



TECHNISCHE UNIVERSITÄT
BERGAKADEMIE FREIBERG

Die Ressourcenuniversität. Seit 1765.

Foundation Vibration Analysis with Simple Physical Models



Dr. N.Tamaskovics

TU Bergakademie Freiberg, Institut für Geotechnik

Chair of Soil Mechanics, Ground Engineering and Mining Geotechnics



Geotechnical design calculations

Geotechnical design calculations (EC7):

**Ultimate limit state design (ULS)
with mostly (trivial) analytic computations
(static force and momentum balance)**

**Serviceability limit state design (SLS)
with mostly analytic but increasingly
also with numerical computations
(deformation analysis)**

Geotechnical design practice:

**For individual design procedure steps
different software tools are available**

**Engineers must master and follow the
development of numerous software tools**

**A unified engineering platform comprising
numerous design computations and
analyses would be of great advantage**

Geotechnical design practice:

Implementation of analytical design methods in GIBIANE and native Cast3M procedures for ULS design

Additional numerical analyses for verification and SLS design with the Finite Element Method in Cast3M

The unification of the design concepts in one system makes Cast3M a very powerful geotechnical engineering environment



The Cone Method (Swiss theory, J.P.Wolf)

Foundation design for dynamic loading:

**Three dimensional problem
with six degrees of freedom
in time or frequency domain**

**High modelling effort for
routine engineering problems**

**Additional questions concern material
damping and absorbing boundaries**

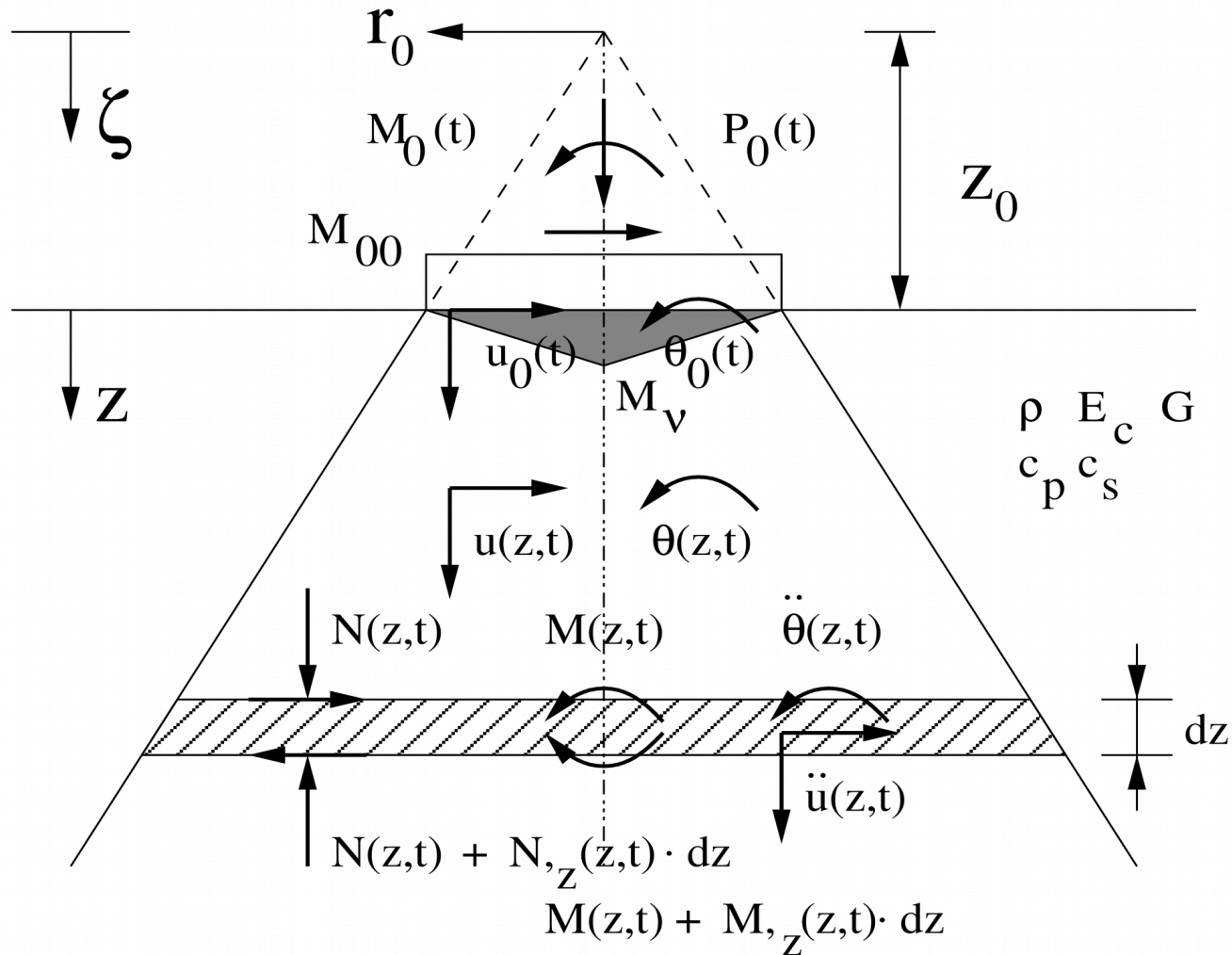
The Cone Method:

A cone with appropriate aperture represents the mechanical behaviour of the dynamically loaded half-space

Individual treatment of translational and rotational degrees of freedom

Construction induced coupling of degrees of freedom is possible

Mechanical concept of the Cone Method for homogeneous half-space



The Cone Method:

Wave equation for translation degrees of freedom in the cone frustum:

$$\left(\xi \mathbf{u} \right)_{,tt} = c_{c/s}^2 \left(\xi \mathbf{u} \right)_{,\xi\xi}$$

Wave equation for rotation degrees of freedom in the cone frustum:

$$\vartheta_{,tt} = c_{c/s}^2 \left(\vartheta_{,\xi\xi} + \frac{4}{\xi} \vartheta_{,\xi} \right)$$

The Cone Method:

