Fissuration en milieux isotrope et orthotrope via les intégrales invariantes: prise en compte des effets environnementaux

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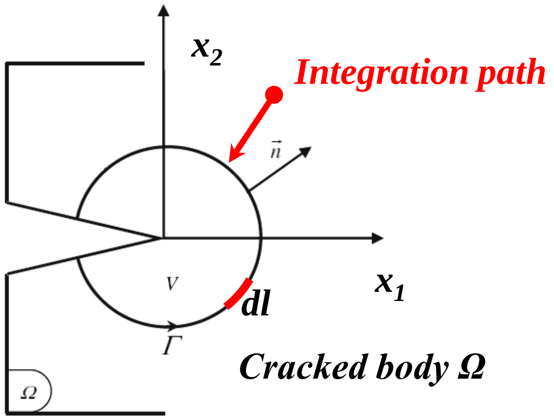
Outline

1. Path independent integrals formulation *(J, M, A)*
2. Validation on elastic isotropic material
3. Generalization to elastic orthotropic material
4. Generalization to viscoelastic material
5. Conclusions and perspectives



Strain energy deformation

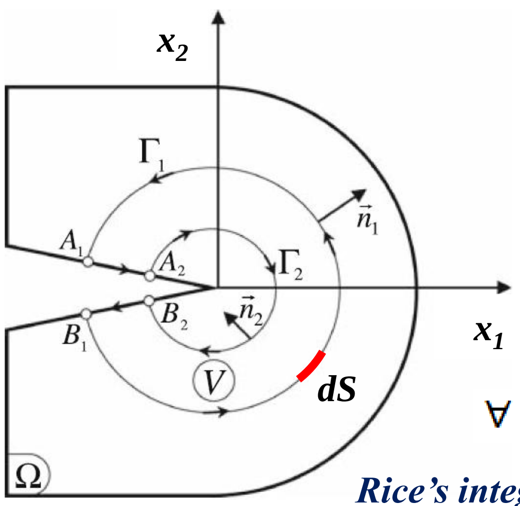
$$\omega(\varepsilon_{ij}) = \frac{1}{2}\lambda\,\varepsilon_{kk}\varepsilon_{hh} + \mu\,\varepsilon_{ij}\varepsilon_{ij}$$

$$J = \int_{\Gamma} \left[\omega(\varepsilon_{ij})\, n_1 - \sigma_{ij} n_j u_{i,1}\right] dl$$

p_{j1} : **Energy momentum tensor**

Noether's theorem

$$L = \int_{V} \omega(\varepsilon_{ij})\, dV \;\Rightarrow\; \delta L = \int_{\partial V} \left[\omega\delta_{jk} - \sigma_{ij}u_{i,k}\right] n_j\, \delta x_k\, dS = \int_{\partial V} p_{jk}\, n_j\, \delta x_k\, dS = 0$$



(1) Virtual displacement $\overrightarrow{x_1}$

$$\begin{cases} \delta u_1 = \delta x_1 = \delta a \\ \delta u_2 = \delta x_2 = 0 \end{cases}$$

(2) Boundary conditions on crack lips

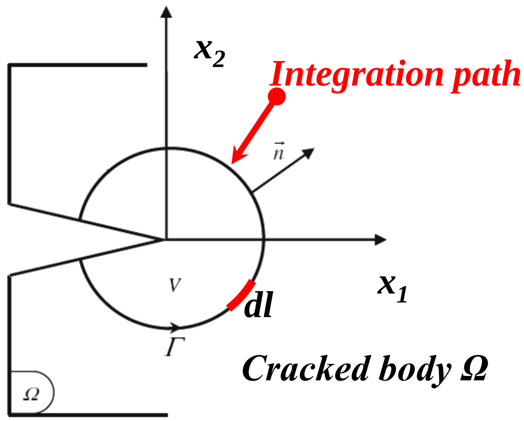
$$\begin{cases} n_1 = 0 \\ \sigma_{ij} n_j = 0 \end{cases}$$

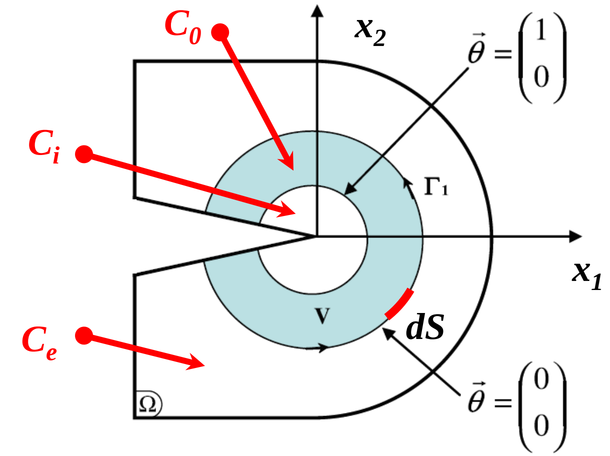
Integration on surface V enclosed by $\partial V = \Gamma_1 + A_1A_2 - \Gamma_2 - B_1B_2$

$$\forall\, \Gamma_1, \Gamma_2 \quad \int_{\Gamma_1} p_{j1}\, n_j\, dS + \int_{A_1A_2} p_{j1}\, n_j\, dS = \int_{\Gamma_2} p_{j1}\, n_j\, dS + \int_{B_1B_2} p_{j1}\, n_j\, dS$$

(2) $=0$ $=0$

Rice's integral





$\vec{\theta} = (\theta_1 \quad \theta_2)$

- C_0 : continuously varying from (1,0) to (0,0)
- C_i : θ=(1,0)
- C_e : θ=(0,0)

Integration on **V** *enclosed by* $\partial V = \Gamma_1 + A_1A_2 - \Gamma_2 - B_1B_2$

$$\int_{\Gamma_2 + A_1A_2 - B_1B_2} p_{jk}\, \theta_k\, n_j\, dS = 0 \quad (1)$$

(1) and *Green-Ostrogradsky theorem's*

$$J = \int_{\Gamma} \left[\omega(\varepsilon_{ij}) n_1 - \sigma_{ij} n_j u_{i,1}\right] dl \;\Rightarrow\; G_\theta = \int_V \left(-p_{jk}\theta_k\right)_{,j} dV = \int_V \left[-\omega\, \theta_{k,k} + \sigma_{ij}\, u_{i,k}\, \theta_{k,j}\right] dV$$

Energy release rate

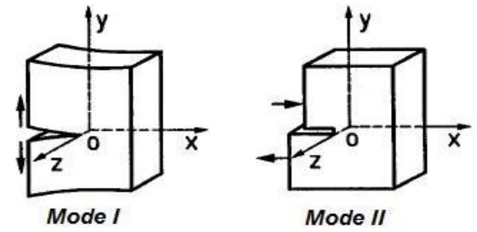
$$J = G = \frac{K_I^2 + K_{II}^2}{E'} \qquad \begin{cases} E' = E & \textbf{plan } \boldsymbol{\sigma} \\ E' = E/(1-\nu^2) & \textbf{plan } \boldsymbol{\varepsilon} \end{cases}$$

Fracture modes separation

$$G_I = \frac{K_I^2}{\kappa} \qquad G_{II} = \frac{K_{II}^2}{\kappa}$$

$$u = u_I + u_{II}$$

$$\sigma = \sigma_I + \sigma_{II}$$



M-integral

Noether theorem's

$$\delta L = \int_V \delta\omega(\varepsilon_{ij}^u, \varepsilon_{ij}^v) dV = 0$$

Bilinear expression of the strain energy density

$$\omega(\varepsilon_{ij}^u, \varepsilon_{ij}^v) = \frac{1}{2}\lambda\, \varepsilon_{kk}^u\, \varepsilon_{hh}^v + \mu\, \varepsilon_{ij}^u\, \varepsilon_{ij}^v$$

