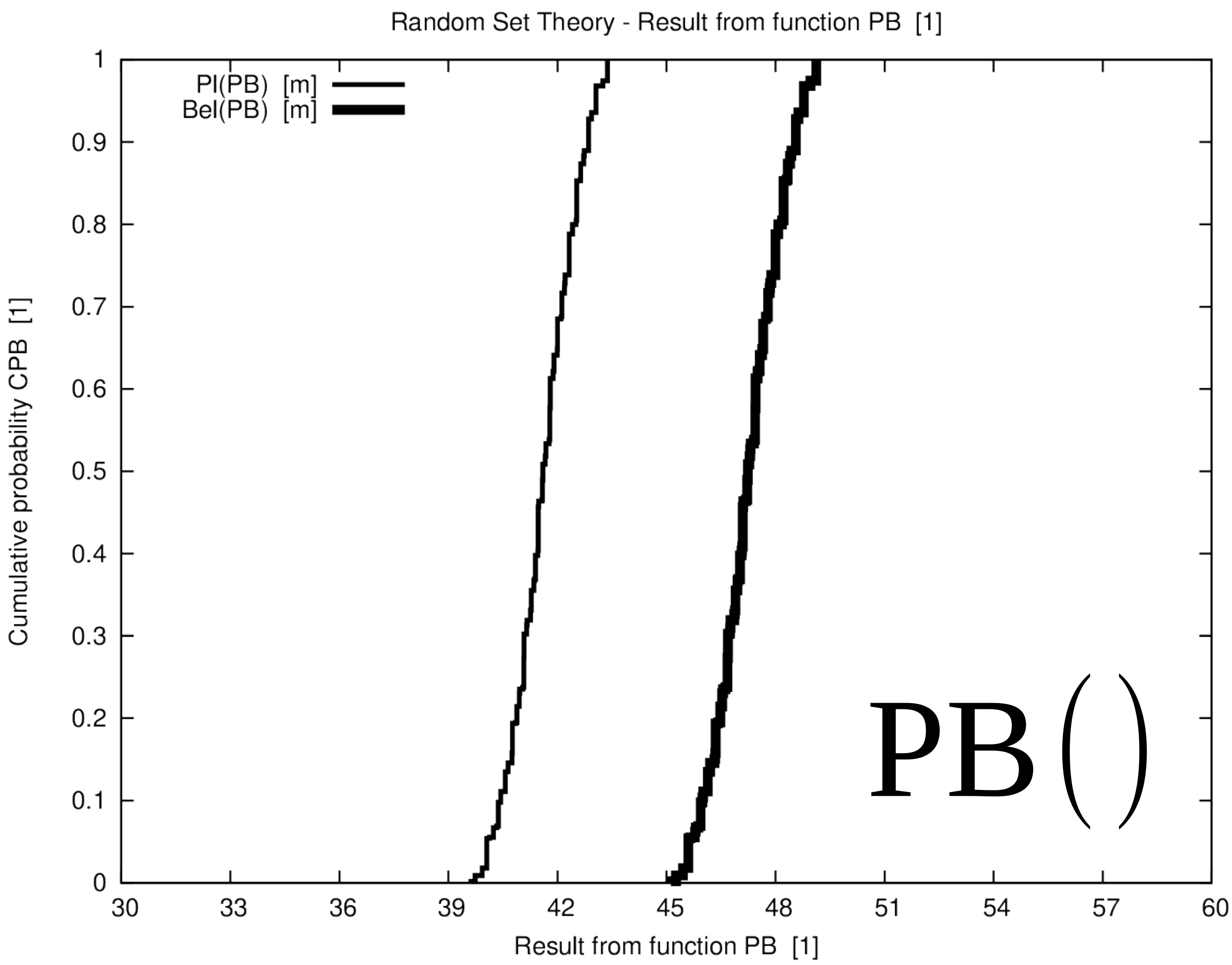
Number of required operations in order to solve the analytical problem:

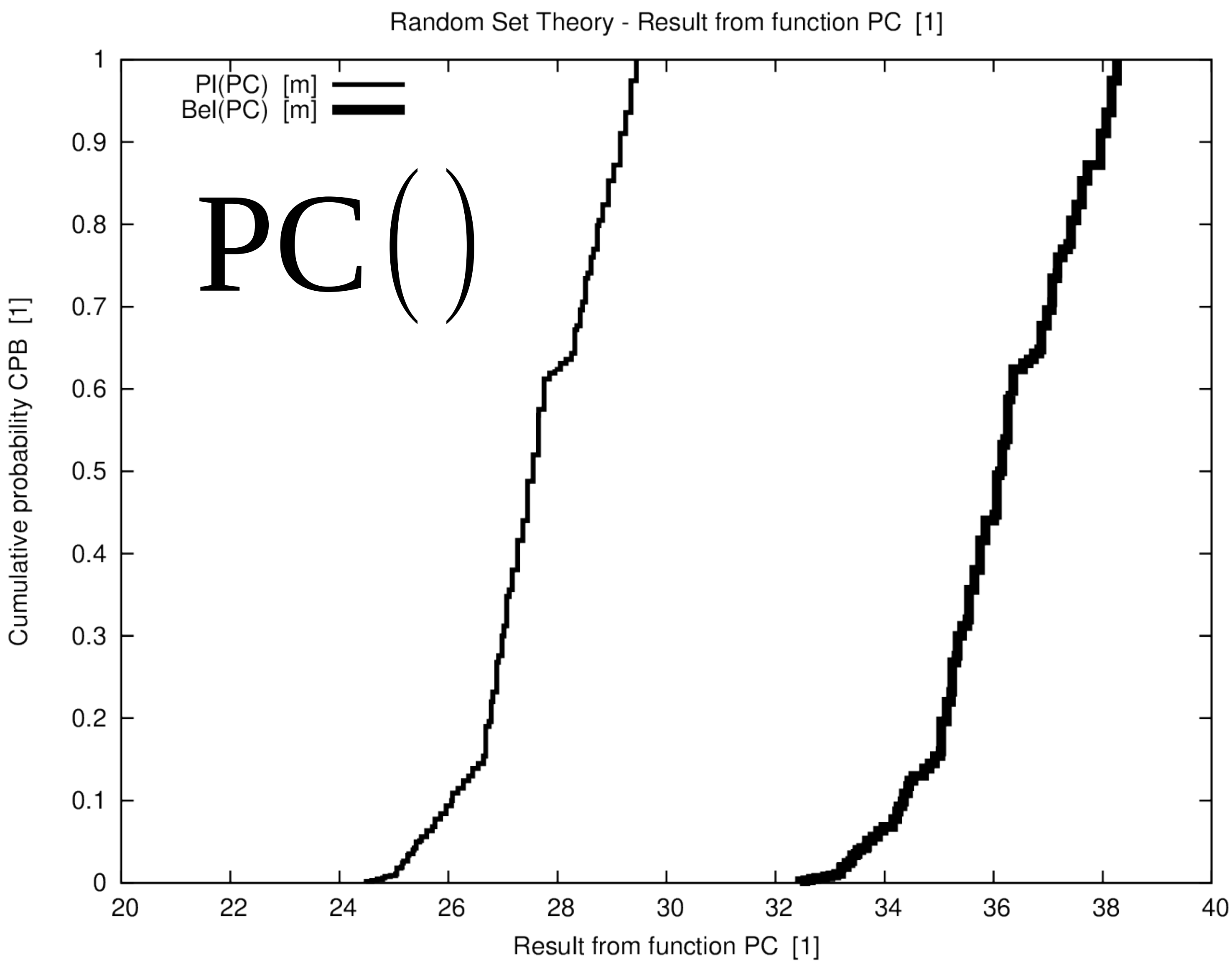
$$N = 2^n \cdot \prod_{j=1}^{n} n_j(m_i)$$