Time domain analysis for translation:

$$\left(M_{0u} + \Delta M_u \right) \ddot{u}_0 + C_u \dot{u}_0 + K_u u_0 = P_0(t)$$

Time domain analysis for rotation:

$$\left(M_{0\vartheta} + \Delta M_\vartheta \right) \ddot{\vartheta}_0 + C_\vartheta \left(\dot{\vartheta}_0 - \dot{\vartheta}_1 \right) + K_\vartheta \vartheta_0 = M_0(t)$$

$$M_{\vartheta 1} \left(\dot{\vartheta}_0 - \dot{\vartheta}_1 \right) + C_{\vartheta 1} \vartheta_1 = 0$$

The Cone Method:

Frequency domain analysis for translation:

$$S_u(\omega) \cdot u(\omega) = P_0(\omega)$$

$$S_u(\omega) = K'_u - M_{u0}\omega^2 + iC'_u\omega$$

Frequency domain analysis for rotation:

$$S_\vartheta(\omega) \cdot \vartheta(\omega) = M_0(\omega)$$

$$S_\vartheta(\omega) = K'_\vartheta - M_{\vartheta0}\omega^2 + iC'_\vartheta\omega$$

The Cone Method:

Frequency domain analysis for translation:

$$K'_u = \frac{Q c_{c/s}^2 A_0}{Z_0} = K_u$$

$$C'_u = Q c_{c/s} A_0 = C_u$$

$$\Delta M_u = 2.4 \pi \left(\nu - \frac{1}{3} \right) Q r_0^3 \geq 0$$

The Cone Method:

Fundamental equation for cone height results from elementary solutions of elasticity theory for stiff disk foundations:

$$\frac{Z_0}{r_0} = \frac{(1 - \nu) \pi}{4} \left(\frac{C_c}{C_s} \right)^2$$

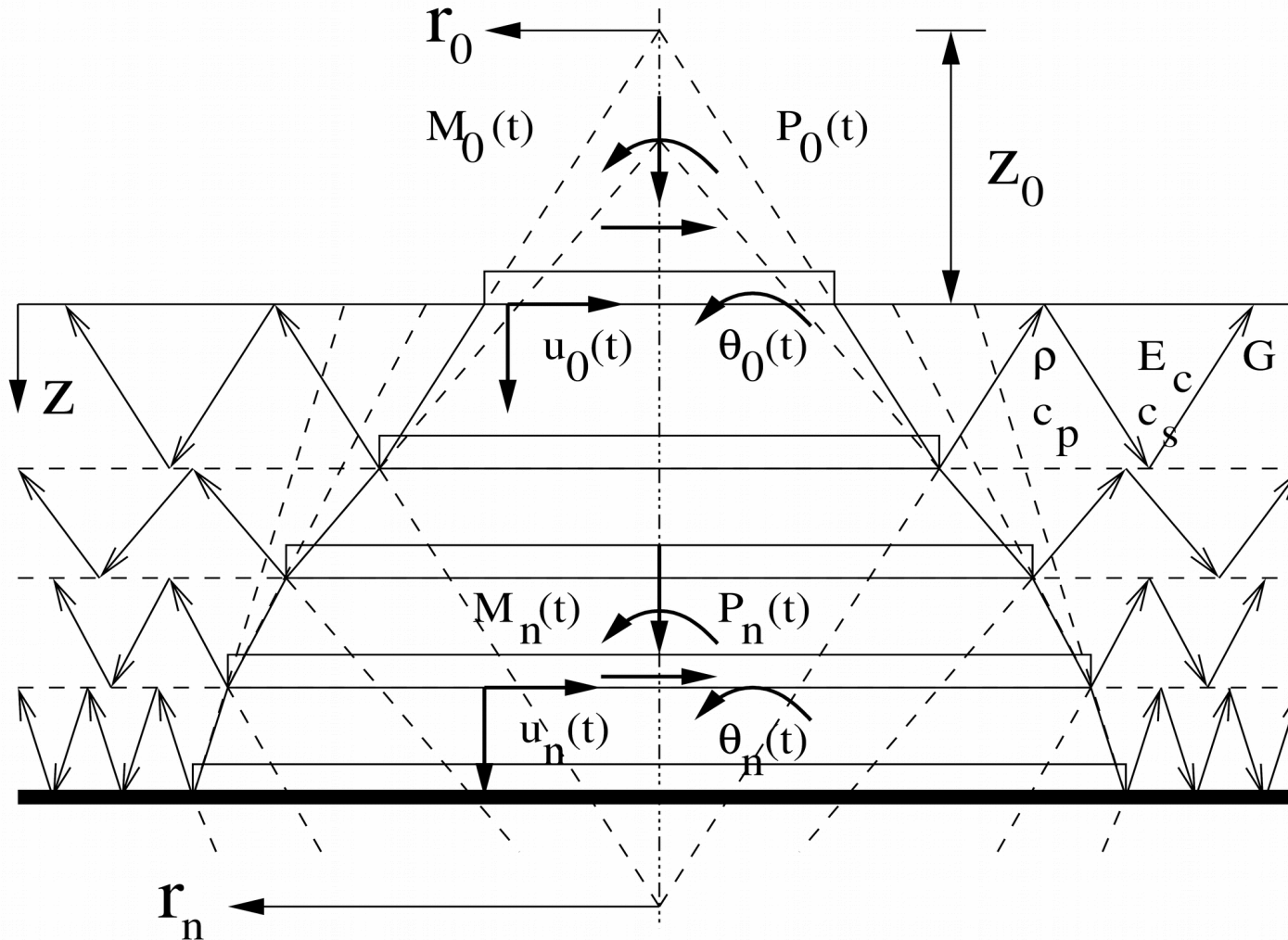
The Cone Method:

Double asymptotic theory converting into rigorous solutions at vanishing and at infinite frequencies

It has been successfully extended for horizontally layered half-space, embedded foundations and pile foundations

The Cone Method delivers results with reliable accuracy for engineering design

Mechanical concept of the Cone Method for half-space with horizontal layering





The GIBIANE Complex Procedure Collection

The Complex Procedure Collection:

@COMPLEX.procedure
@COMPLEX.notice

**Comprehensive GIBIANE library
implementing the most important analysis
operations for complex numbers**

**Inclusion into the 'main branch' of Cast3M
after final validation and verification**

The Complex Procedure Collection:

**Complex procedures have
a distinguishable name: @Cxxx**

'@' symbol marks external contribution

**Procedures treat all internal variables
as local (decorated with '!' symbol)**

**Procedures strictly avoid internal
generation and passing of references**

The Complex Procedure Collection:

Complex numbers are simply represented with TABLE objects

'RE' table entry for real part (R.'RE')

'IM' table entry for imaginary part (R.'IM')

The GIBIANE implementation could be significantly improved with native Cast3M procedures in ESOPE

The Complex Procedure Collection:

Usage example:

```
A=TABLE;      B=TABLE;      R=TABLE;  
A.'RE'=1.0;   B.'RE'=3.0;   R.'RE'=0.0;  
A.'IM'=2.0;   B.'IM'=4.0;   R.'IM'=0.0;
```

```
Z = @CADD R A B ;
```

```
R.'RE'=4.0;  R.'IM'=6.0;
```

Z on the left side has the reference of R (!)

The Complex Procedure Collection:

Basic operators:

**@CABS , @CARG , @CNIL ,
@CEQT , @CCJG , @CNGT ,
@CCRT , @CPLR , @CCMP**

Elemental mathematics:

@CADD , @CSUB , @CMUL , @CDIV

The Complex Procedure Collection:

Advanced mathematics:

**@CEXP , @CLOG , @CPOW ,
@CSQR , @CCBR , @CRTN**

Trigonometric and hyperbolic operators:

**@CSIN , @CCOS , @CTAN ,
@CASN , @CACCS , @CATN ,
@CSNH , @CCSH , @CTNH ,
@CASH , @CACH , @CATH**

The Complex Procedure Collection:

**Implementation is based on
open-source code examples from
Perl, Javascript and Node.js**

**Verification and validation is based on
systematic test routines in Python
(cmath package)**

Analysis of a foundation under vertical dynamic loading in frequency domain

The Cone Method:

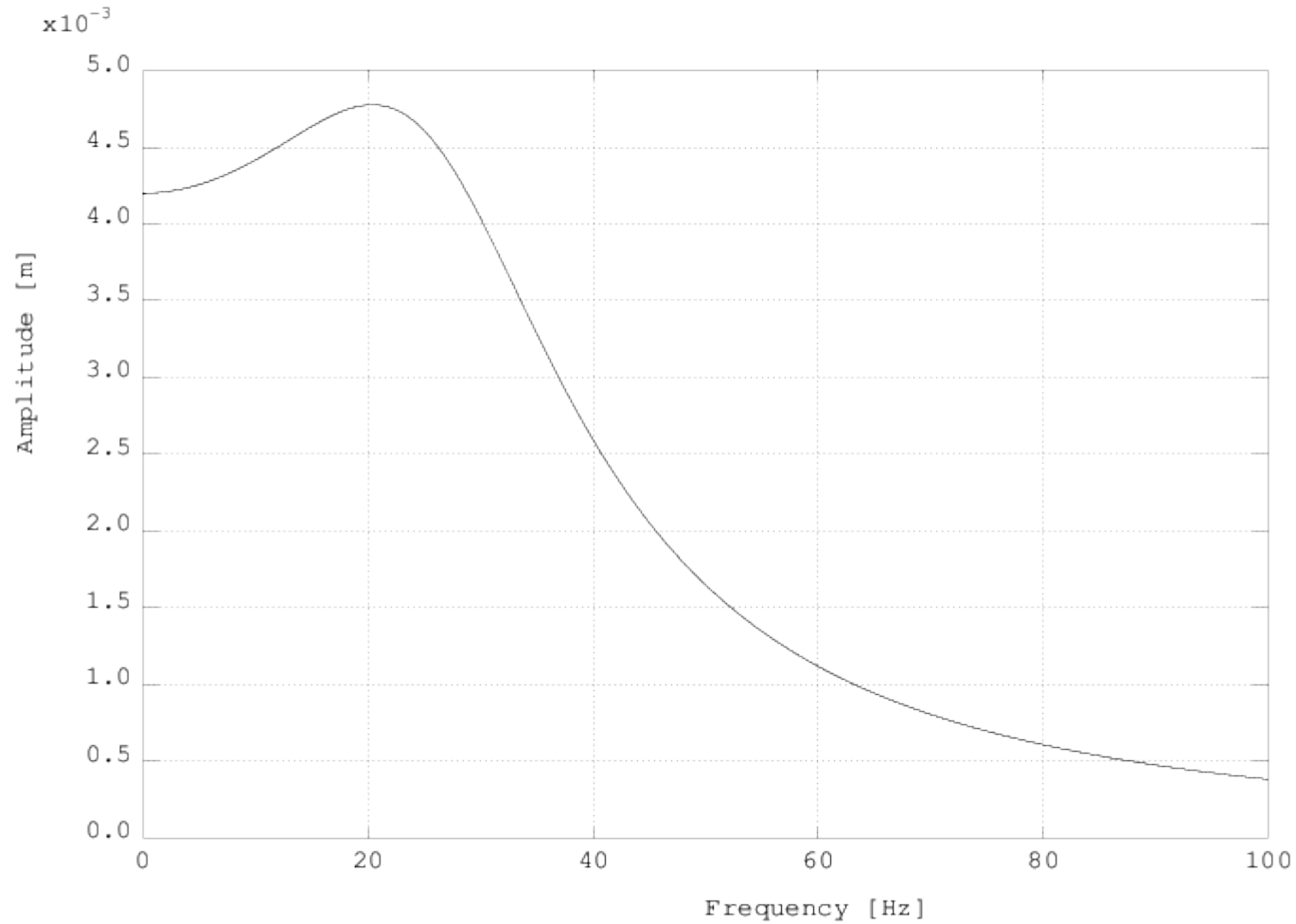
Frequency domain analysis for translation:

$$r_0 = 1.0 \text{ [m]}, \quad m = 6.0e3 \text{ [kg]}, \quad \Delta M_u = 923.6 \text{ [kg]}$$
$$E = 1.0e8 \text{ [N/m}^2\text{]}, \quad \nu = 0.4 \text{ [1]}, \quad \rho = 1750.0 \text{ [kg]}$$

$$P(\omega) = 1.0e6 + 0i \text{ [N]}$$

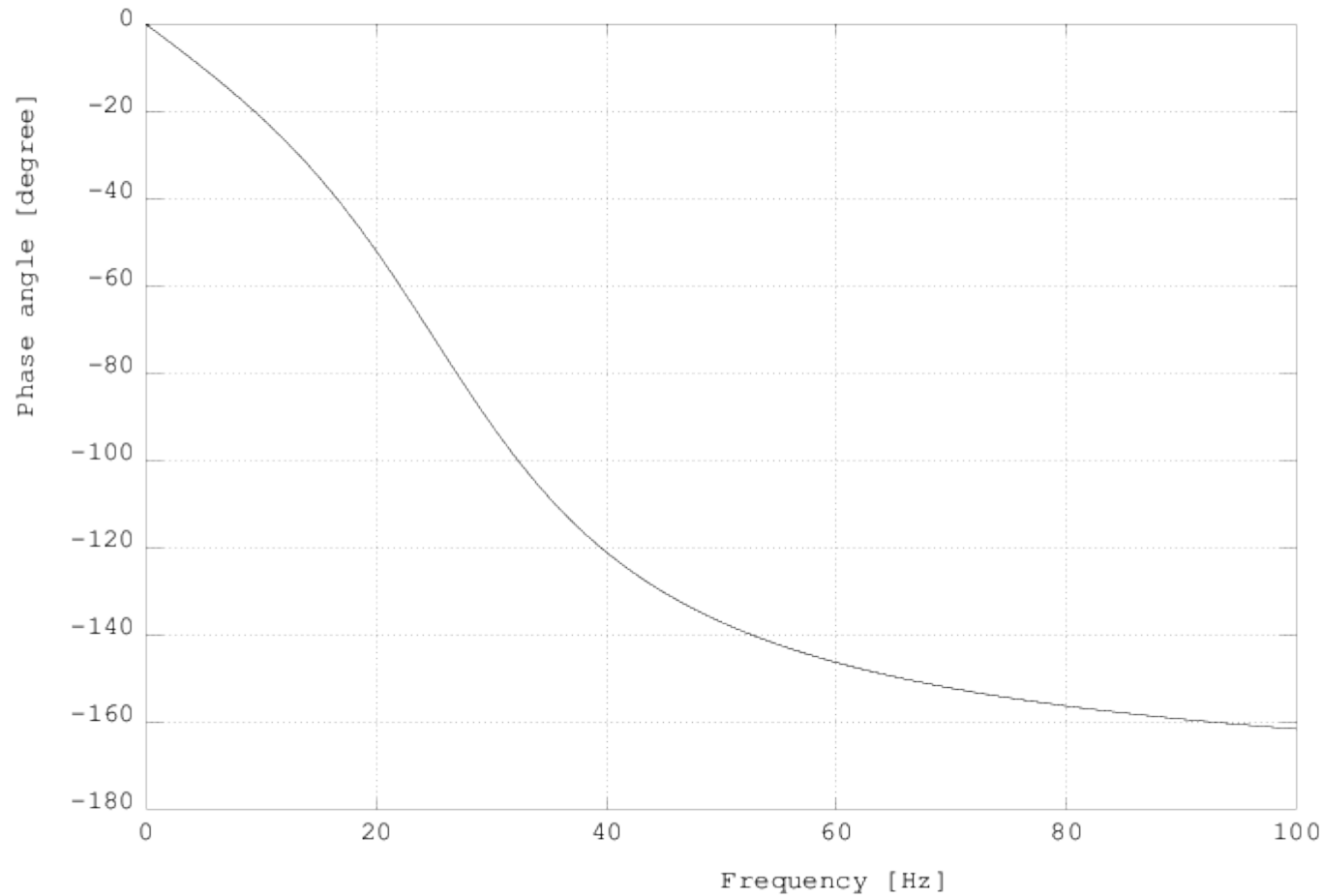
$$u(\omega) = S^{-1}(\omega) \cdot P(\omega)$$

The Cone Method - Amplitude:



The Cone Method

The Cone Method – Phase angle:



The Cone Method

Conceptional Case Study (qualitative analysis)