Real fields(FEM)

$$\varepsilon_{ij}^u = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\sigma_{ij}^u = \lambda\, \delta_{ij} u_{k,k} + \mu(u_{i,j} + u_{j,i})$$

Virtual fields (auxiliary problem)

$$\varepsilon_{ij}^v = \frac{1}{2}(v_{i,j} + v_{j,i})$$

$$\sigma_{ij}^v = \lambda\, \delta_{ij} v_{k,k} + \mu(v_{i,j} + v_{j,i})$$

M-integral formulation

$$M = \int_\Gamma \frac{1}{2}[\sigma_{ij,1}^v\, u_i - \sigma_{ij}^u\, v_{i,1}] n_j dl$$

$$M_\theta = \int_V -\frac{1}{2}[\sigma_{ij,1}^v\, u_i - \sigma_{ij}^u\, v_{i,1}] \theta_{1,j} dV$$

Relation between M-integral and SIF K_I et K_{II}

$$M = \frac{K_I^u K_I^v + K_{II}^u K_{II}^v}{E'}$$

$$(K_I^v = 1, K_{II}^v = 0) \Rightarrow K_I^u$$

$$(K_I^v = 0, K_{II}^v = 1) \Rightarrow K_{II}^u$$

T and A integrales Temperature variation $\Delta T = T - T_0$

Noether theorem's

$$\delta L = \int_V \delta\omega\left(\varepsilon_{ij}^u, \varepsilon_{ij}^v, \Delta T\right) dV = 0$$

Bilinear expression of the strain energy density

$$\omega\left(\varepsilon_{ij}^u, \varepsilon_{ij}^v, \Delta T\right) = \frac{1}{2}\lambda\, \varepsilon_{kk}^u\, \varepsilon_{hh}^v + \mu\, \varepsilon_{ij}^u\, \varepsilon_{ij}^v - \beta \Delta T \varepsilon_{hh}^v$$

Real fields(FEM)

$$\varepsilon_{ij}^u = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$\sigma_{ij}^u = \lambda\, \delta_{ij} u_{k,k} + \mu\left(u_{i,j} + u_{j,i}\right)$$

$$T^u = \Delta T = T - T_0$$

Virtual fields (auxiliary problem)

$$\varepsilon_{ij}^v = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right)$$

$$\sigma_{ij}^v = \lambda\, \delta_{ij} v_{k,k} + \mu\left(v_{i,j} + v_{j,i}\right)$$

$$T^v = 0$$

T-integral formulation

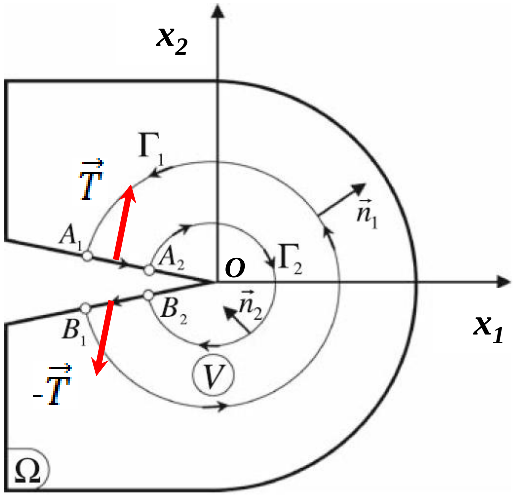
$$T = \int_\Gamma \frac{1}{2}\left[\sigma_{ij,1}^v\, u_i - \sigma_{ij}^u\, v_{i,1} - \gamma \Delta T\left(v_{1,j} - \psi_{1,j}\right) + \gamma \Delta T_{,j}\left(v_1 - \psi_1\right)\right] n_j dl$$

A-integral formulation

$$A = T_\theta = \int_V -\frac{1}{2}\left[\sigma_{ij,1}^v\, u_i - \sigma_{ij}^u\, v_{i,1} - \gamma \Delta T\left(v_{1,j} - \psi_{1,j}\right) + \gamma \Delta T_{,j}\left(v_1 - \psi_1\right)\right] \theta_{1,j} dV$$

A_1: Classical term — A_2 : temperature variation effect

Improvement of the A-integral formulation



Applied forces on the crack lips $\vec{T} = \begin{Bmatrix} p(x_1) \\ q(x_1) \end{Bmatrix}$

B.C. on the crack lips

- A_1A_2 et B_2B_1 : $\sigma_{ij} n_j = T_i \quad \sigma^{v}_{ij} = 0$
- A_1A_2 : $n_1 = 0, n_2 = -1, T_1 = p, T_2 = q$
- B_2B_1 : $n_1 = 0, n_2 = 1, T_1 = -p, T_2 = -q$

Energy momentum tensor

$$p_{j1} = \frac{1}{2}\left[\sigma^{v}_{ij,1}\, u_i - \sigma^{u}_{ij}\, v_{i,1} - \gamma \Delta T \left(v_{1,j} - \psi_{1,j}\right) + \gamma \Delta T_{,j}(v_1 - \psi_1)\right]$$

Noether theorem's

$$\int_{\Gamma_1 - \Gamma_2} p_{jk}\, n_j\, dS - \int_{A_1A_2 + B_2B_1} \frac{1}{2}\left[T_1 v_{1,1} + T_2 v_{2,1}\right] dx_1 = 0$$

A-integral formulation

$$A = T_\theta = \int_V -\frac{1}{2}\left[\sigma^{v}_{ij,1}\, u_i - \sigma^{u}_{ij}\, v_{i,1} - \gamma \Delta T \left(v_{1,j} - \psi_{1,j}\right) + \gamma \Delta T_{,j}(v_1 - \psi_1)\right] \theta_{1,j}\, dV$$

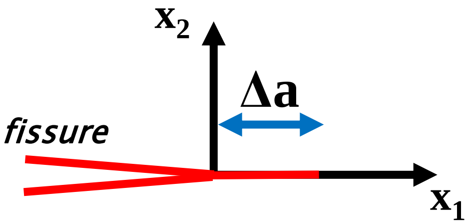
A_1: Classical term — A_2 : temperature variation effect

$$- \int_{A_1A_2 + B_2B_1} T_i\, v_{i,j}\, \theta_j\, dx_1$$

A_3 : effect of pressure applied on the crack lips

Improvement of the A-integral formulation

Crack growth process



$$f\left[\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}}\right] = 1 \rightarrow \textit{crack growth}$$

$$f\left[\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}}\right] < 1 \rightarrow \textit{no crack growth}$$

A-integral formulation

$$A = T_\theta = \int_V -\frac{1}{2}\left[\sigma^v_{ij,1}\, u_i - \sigma^u_{ij}\, v_{i,1} - \gamma\Delta T\left(v_{1,j} - \psi_{1,j}\right) + \gamma\Delta T_{,j}\left(v_1 - \psi_1\right)\right]\theta_{1,j} dV$$

A_1: Classical term — A_2 : temperature variation effect

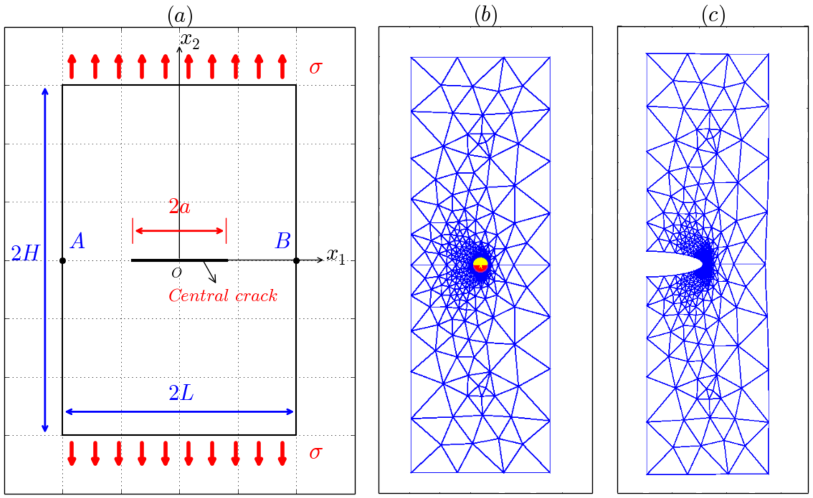
$$-\int_{A_1A_2+B_2B_1} T_i\, v_{i,j}\, \theta_j\, dx_1$$