$$N = 2^4 \cdot 3 \cdot 2 \cdot 3 \cdot 4 = \mathbf{1152}$$

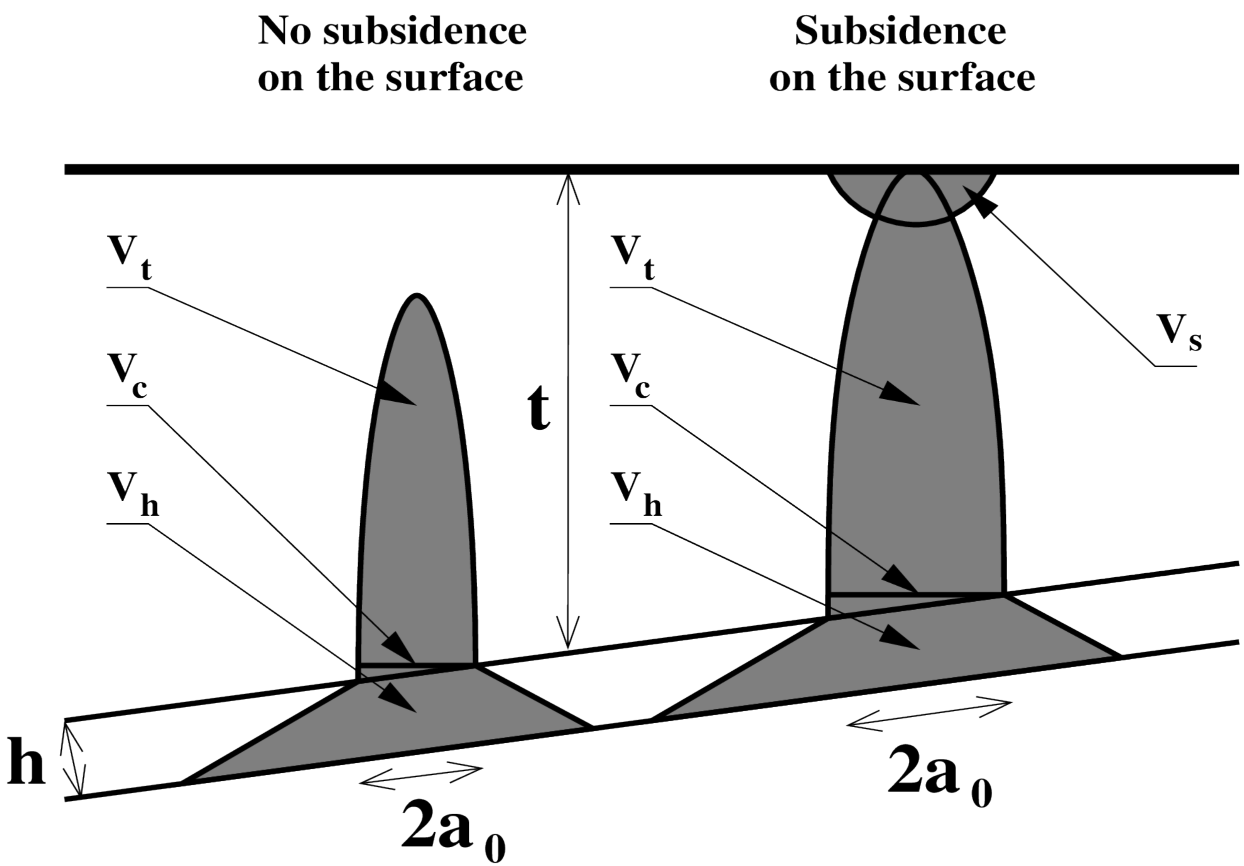
Cast3M


Random Set Theory - Result from function PA [1]
Pl(PA) [m]
Bel(PA) [m]
Cumulative probability CPB [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
110
115
120
125
130
135
140
145
150
155
160
Result from function PA [1]
PA( )


Random Set Theory - Result from function PB [1]
Pl(PB) [m]
Bel(PB) [m]
Cumulative probability CPB [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
30
33
36
39
42
45
48
51
54
57
60
Result from function PB [1]
PB()


Random Set Theory - Result from function PC [1]
Pl(PC) [m]
Bel(PC) [m]
PC()
Cumulative probability CPB [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
20
22
24
26
28
30
32
34
36
38
40
Result from function PC [1]

Applied analytical example



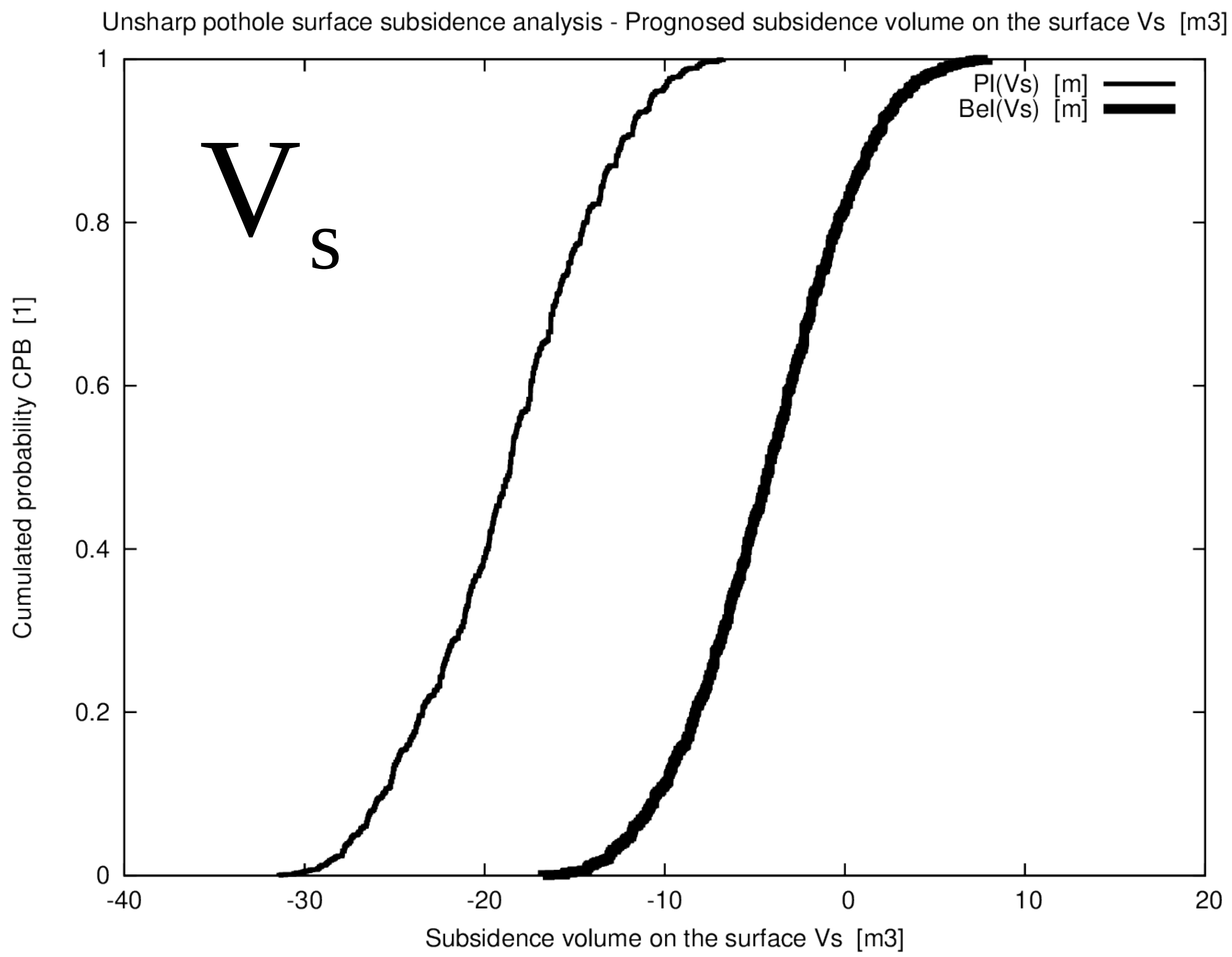
Analytical pothole subsidence analysis and prognosis with the failure mass volume balance method including six parameters for a single layer problem