Vibrations of a tall viaduct bridge on non-flexible foundations

Non-flexible foundations Vertical displacements under self-weight

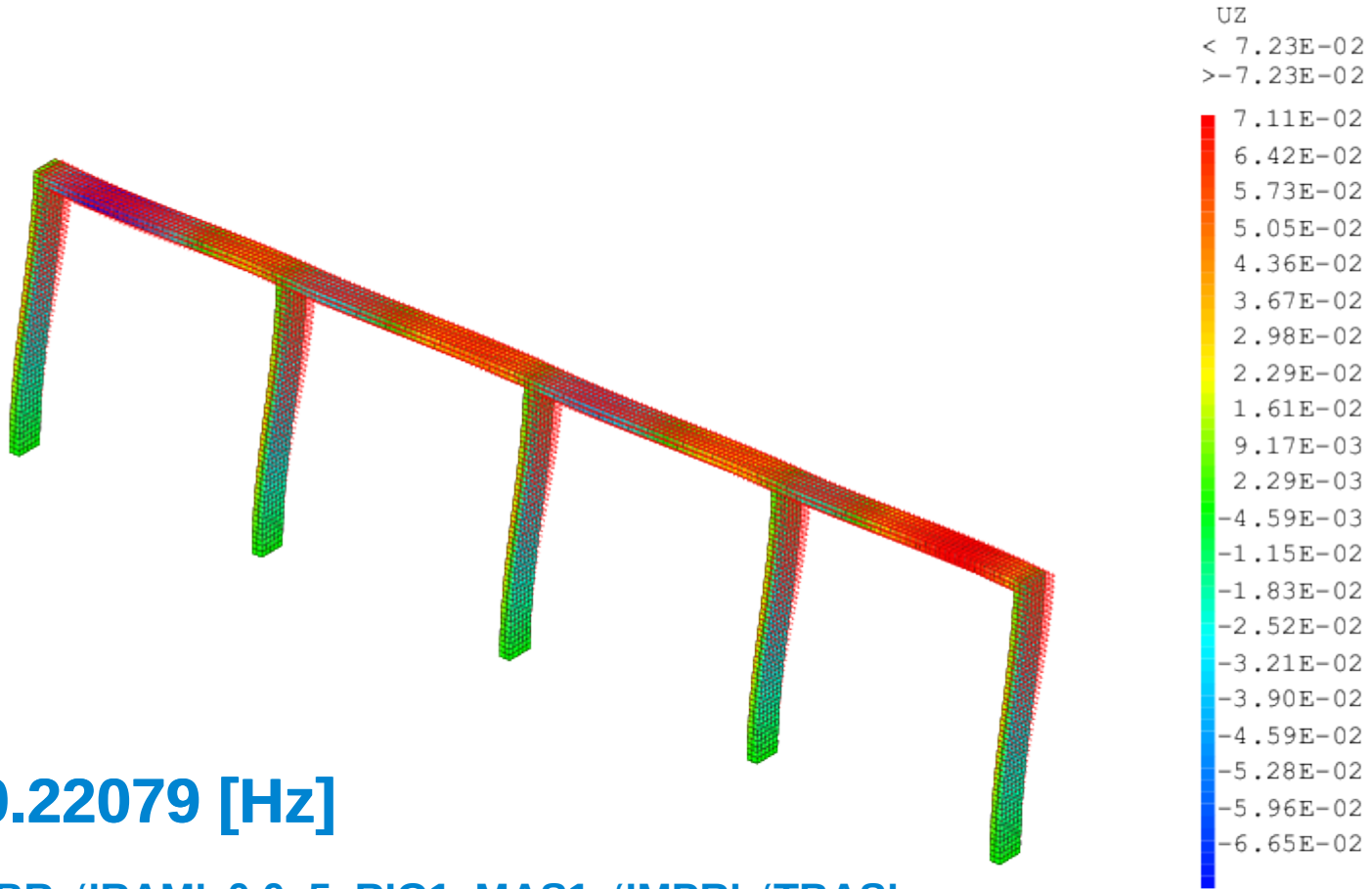


RES1 = RESO RIG1 LICHP1 ;

GIBI FECIT

AMPLITUDE
 DEFORMEE
 COORDONATES

Non-flexible foundations First eigenfrequency and eigenmode



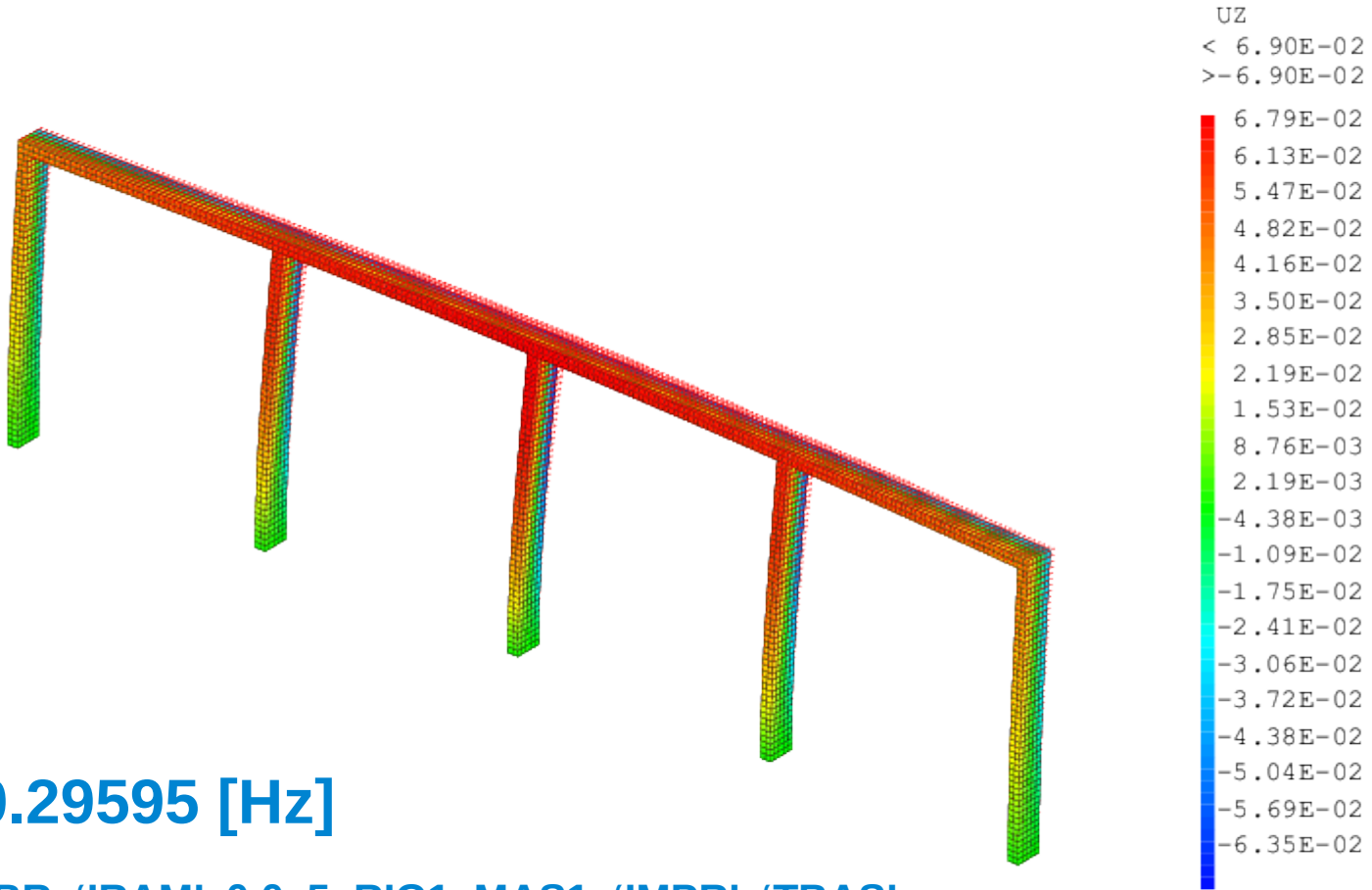
$F_{n,1} = 0.22079$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
1	1	2.20790E-01 HZ