A_3 : effect of pressure applied on the crack lips

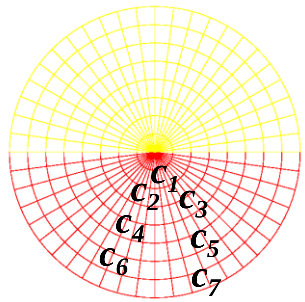
$$-\int_V \left[\sigma^v_{ij,k} u_{i,j} + \sigma^u_{ij,k} v_{i,j} + \beta\delta_{ij} u_{i,jk}\Delta T\right]\theta_k\, dV$$

A_4 : effect of crack growth

Mode I



Rectangular plate with central crack subjected to a far-field tensile stress: (a) – Geometry and loads, (b) – Finite elements mesh, (c) – Deformed shape



- **Material properties**

  $E = 20000\ daN/mm^2 \quad \nu = 0{,}3$

- **Applied load**

  $\sigma = 1\ daN/mm^2$

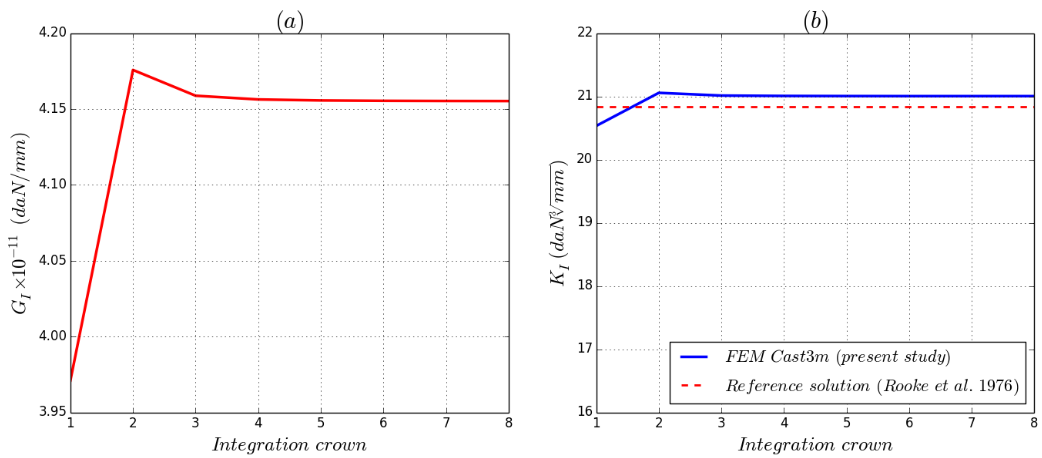
- **Geometry parameters**

  $2L = 400\ mm \quad 2H = 1200\ mm \quad 2a = 200\ mm$

- **Results for plan strain condition**

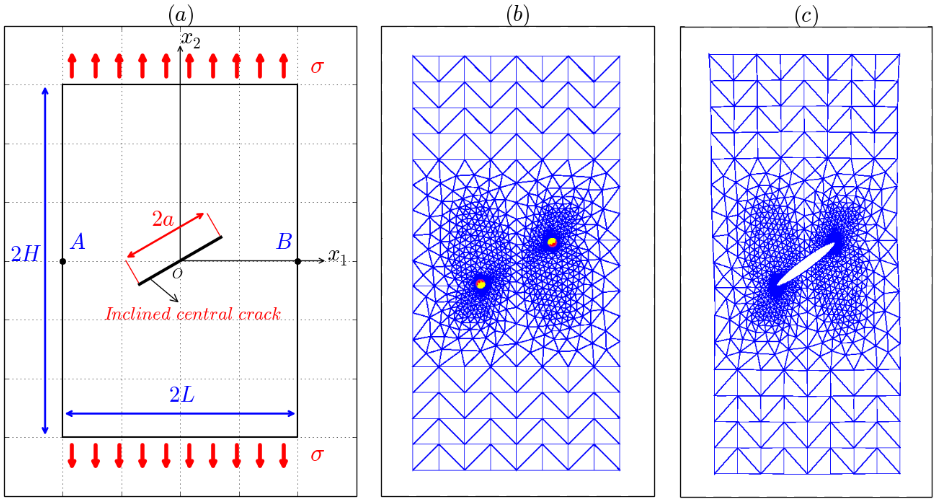
  (Rooke et al. 76; Wilson, IJF 79)

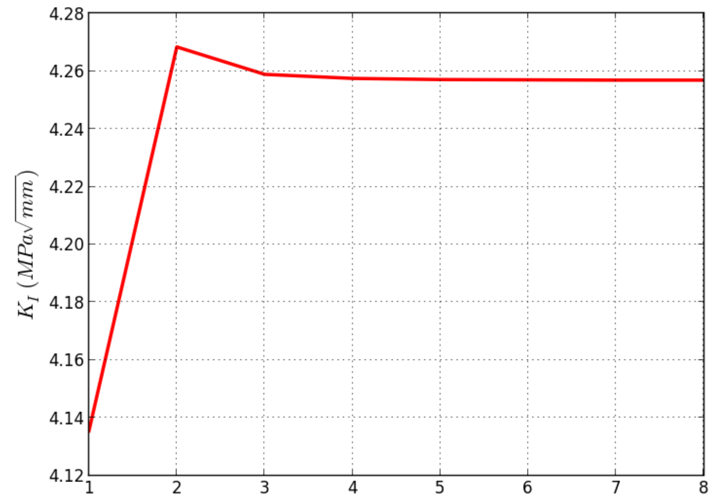
$K_I\ (daN/mm^{3/2})$		
Analytical	**FEM**	**Ecart %**
20,843	21,012	0,8



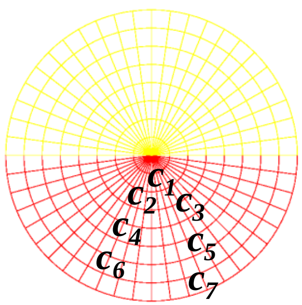
Path independence verification of
(a) – the energy release rate GI,
(b) – the stress intensity factor KI

Mixed mode (I and II)





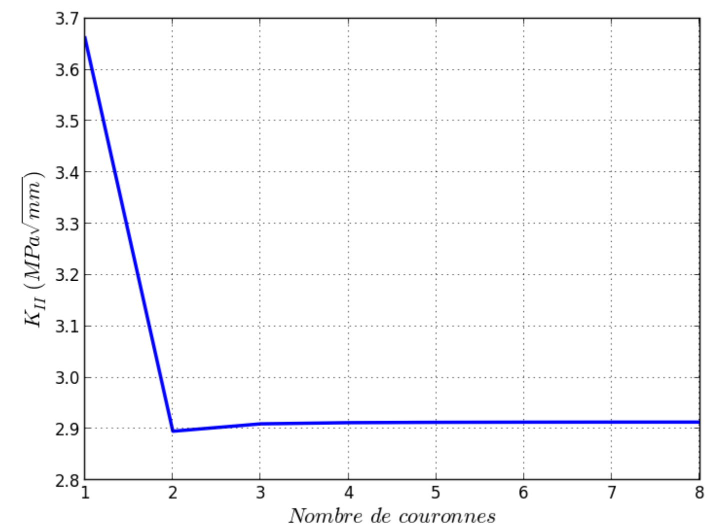
Rectangular plate with central inclined crack subjected to a tensile stress (a) – Geometry and loads, (b) – Finite elements mesh, (c) – Deformed shape