$$V_s = V_s\,(h, a_0, \alpha, \varphi, s, t)$$

Number of required operations in order to solve the analytical problem with 6 parameters and 3 focal elements each:

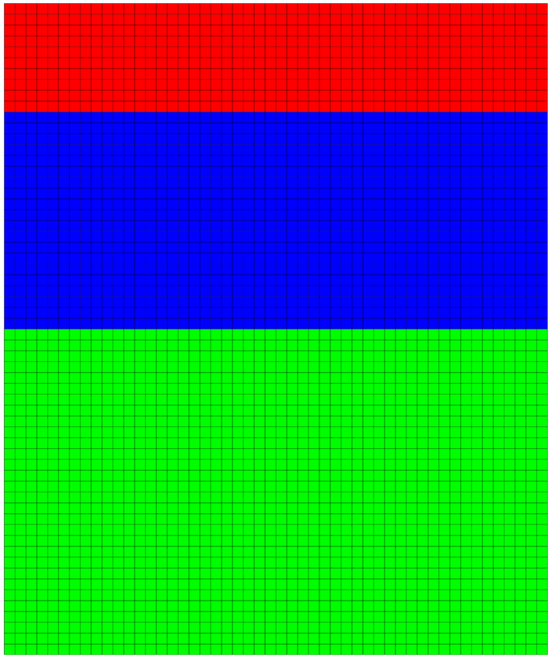
$$N = 2^n \cdot \prod_{j=1}^{n} n_j(m_i)$$

$$N = 2^6 \cdot 3^6 = \mathbf{46656}$$


Unsharp pothole surface subsidence analysis - Prognosed subsidence volume on the surface Vs [m3]
Pl(Vs) [m]
Bel(Vs) [m]
V_s
Cumulated probability CPB [1]
0
0.2
0.4
0.6
0.8
1
-40
-30
-20
-10
0
10
20
Subsidence volume on the surface Vs [m3]

Numerical example with a post-modern geotechnical design approach

Cast3M

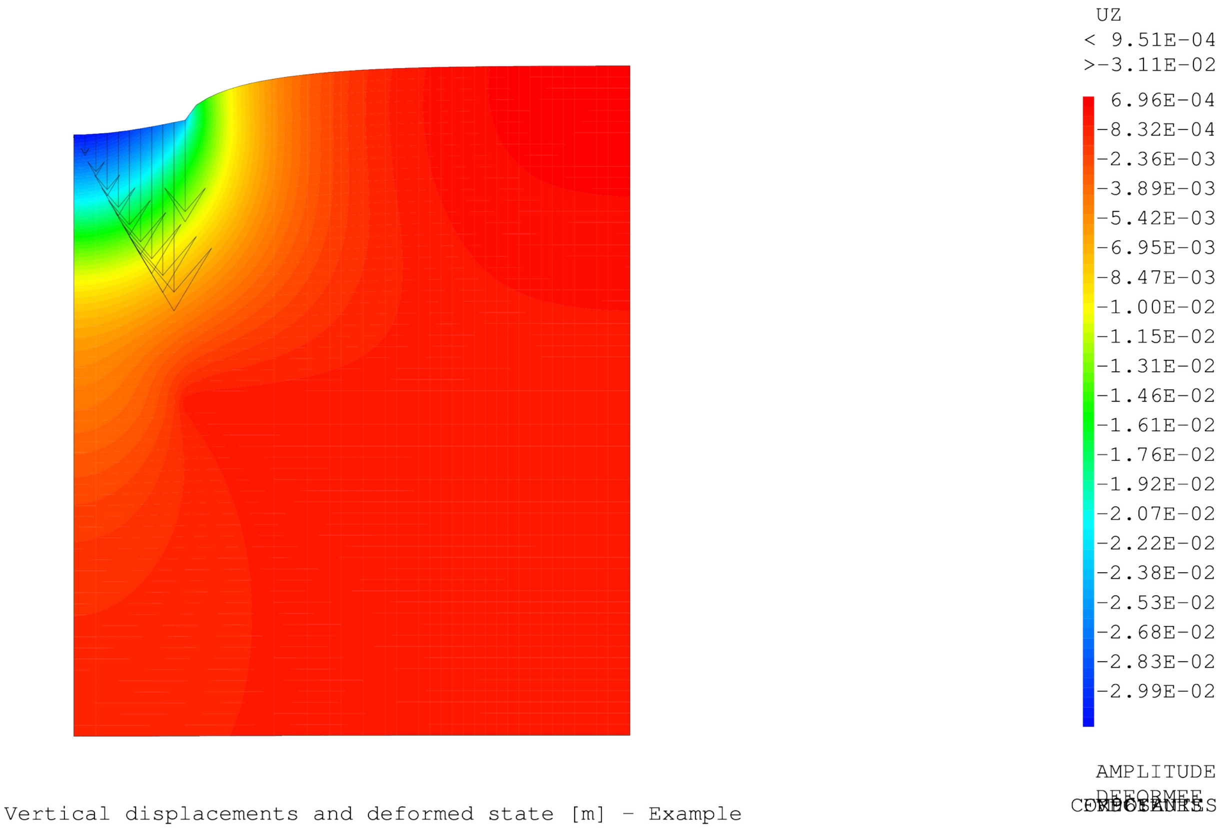


Random-set Theory - Foundation settlement analysis - Mesh

Demonstration of the Random Set Fininte Element Method (RS-FEM) on a very simple example problem (test case):