AMPLITUDE
DEFORMEE
COMBUSTION

Non-flexible foundations Second eigenfrequency and eigenmode



$F_{n,2} = 0.29595$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	2	2.95954E-01 HZ

AMPLITUDE
DEFORMEE
COORDONATE 8

Non-flexible foundations Third eigenfrequency and eigenmode



$F_{n,3} = 0.35570$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	3	3.55700E-01 HZ

AMPLITUDE
 DEFORMEE
 COORDONATES

Non-flexible foundations Fourth eigenfrequency and eigenmode



$F_{n,4} = 0.47951$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	4	4.79506E-01 HZ

AMPLITUDE
DEFORMEE
COMBUSTION

Non-flexible foundations Fifth eigenfrequency and eigenmode



$F_{n,5} = 0.74556$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	5	7.45556E-01 HZ

AMPLITUDE
DEFORMEE
COMBINAISON

Conceptional Case Study (qualitative analysis)

Vibrations of a tall viaduct bridge on flexible foundations (low stiffness in the middle)

Non-flexible foundations First eigenfrequency and eigenmode



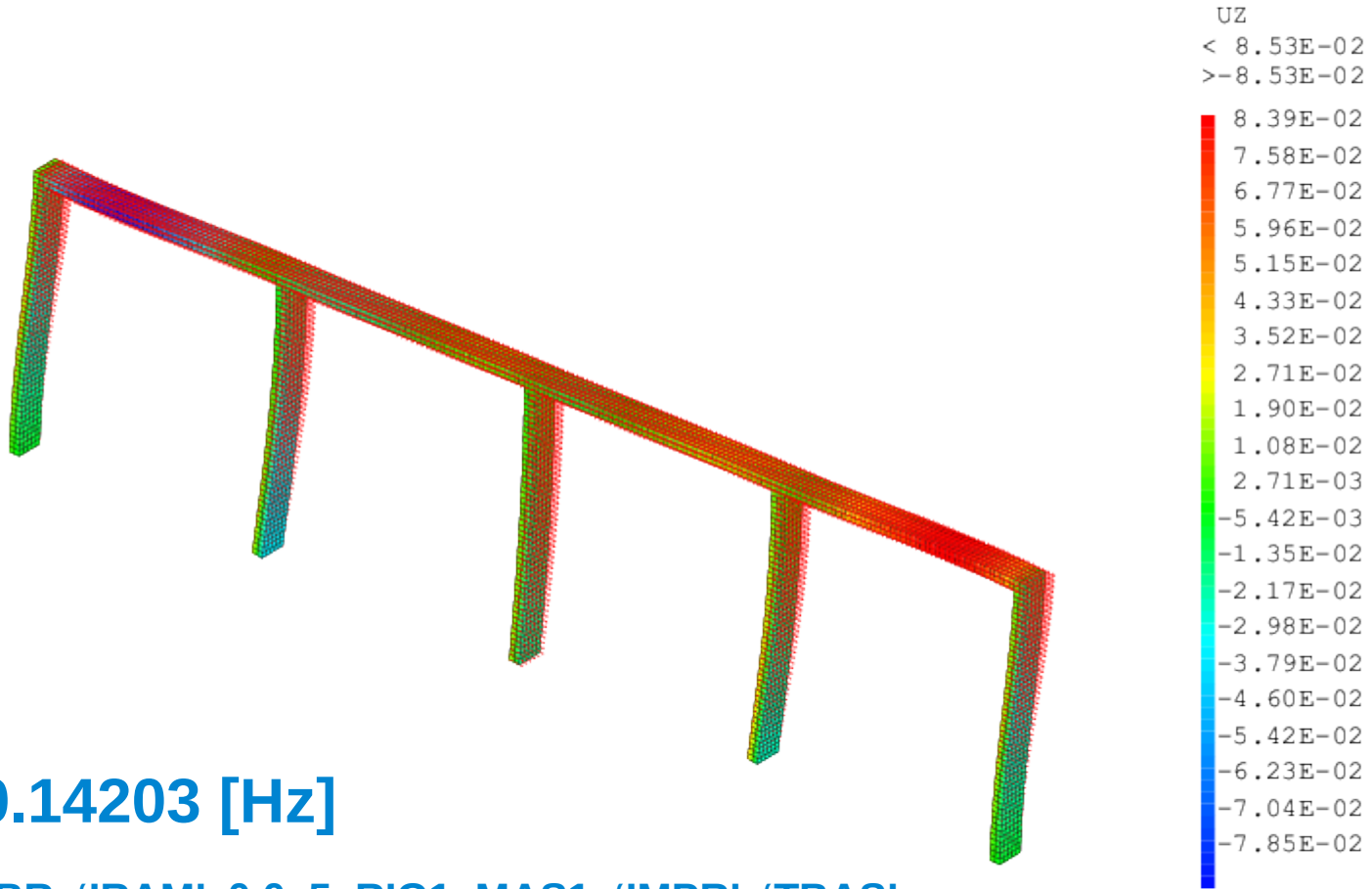
$F_{n,1} = 0.11115$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
1	1	1.11153E-01 HZ

AMPLITUDE
DEFORMEE
COMBINAISON

Non-flexible foundations Second eigenfrequency and eigenmode



$F_{n,2} = 0.14203$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	2	1.42029E-01 HZ