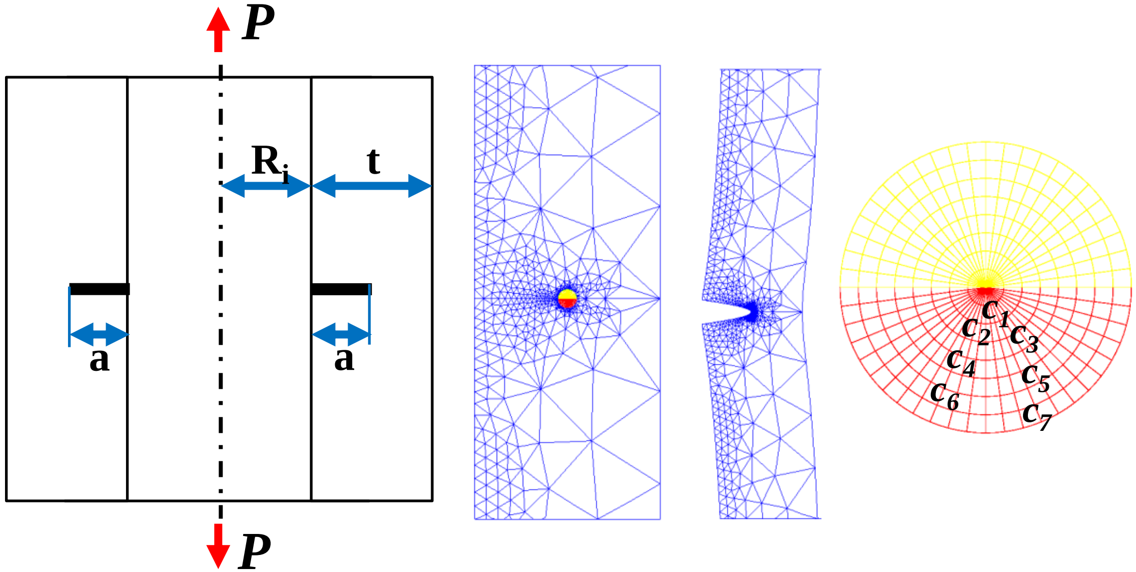
- **Results for plan stress condition**

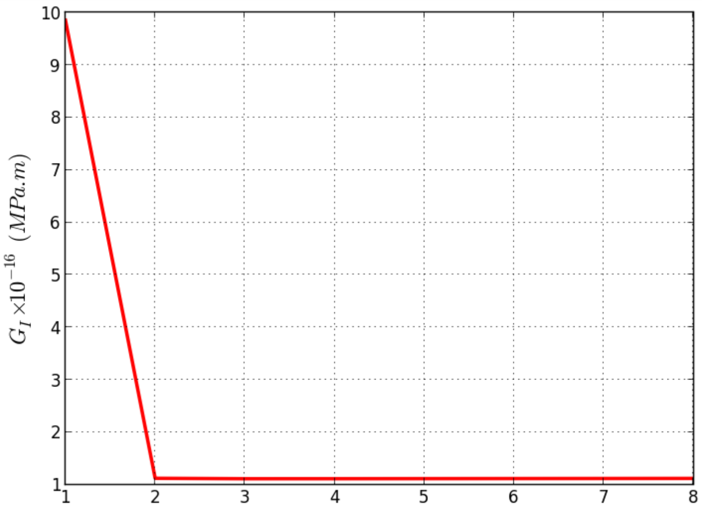
K_I $(MPa\sqrt{mm})$		
Analytical	**FEM**	**Ecart %**
4,247	4,257	0,2

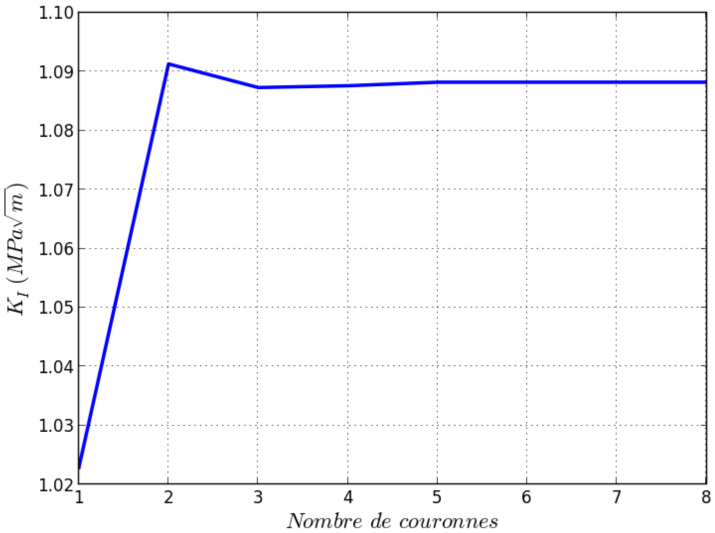
K_{II} $(MPa\sqrt{mm})$		
Analytical	**FEM**	**Ecart %**
2,877	2,913	1,2



Axisymetric problem







- **Material properties**

  $E = 2\ 10^{11}\ Pa \quad \nu = 0,3$

- **Applied load**

  $P = 1\ 10^{6}\ N$

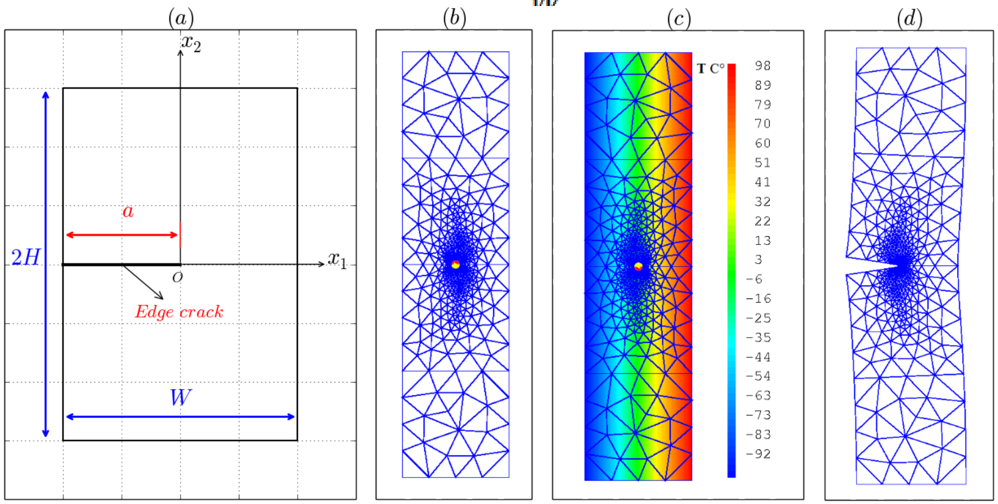
- **Geometry parameters**

  $R_i = 1\ m \quad t = 0,1\ m \quad a = 0,05\ m$

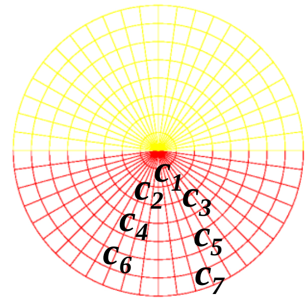
K_I $(MPa\sqrt{m})$		
Analytical	FEM	Ecart %
1,085	1,087	0,2

Thermal load

Temperature field $T(x) = 2T_0 \frac{x}{W}$



Rectangular plate with edge crack subjected to temperature field (a) – Geometry, (b) – Finite elements mesh, (c) – Thermal load, (d) – Deformed shape