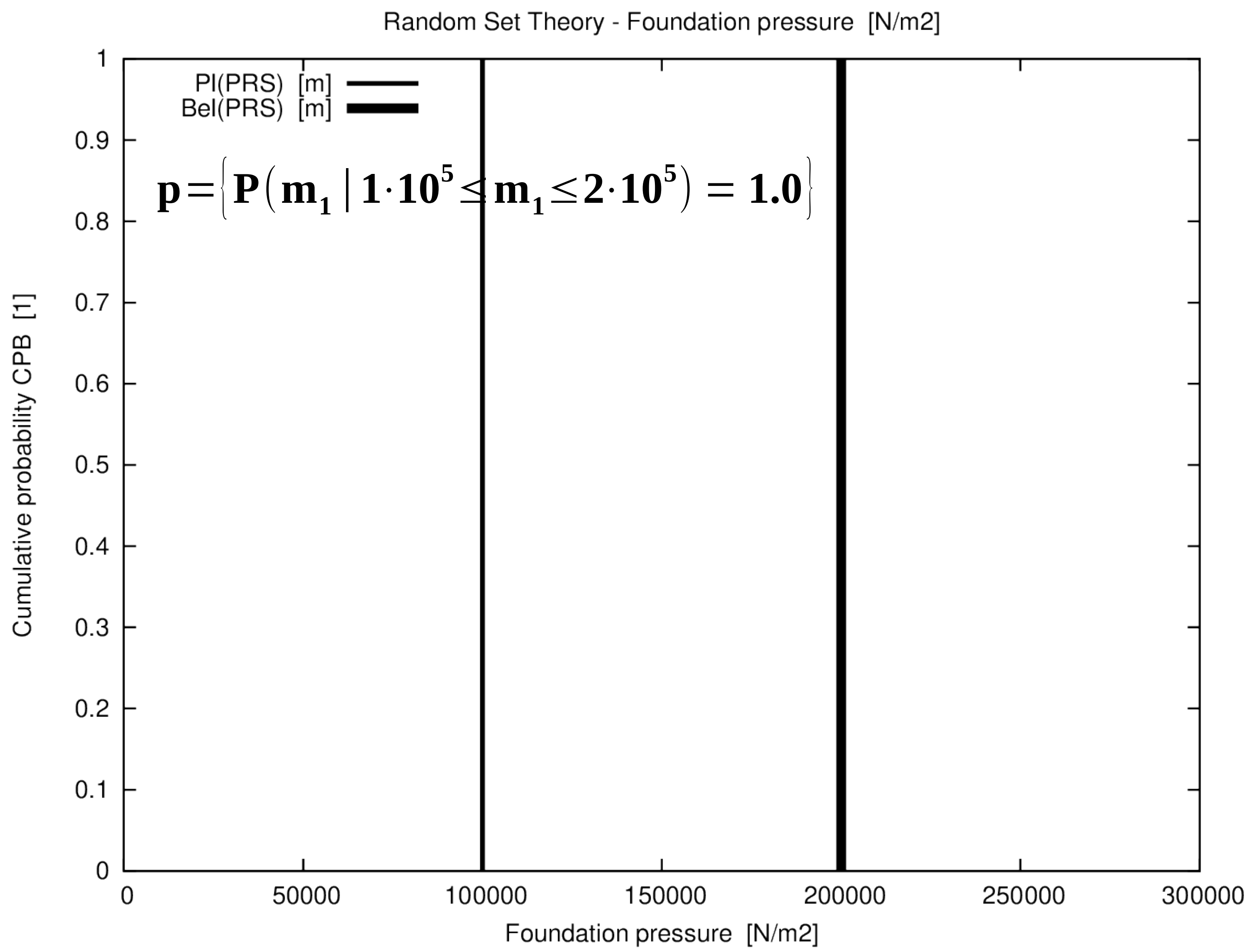
- **Axially symmetric mechanical model**
- **Stiff disk foundation on a three layer subsoil with isotropic elastic behaviour**
- **Thin upper SAND layer (index Sa)**
- **Thick middle SILT layer (index Si)**
- **Very thick lower CLAY layer (index Cl)**
- **Only deformations resulting from the surface load are considered**
- **Young modulus, Poisson ratio and the foundation load are considered to be STOCHASTIC and UNSHARP**
- **Resulting foundation settlement is also STOCHASTIC and UNSHARP**
- **Number of computations is feasible for elastic deformation process**