AMPLITUDE
DEFORMEE
COMBINAISON

Non-flexible foundations Third eigenfrequency and eigenmode



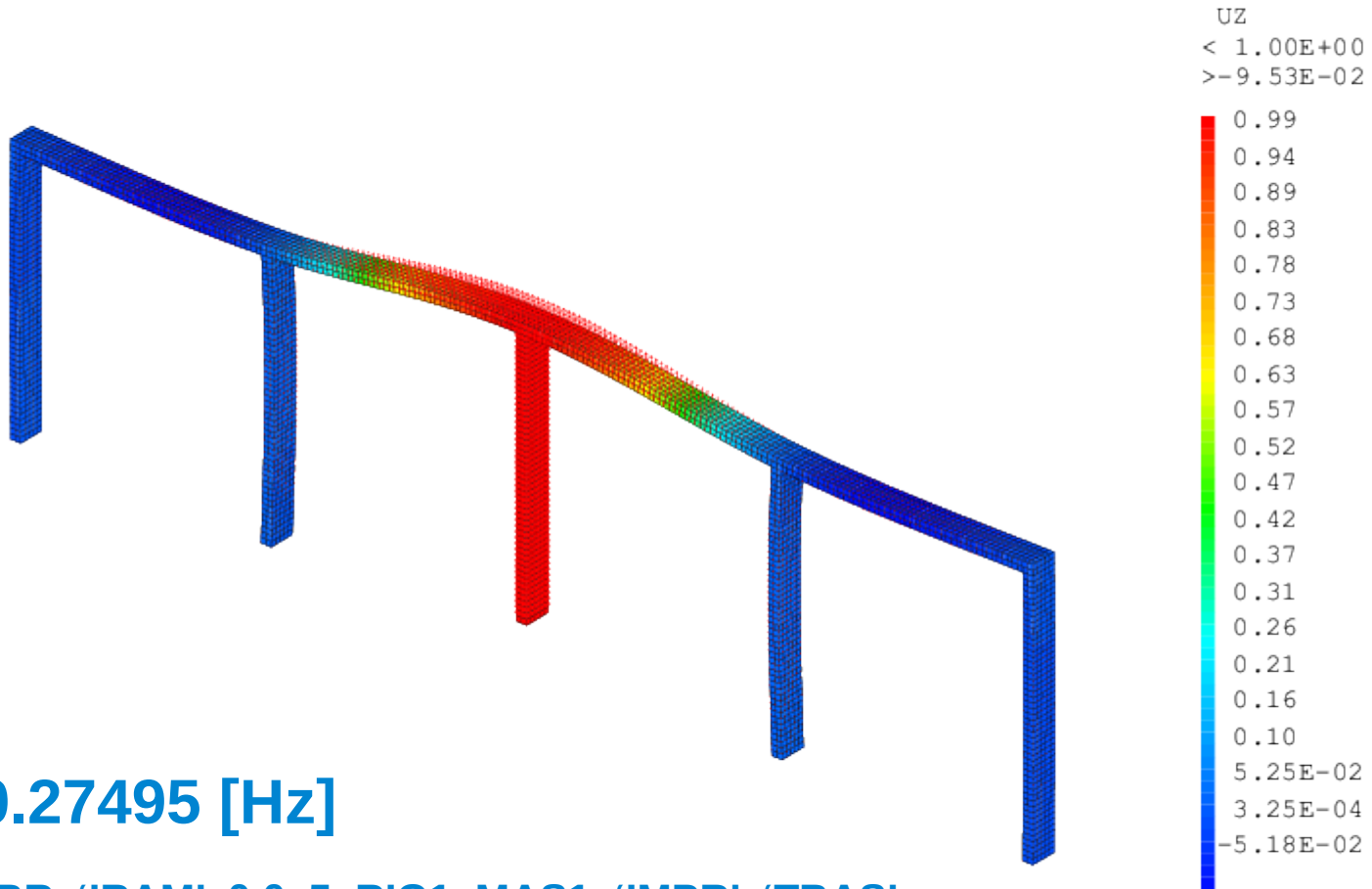
$F_{n,3} = 0.25745$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCE
	3	2.57449E-01 HZ