- **Applied load**

  T0 = 100 °C

- **Material properties**

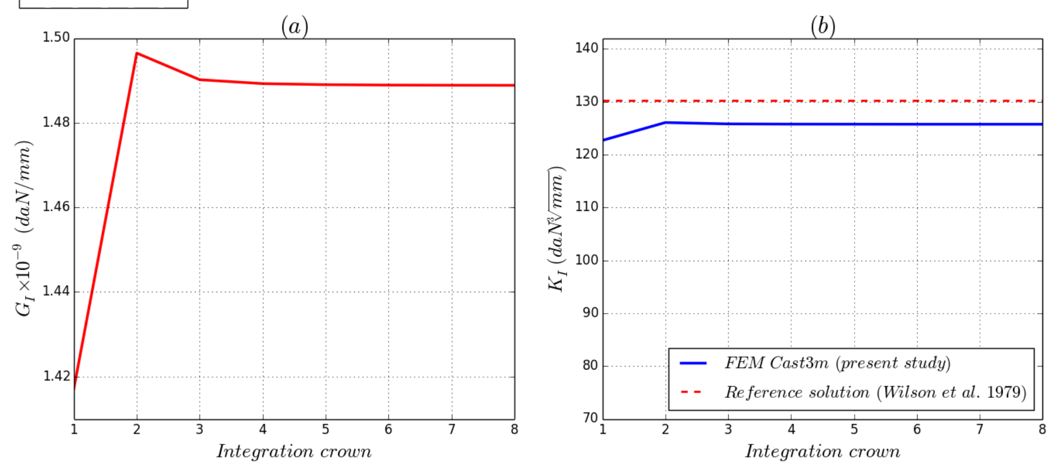
  E = 20000 daN/mm²
  ν = 0,3 α = 5 10⁻⁶ °C⁻¹

- **Geometry parameters**

  2H = 800 mm 2W = 200 mm
  a = 100 mm

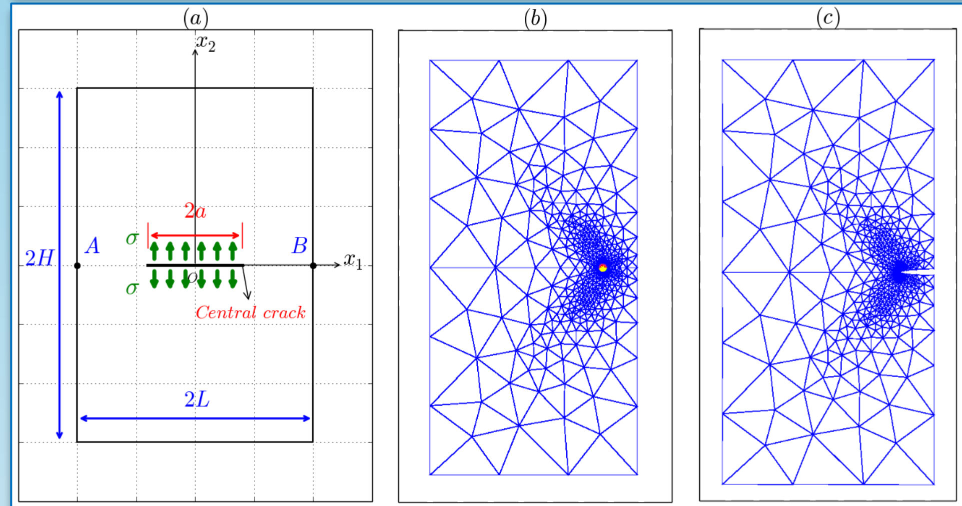
- **Computing in plan strain**

$K_I\,(daN/mm^{3/2})$		
Analytique	Numérique	Ecart %
130,15	125,72	3,4

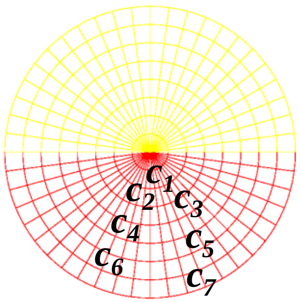


Path independence verification of (a) – the energy release rate GI, (b) – the stress intensity factor K_I

Pressure on the crack lips



Rectangular plate with central crack subjected to a tensile stress (a) – Geometry and loads, (b) – Finite elements mesh, (c) – Deformed shape



- **Applied load**

  $\sigma = 1\ daN/mm^2$

- **Material properties**

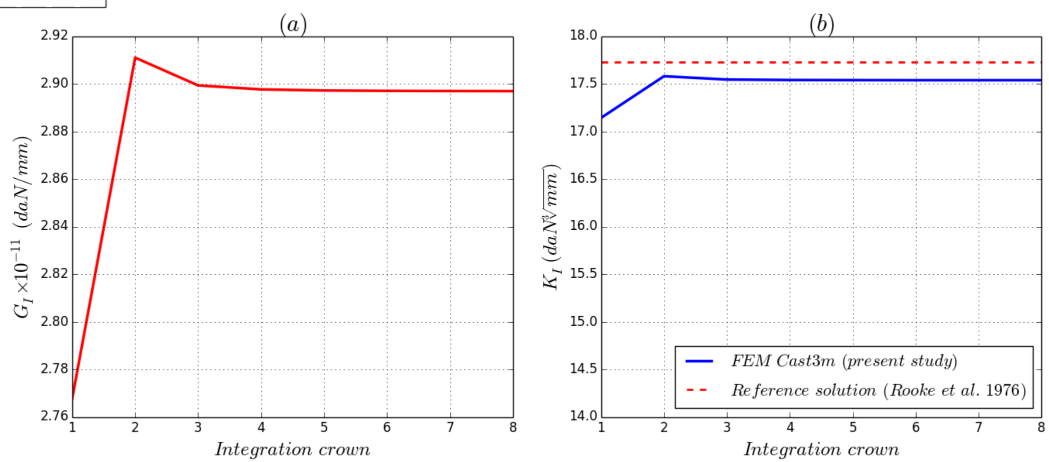
  $E = 20000\ daN/mm^2 \qquad \nu = 0{,}3$