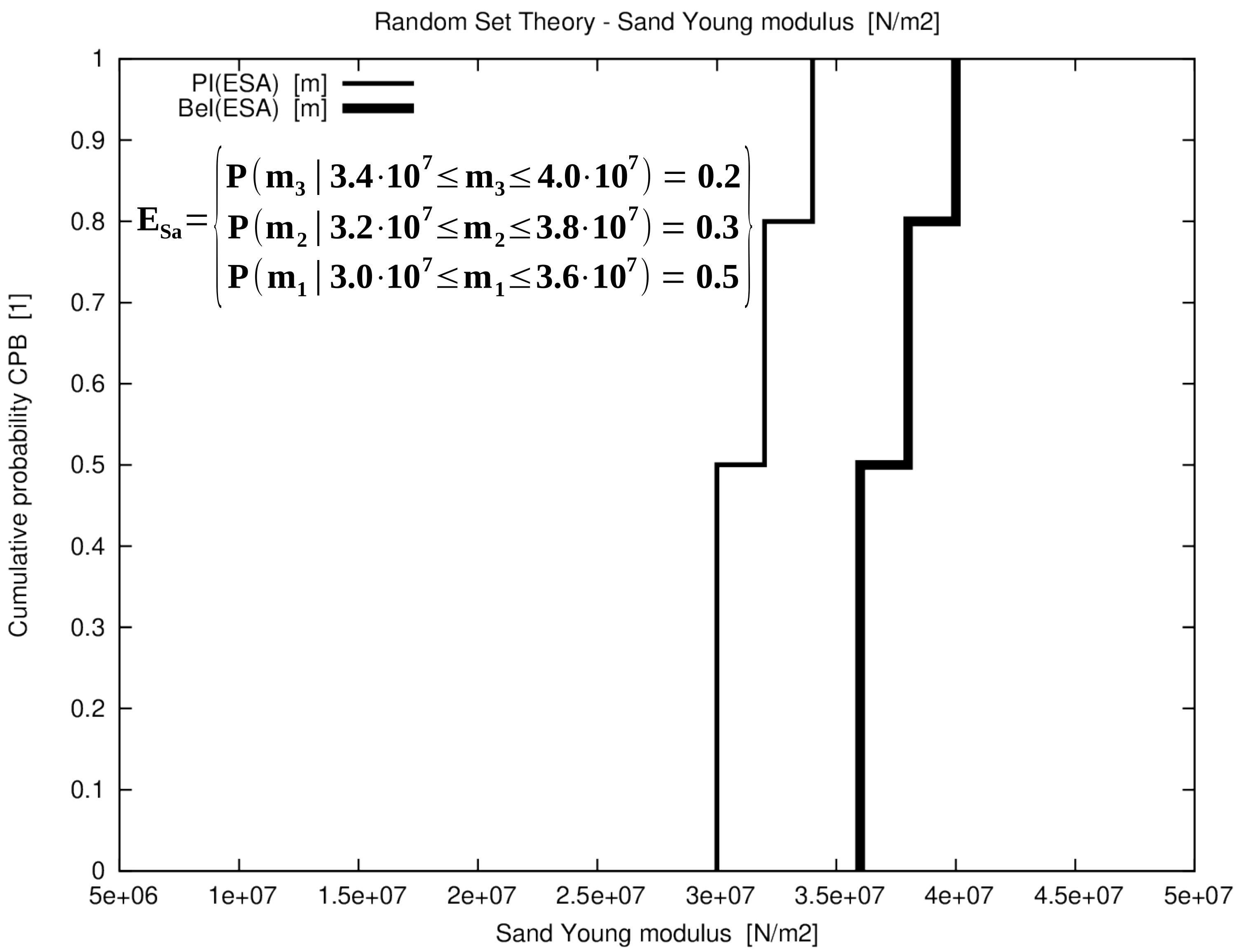
Cast3M

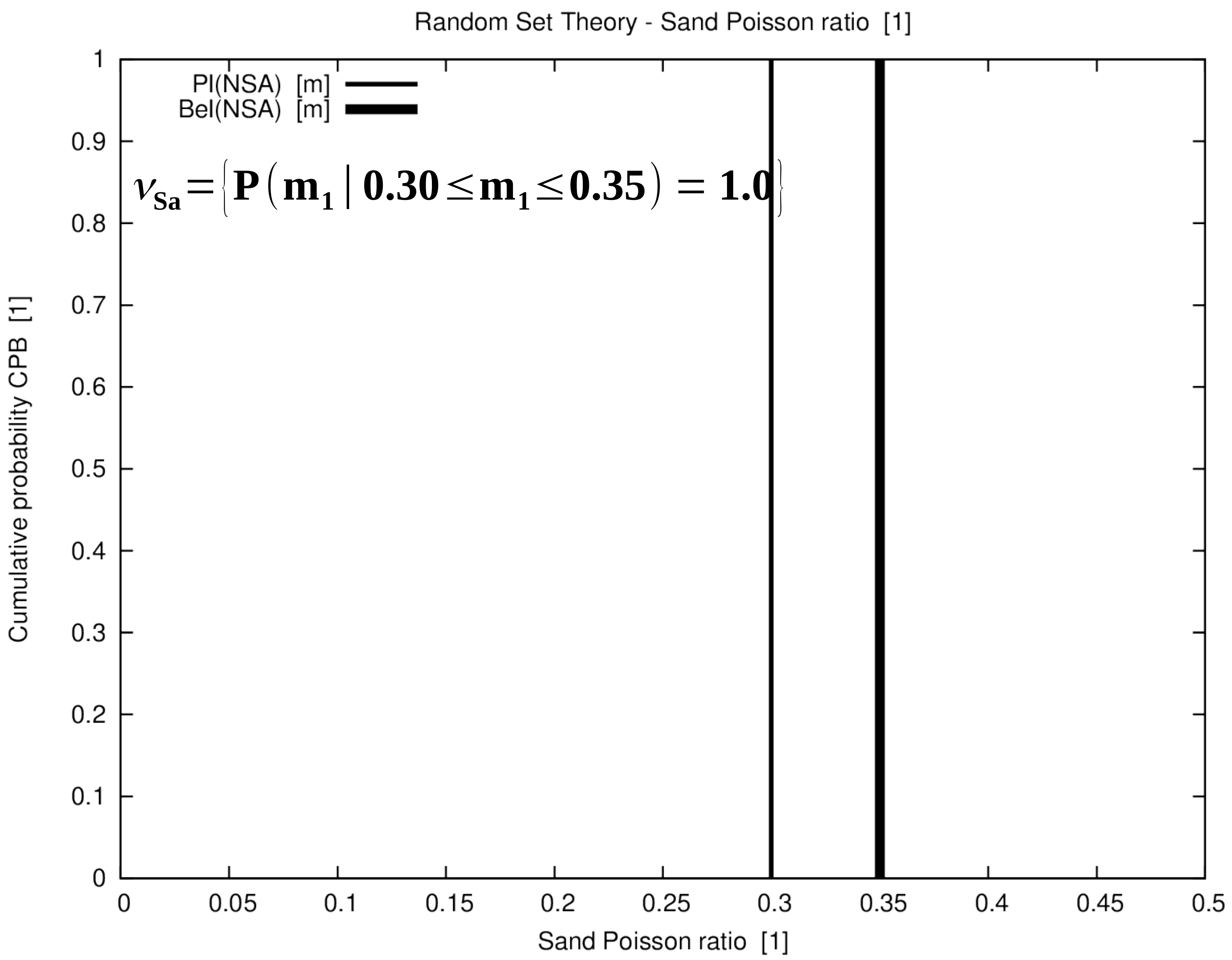


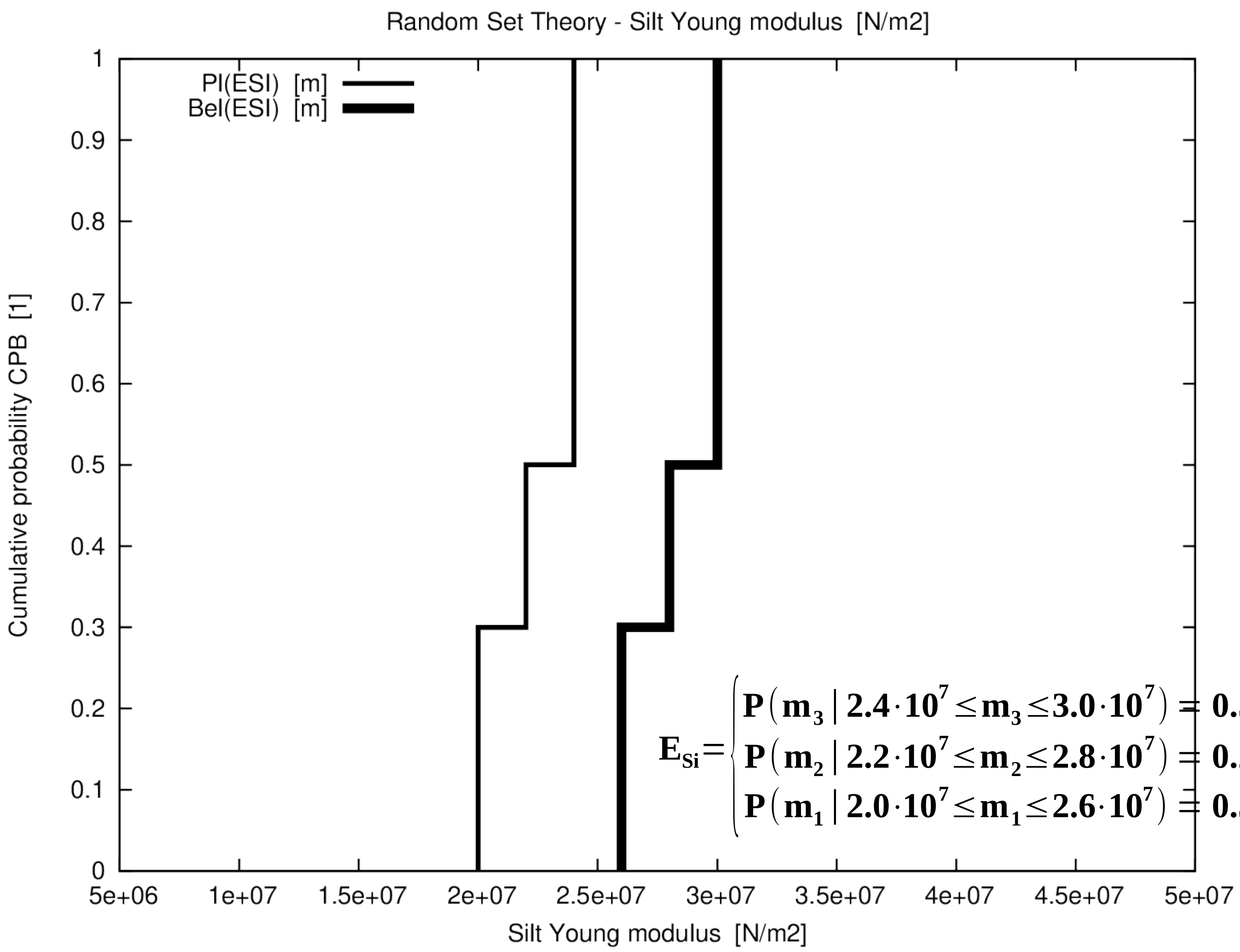
Example vertical displacement field from a single numerical analysis