AMPLITUDE
DEFORMEE
COORDONATE 3

Non-flexible foundations Fourth eigenfrequency and eigenmode



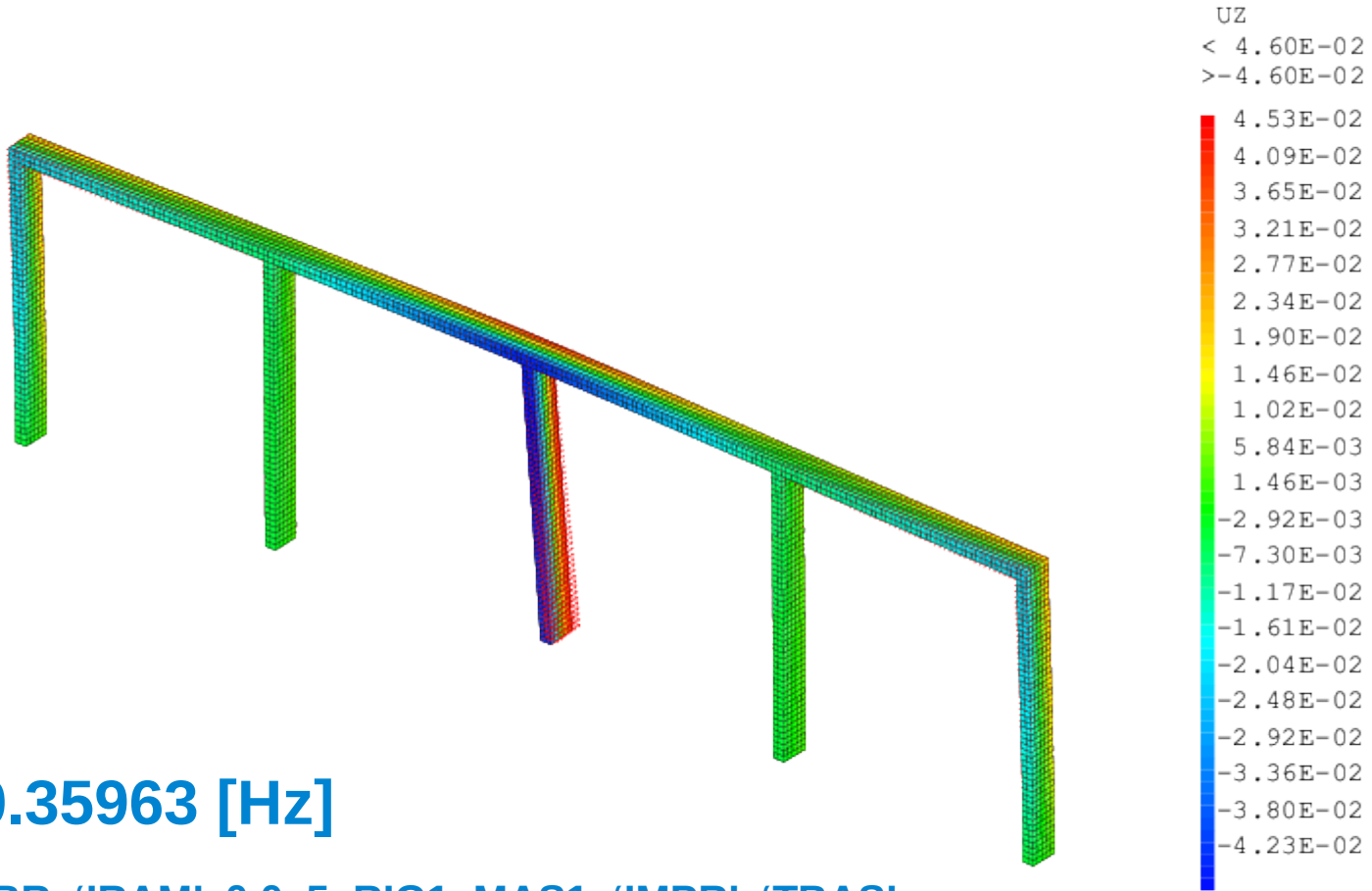
$F_{n,4} = 0.27495$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	4	2.74948E-01 HZ

AMPLITUDE
DEFORMEE
COMBUSTION

Non-flexible foundations Fifth eigenfrequency and eigenmode



$F_{n,5} = 0.35963$ [Hz]

SOL1 = VIBR 'IRAM' 0.0 5 RIG1 MAS1 'IMPR' 'TBAS';

MODE	NUMERO	FREQUENCY
	5	3.59628E-01 HZ

AMPLITUDE
 DEFORMEE
 COORDONATES

Foundation design for dynamic loading:

**In the case of long bridges,
the numerical simulation of the
subsoil behaviour can represent a
very high effort (huge geometry)**

**The Cone Method can deliver
reliable and validated foundation
stiffness and damping properties for
time and frequency domain analysis**



Summary and conclusions

Summary and conclusions:

Cast3M offers the unique opportunity for geotechnical design in ultimate (ULS) and serviceability (SLS) limit state in one engineering environment

An extensive complex analysis procedure library has been implemented primarily for frequency domain analysis

Summary and conclusions:

The Cone Method is a very useful practical engineering method for the design of foundations under dynamic loading

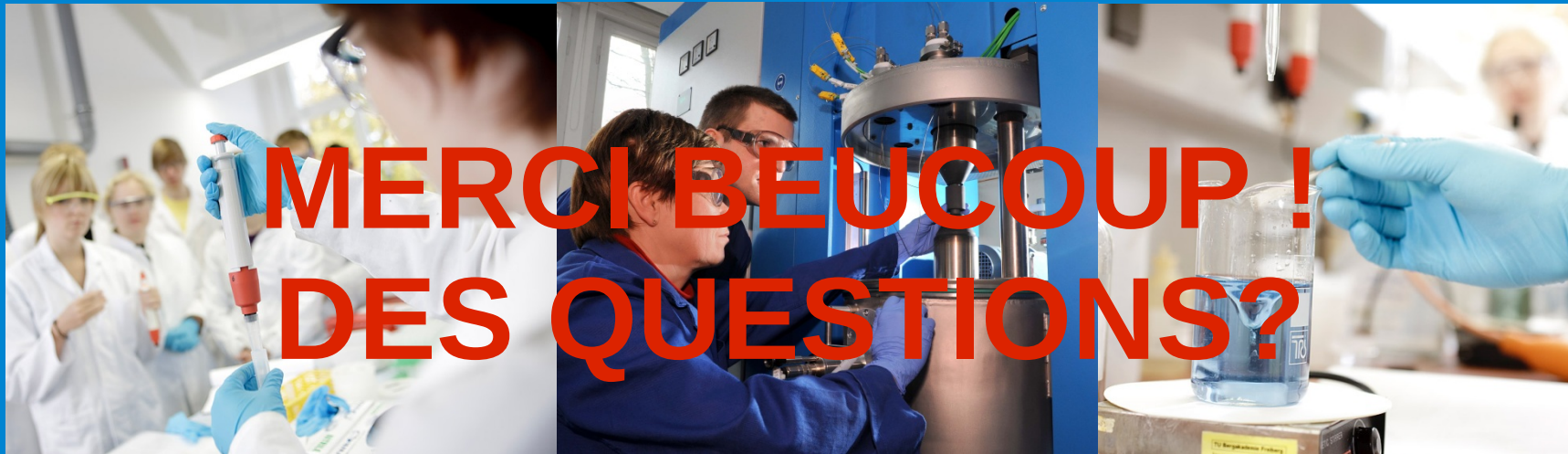
Based on the presented complex analysis procedure library, the Cone Method can be implemented into the Cast3M engineering environment



TECHNISCHE UNIVERSITÄT
BERGAKADEMIE FREIBERG

Die Ressourcenuniversität. Seit 1765.

Foundation Vibration Analysis with Simple Physical Models



Dr. N.Tamaskovics

TU Bergakademie Freiberg, Institut für Geotechnik

Chair of Soil Mechanics, Ground Engineering and Mining Geotechnics