- **Geometry parameters**

  $2b = 400\ mm \quad 2h = 1200\ mm \quad 2a = 200\ mm$

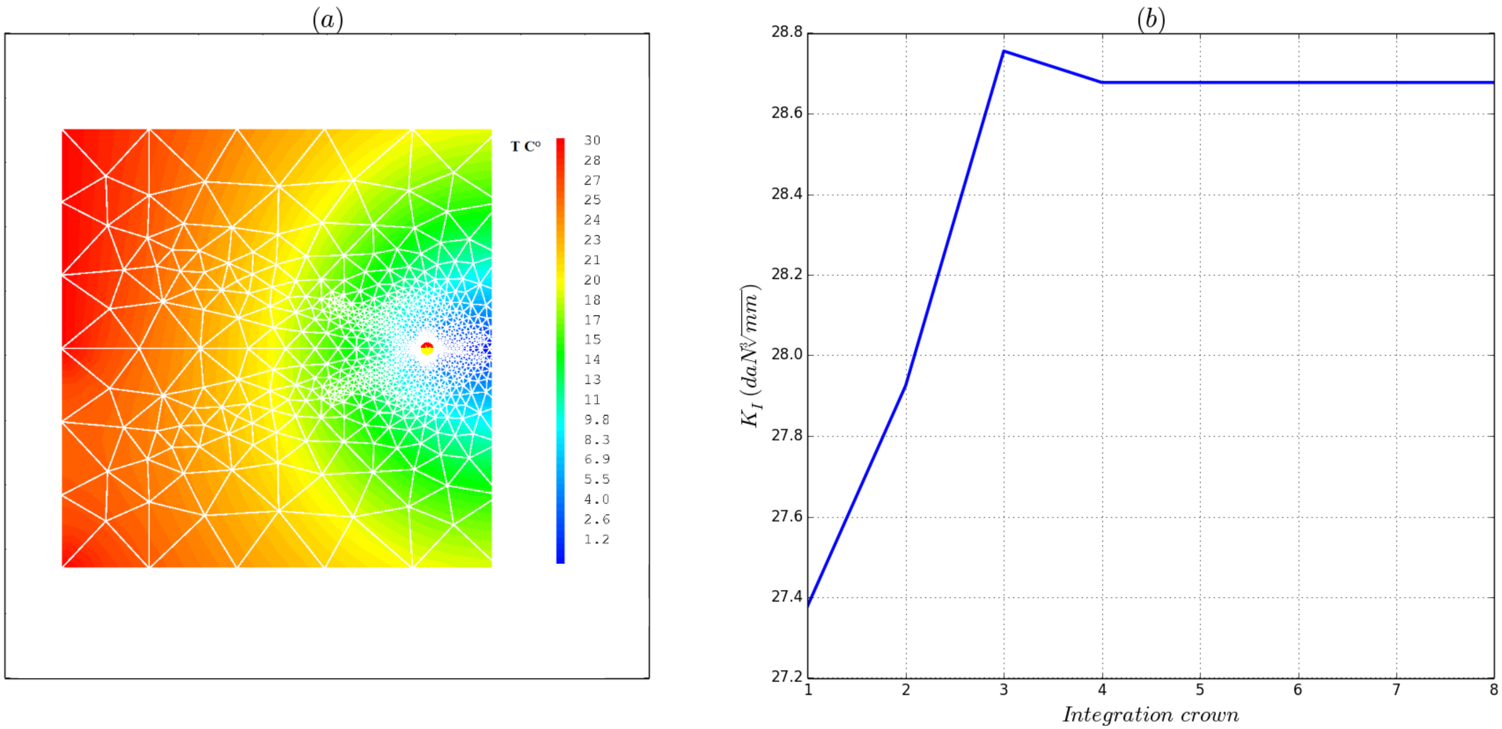
- **Results for plan strain condition**

$K_I\ (daN/mm^{3/2})$		
Analytical	**FEM**	**Ecart %**
17,725	17,544	1



Path independence verification of (a) – the energy release rate G_I, (b) – the stress intensity factor K_I

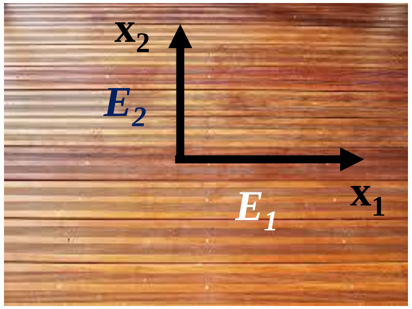
Effect of thermal load and pressure on the crack lips



(a) Temperature field distribution,
(b) (b) Path independence verification of the stress intensity factor K_I

- T_0 = 0°C à T_1= 30°C

Generalization to elastic orthotropic material



Plan stress condition

Temperature variation

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} + \begin{bmatrix} \alpha_1 \Delta T \\ \alpha_2 \Delta T \\ 0 \end{bmatrix}$$

Plan strain condition

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} (1-\nu_{31}\nu_{13})/E_1 & -(\nu_{12}+\nu_{13}\nu_{32})/E_1 & 0 \\ -(\nu_{12}+\nu_{13}\nu_{32})/E_1 & (1-\nu_{23}\nu_{32})/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} + \begin{bmatrix} (\nu_{31}\alpha_3+\alpha_1)\Delta T \\ (\nu_{32}\alpha_3+\alpha_2)\Delta T \\ 0 \end{bmatrix}$$

Hyp1 : $\gamma = f(E_1, \nu_{12}, \alpha_1)$

$$A = T_\theta = \int_V -\frac{1}{2}\left[\sigma^v_{ij,1}\, u_i - \sigma^u_{ij}\, v_{i,1} + \gamma \Delta T \left(v_{1,j} - \psi_{1,j}\right) + \gamma \Delta T_{,j} \left(v_1 - \psi_1\right)\right]\theta_{1,j} dV$$

A_1: Classical term — **A_2 : temperature variation effect**

Hyp2 : $\beta = g(E_1, \nu_{12}, \alpha_1)$

$$- \int_{A_1A_2+B_2B_1} T_i\, v_{i,j}\, \theta_j\, dx_1 \qquad - \int_V \left[\sigma^v_{ij,k} u_{i,j} + \sigma^u_{ij,k} v_{i,j} + \beta \delta_{ij} u_{i,jk} \Delta T\right]\theta_k\, dV$$

A_3 : effect of pressure applied on the crack lips — **A_4 : effect of crack growth**

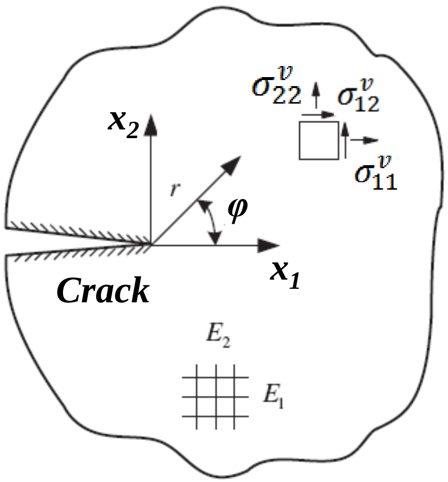
Virtual fields computation

Anisotropic material

$$c_{11}\mu^4 - 2c_{16}\mu^3 + (2c_{12} + c_{66})\mu^2 - 2c_{26}\mu + c_{22} = 0$$

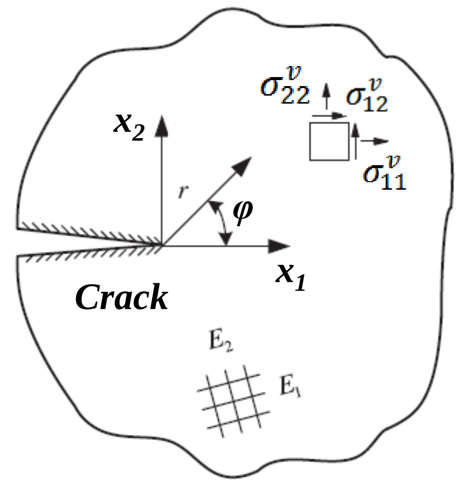
Orthotropic material $c_{16} = c_{26} = 0$

$$\mu^4 + \underbrace{\frac{2c_{12} + c_{66}}{c_{11}}}_{A}\mu^2 + \underbrace{\frac{c_{22}}{c_{11}}}_{B} = 0$$



Compliance matrix

$$[C] = \begin{bmatrix} c_{11} & c_{12} & c_{16} \\ c_{12} & c_{22} & c_{26} \\ c_{16} & c_{26} & c_{66} \end{bmatrix}$$



$$R_{crack} \equiv R_{orthotropy}$$

$$R_{crack} \neq R_{orthotropy}$$

Case I

$$\mu_1 = i\sqrt{A + \sqrt{A^2 - B}}$$

$$\mu_2 = i\sqrt{A - \sqrt{A^2 - B}}$$

Case II

$$\mu_1 = i\sqrt{\sqrt{B}}$$

$$\mu_2 = \mu_1$$

Case III

$$\mu_1 = i\sqrt{-A + i\sqrt{A^2 - B}}$$

$$\mu_2 = -\Re[\mu_1] + i\Im[\mu_1]$$

Case VI

$$\mu_1' = \frac{\mu_1 \cos\theta - \sin\theta}{\cos\theta + \mu_1 \sin\theta}$$

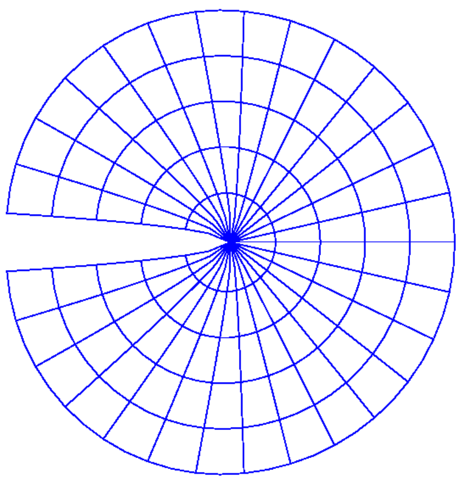
$$\mu_2' = \frac{\mu_2 \cos\theta - \sin\theta}{\cos\theta + \mu_2 \sin\theta}$$