Unsharp Finite Element Analysis based on Random Set Theory



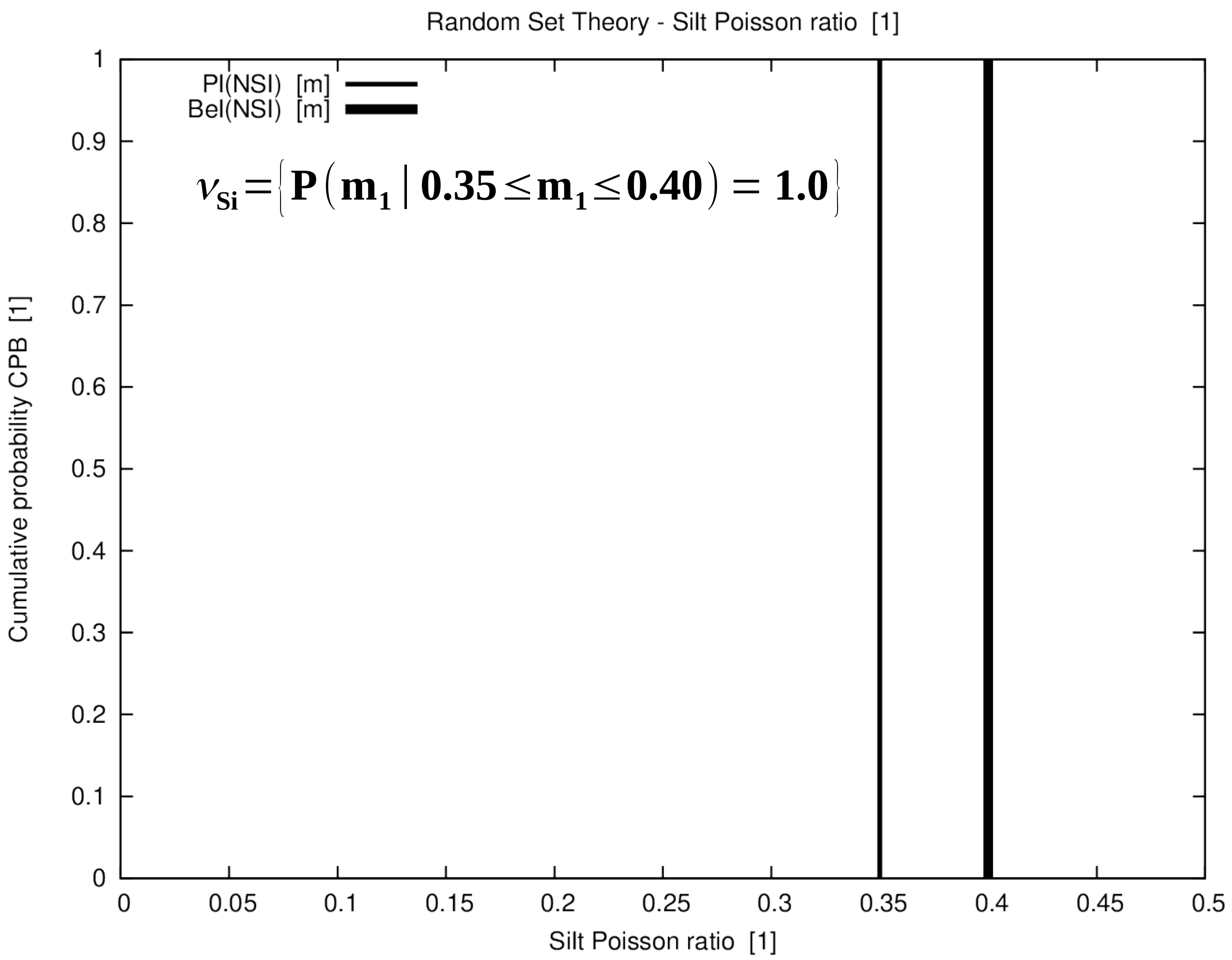

Random Set Theory - Sand Young modulus [N/m2]
Pl(ESA) [m]
Bel(ESA) [m]
$E_{Sa} = \left\{ \begin{array}{l} P(m_3 \mid 3.4\cdot10^7 \le m_3 \le 4.0\cdot10^7) = 0.2 \\ P(m_2 \mid 3.2\cdot10^7 \le m_2 \le 3.8\cdot10^7) = 0.3 \\ P(m_1 \mid 3.0\cdot10^7 \le m_1 \le 3.6\cdot10^7) = 0.5 \end{array} \right\}$
Cumulative probability CPB [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5e+06
1e+07
1.5e+07
2e+07
2.5e+07
3e+07
3.5e+07
4e+07
4.5e+07
5e+07
Sand Young modulus [N/m2]