Virtual fields computation

Virtual displacement field

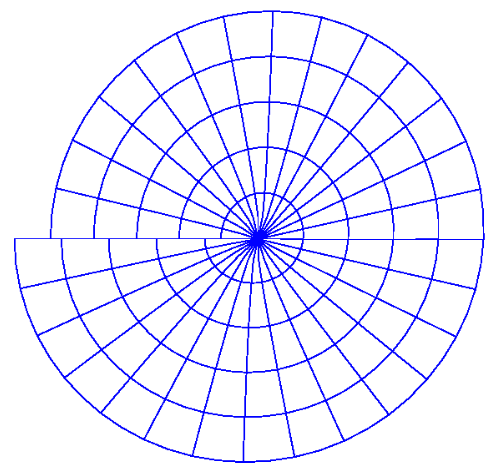
$$v_1 = \sqrt{\frac{2r}{\pi}} \mathrm{Re}\left[\frac{K_I}{\mu_1-\mu_2}\left(\mu_1 p_2\sqrt{z_2}-\mu_2 p_1\sqrt{z_1}\right)+\frac{K_{II}}{\mu_1-\mu_2}\left(p_2\sqrt{z_2}-p_1\sqrt{z_1}\right)\right]$$

$$v_2 = \sqrt{\frac{2r}{\pi}} \mathrm{Re}\left[\frac{K_I}{\mu_1-\mu_2}\left(\mu_1 q_2\sqrt{z_2}-\mu_2 q_1\sqrt{z_1}\right)+\frac{K_{II}}{\mu_1-\mu_2}\left(q_2\sqrt{z_2}-q_1\sqrt{z_1}\right)\right]$$



$v_1^I + v_2^I$

Opening mode



$v_1^{II} + v_2^{II}$

Shear mode

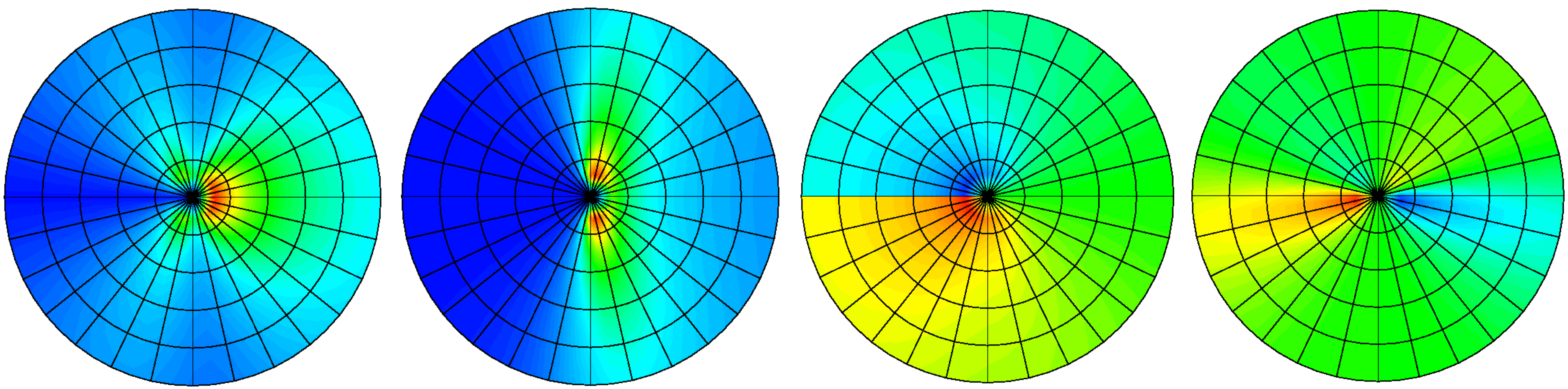
Virtual fields computation

Virtual stress field

$$\sigma_{11}^{v} = \sqrt{\frac{1}{2\pi r}}\mathrm{Re}\left[\frac{K_I \mu_1 \mu_2}{\mu_1-\mu_2}\left(\frac{\mu_2}{\sqrt{z_2}}-\frac{\mu_1}{\sqrt{z_1}}\right)+\frac{K_{II}}{\mu_1-\mu_2}\left(\frac{\mu_2^2}{\sqrt{z_2}}-\frac{\mu_1^2}{\sqrt{z_1}}\right)\right]$$

$$\sigma_{22}^{v} = \sqrt{\frac{1}{2\pi r}}\mathrm{Re}\left[\frac{K_I}{\mu_1-\mu_2}\left(\frac{\mu_1}{\sqrt{z_2}}-\frac{\mu_2}{\sqrt{z_1}}\right)+\frac{K_{II}}{\mu_1-\mu_2}\left(\frac{1}{\sqrt{z_2}}-\frac{1}{\sqrt{z_1}}\right)\right]$$

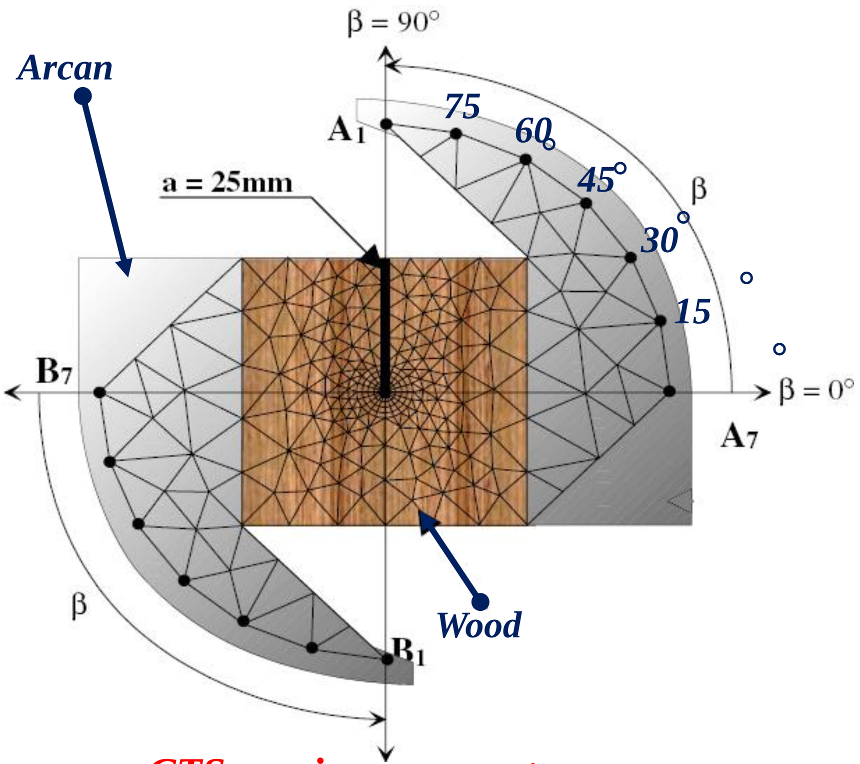
$$\sigma_{12}^{v} = \sqrt{\frac{1}{2\pi r}}\mathrm{Re}\left[\frac{K_I \mu_1 \mu_2}{\mu_1-\mu_2}\left(\frac{1}{\sqrt{z_1}}-\frac{1}{\sqrt{z_2}}\right)+\frac{K_{II}}{\mu_1-\mu_2}\left(\frac{\mu_1}{\sqrt{z_1}}-\frac{\mu_2}{\sqrt{z_2}}\right)\right]$$