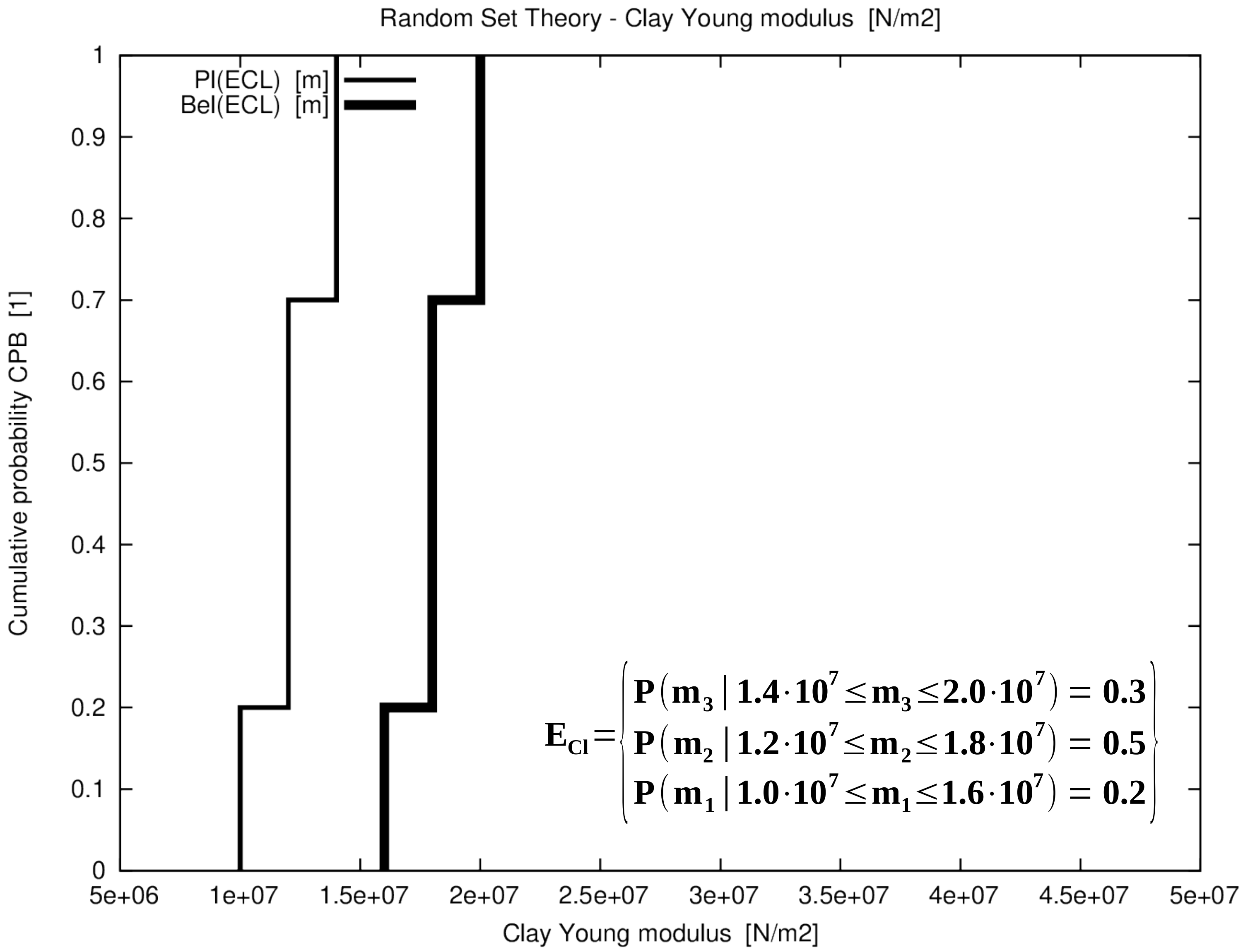

Random Set Theory - Sand Poisson ratio [1]
Pl(NSA) [m]
Bel(NSA) [m]
$\nu_{Sa} = \{ P(m_1 \mid 0.30 \leq m_1 \leq 0.35) = 1.0 \}$
Cumulative probability CPB [1]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Sand Poisson ratio [1]

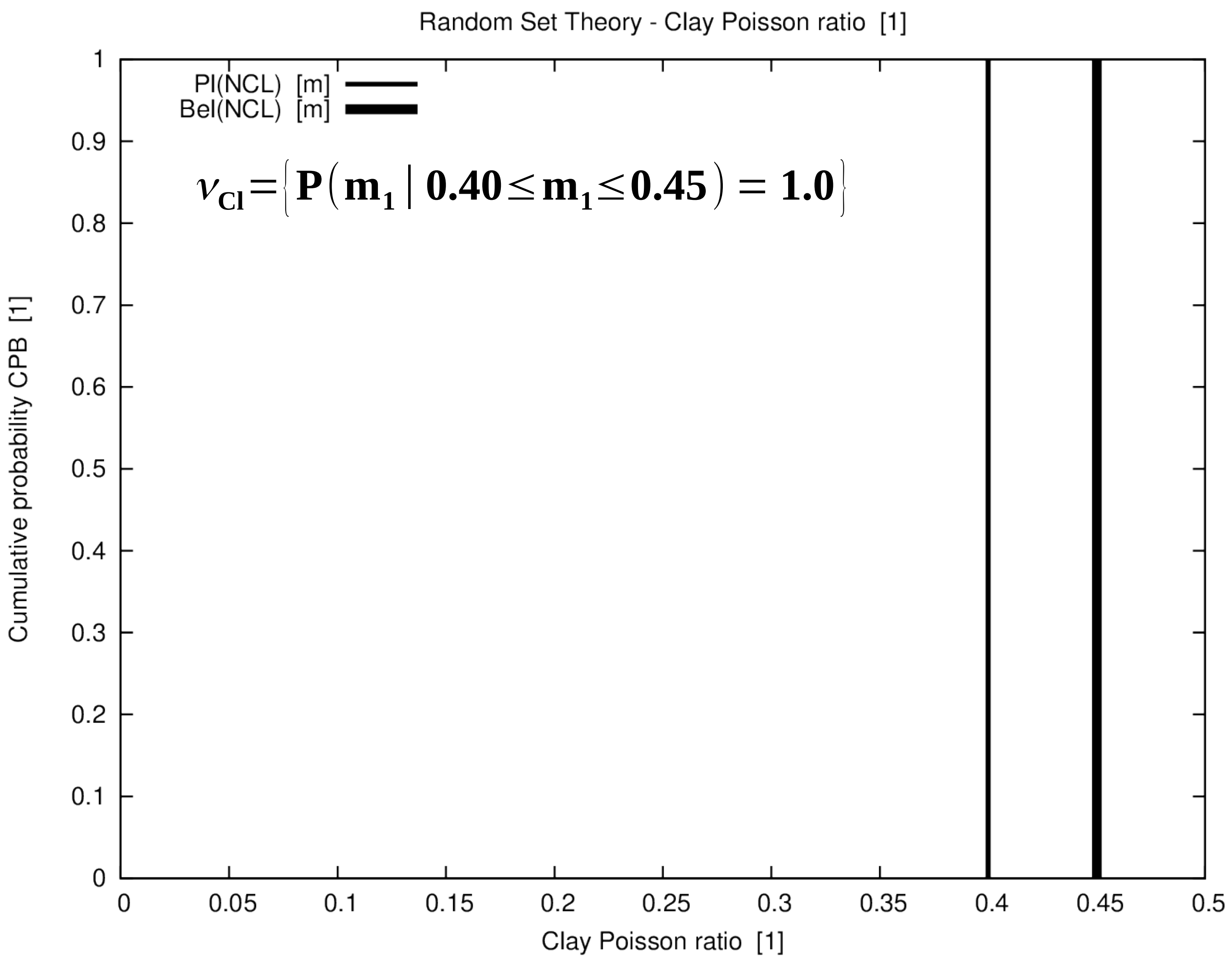


$$E_{Si} = \begin{Bmatrix} P\left(m_3 \mid 2.4\cdot10^7 \le m_3 \le 3.0\cdot10^7\right) = 0.5 \\ P\left(m_2 \mid 2.2\cdot10^7 \le m_2 \le 2.8\cdot10^7\right) = 0.2 \\ P\left(m_1 \mid 2.0\cdot10^7 \le m_1 \le 2.6\cdot10^7\right) = 0.3 \end{Bmatrix}$$

Unsharp Finite Element Analysis based on Random Set Theory




Random Set Theory - Clay Young modulus [N/m2]
Pl(ECL) [m]
Bel(ECL) [m]
Cumulative probability CPB [1]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5e+06
1e+07
1.5e+07
2e+07
2.5e+07
3e+07
3.5e+07
4e+07
4.5e+07
5e+07
Clay Young modulus [N/m2]
$$\mathbf{E_{Cl}} = \left\{ \begin{array}{l} \mathbf{P}(\mathbf{m_3} \mid \mathbf{1.4 \cdot 10^7 \leq m_3 \leq 2.0 \cdot 10^7}) = \mathbf{0.3} \\ \mathbf{P}(\mathbf{m_2} \mid \mathbf{1.2 \cdot 10^7 \leq m_2 \leq 1.8 \cdot 10^7}) = \mathbf{0.5} \\ \mathbf{P}(\mathbf{m_1} \mid \mathbf{1.0 \cdot 10^7 \leq m_1 \leq 1.6 \cdot 10^7}) = \mathbf{0.2} \end{array} \right\}$$


Random Set Theory - Clay Poisson ratio [1]
Pl(NCL) [m]
Bel(NCL) [m]
$\nu_{Cl} = \{ P(m_1 \mid 0.40 \leq m_1 \leq 0.45) = 1.0 \}$
Cumulative probability CPB [1]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Clay Poisson ratio [1]

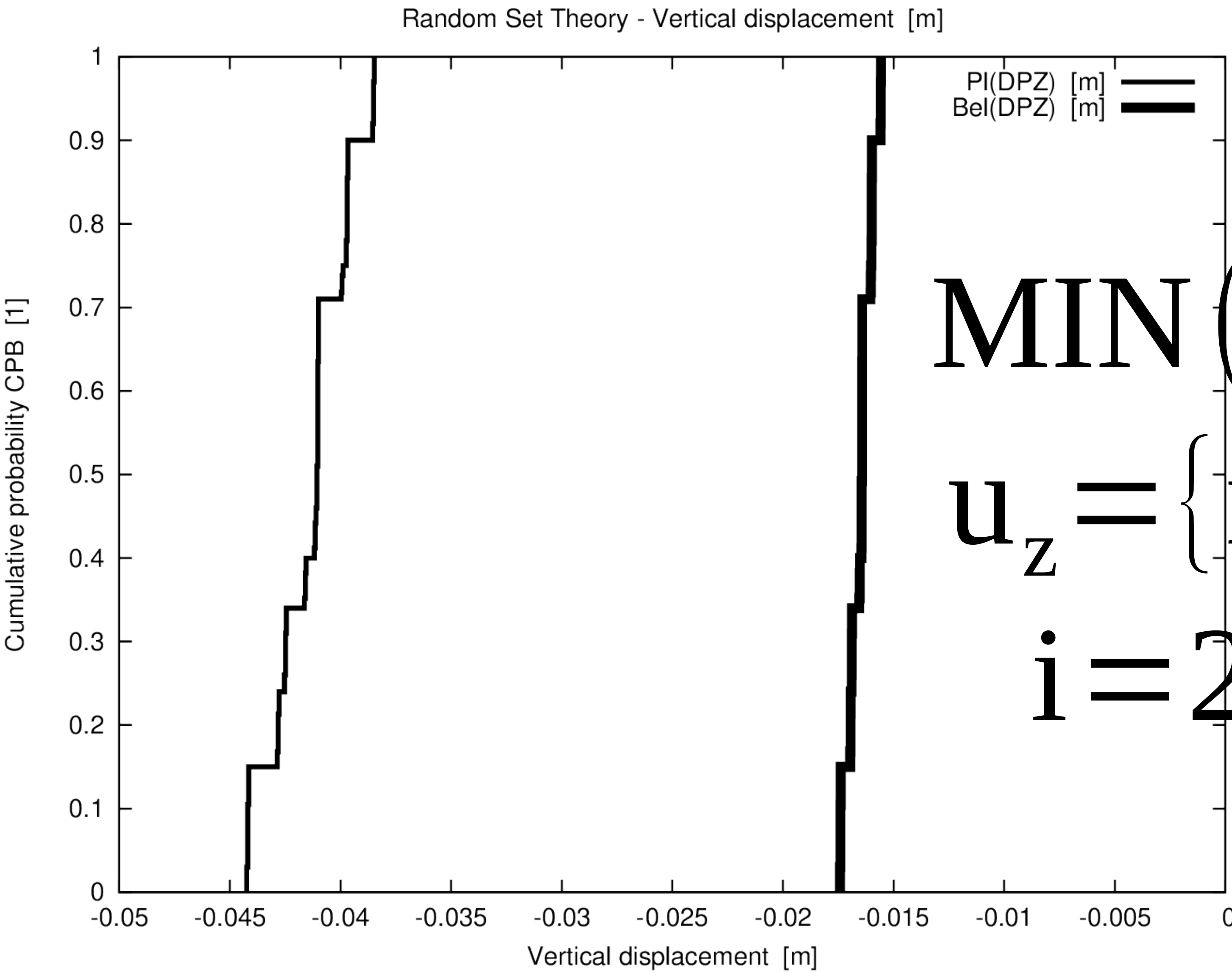
Number of required operations in order to solve the numerical problem:

$$N = 2^n \cdot \prod_{j=1}^{n} n_j(m_i)$$

$$N = 2^7 \cdot 1 \cdot 3 \cdot 1 \cdot 3 \cdot 1 \cdot 3 \cdot 1 = \mathbf{3456}$$

Unsharp Finite Element Analysis based on Random Set Theory

Cast3M



$$MIN\left(u_z\right)$$

$$u_z = \left\{m_i\right\}$$

$$i = 27$$

Summary and conclusions

Summary and conclusions:

The Random Set Theory has been implemented into GIBIANE with object orientation and is available in Cast3M

The Random Set Theory poses no restriction on the physical modelling method and its mathematical formulation

Summary and conclusions:

In the GIBIANE implementation of the Random Set Theory only ONE concurrent simulation can be run in Cast3M

In order to reduce the number of imperatively required computations, a sensitivity analysis should be combined with the application of the powerful Random Set Theory

Summary and conclusions:

The Random Set Theory analysis leads to cumulative probabilities for the calculation results that can be further interpreted with methods of stochastic analysis

In risk analysis, required failure probabilities can be derived from Random Set Theory analysis for arbitrary physical systems under consideration

Some notes and comments on object oriented programming in GIBIANE

Some notes on object oriented programming in GIBIANE:

Object variable and function names should always be protected within object methods with apostrophes: %’VAR’

Objects are based on the concept of TABLE named fields and (public) internal object variables can be readily accessed: OBJ . ’VAR’

Some notes on object oriented programming in GIBIANE:

Variables transmitted to objects are passed by reference and can be modified

Object oriented approach offers a well organized and very recommendable method of code and data management in GIBIANE procedures

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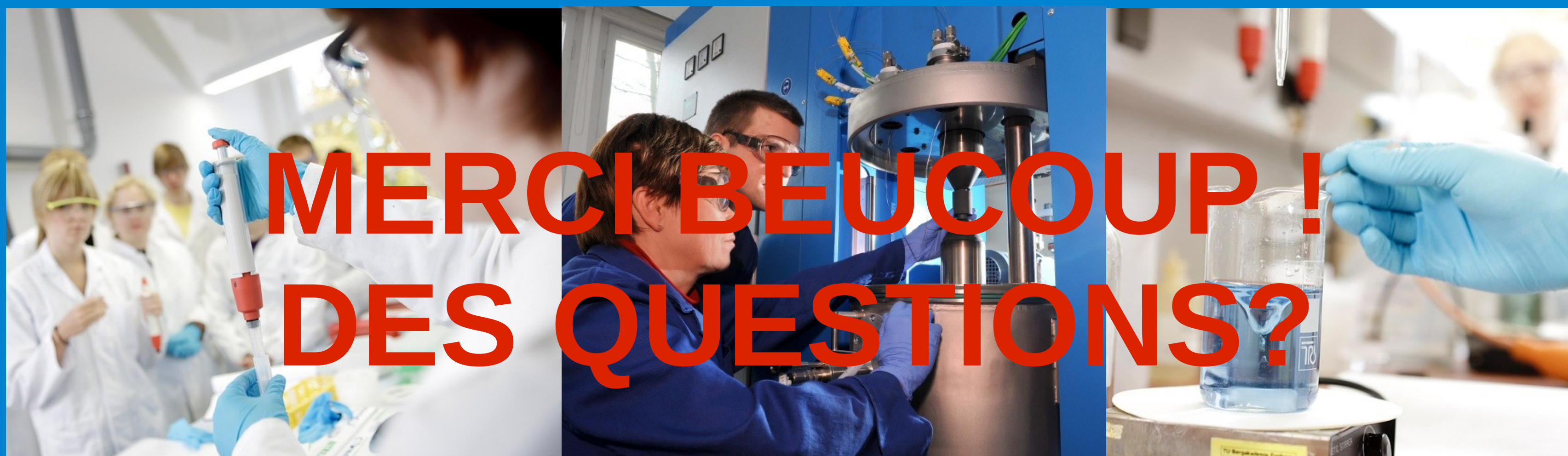
Unsharp Finite Element Analysis Based on Random Set Theory



MERCI BEUCOUP !
DES QUESTIONS?

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Chair of Soil Mechanics and Ground Engineering