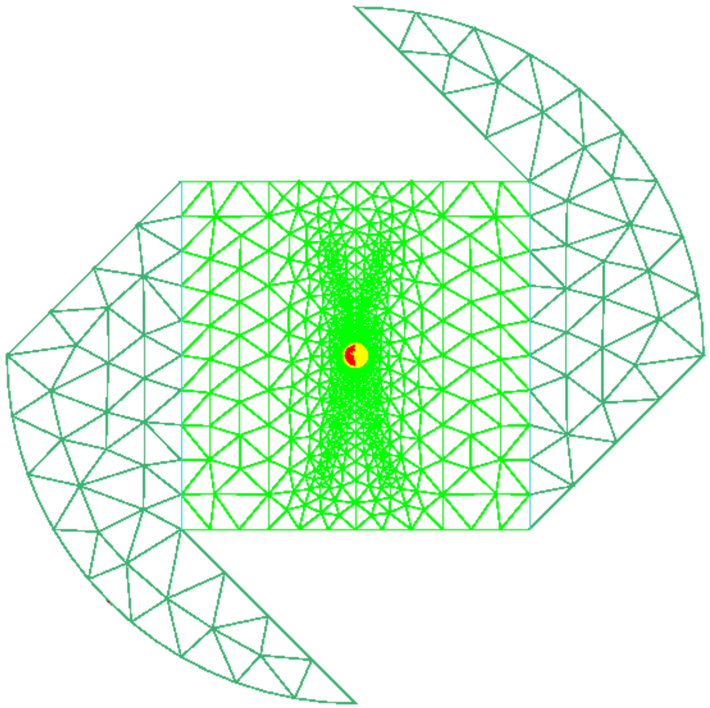
σ_{11}^{I} σ_{22}^{I} σ_{11}^{II} σ_{22}^{II}

Opening mode *Shear mode*

Validation on CTS (Compact Tension Shear) specimen



CTS specimen geometry

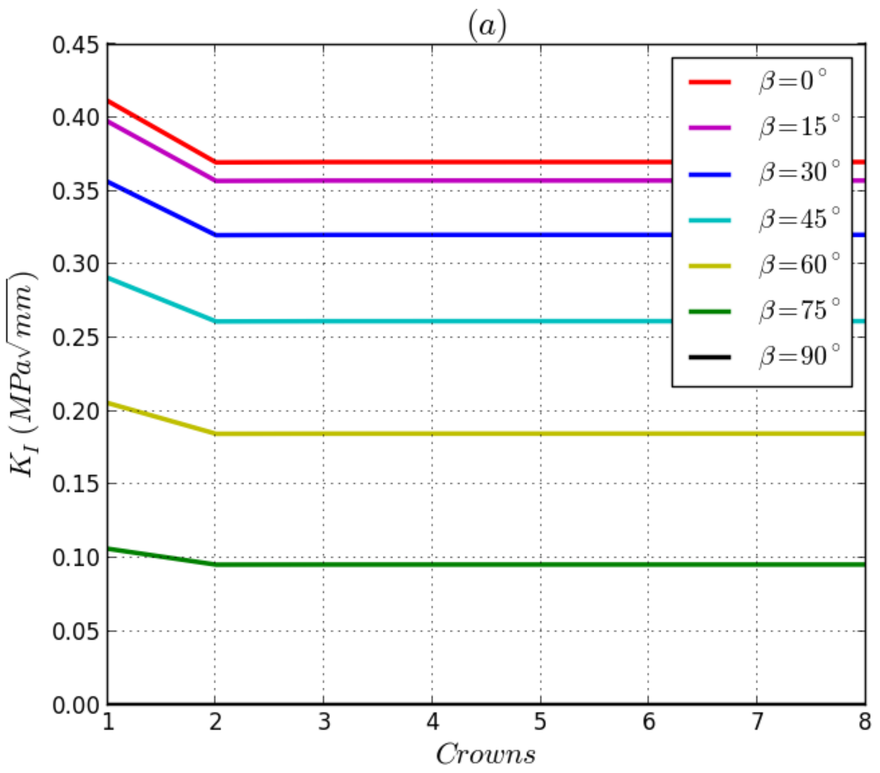


Mesh of the CTS specimen

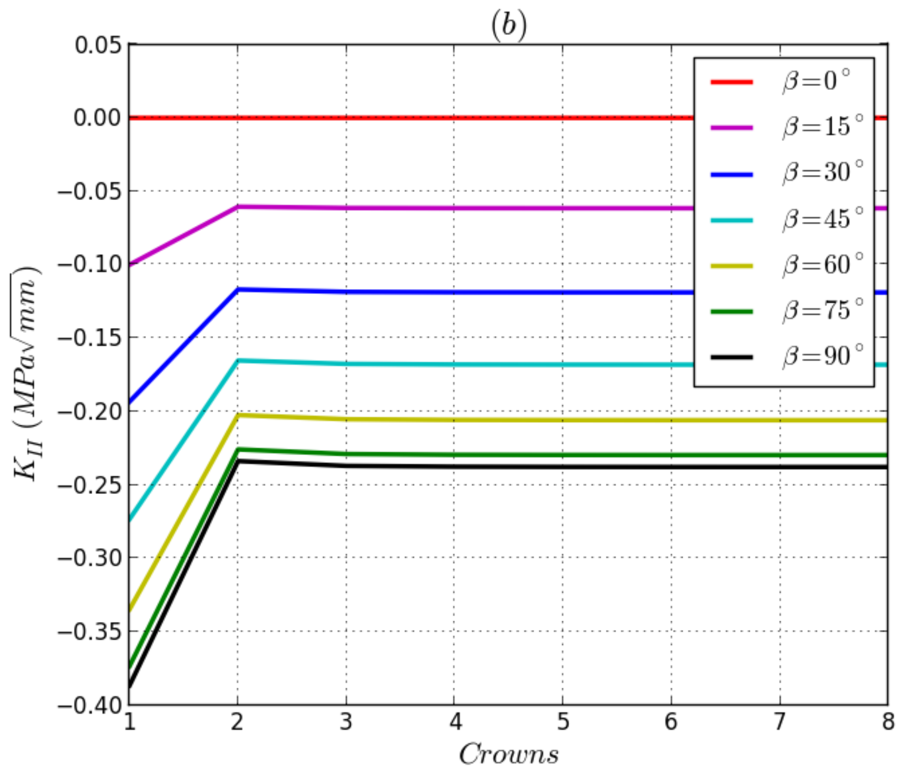
Parameters

$E_1 = 600 MPa$ $E_2 = 15000 MPa$ $G_{12} = 700 MPa$

Numerical results for stress intensity factors without thermal load



Opening mode



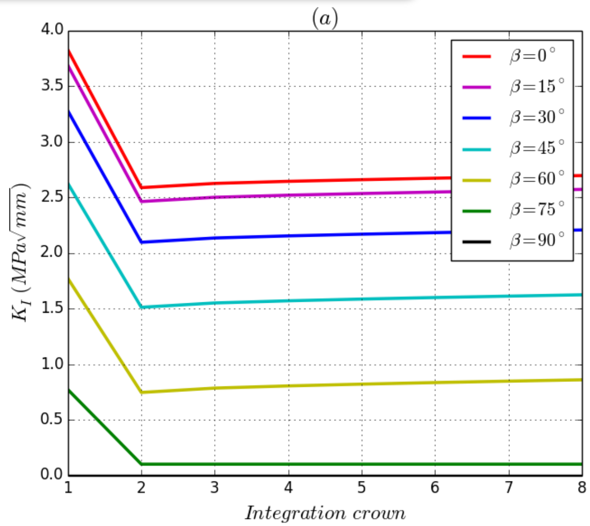
Shear mode

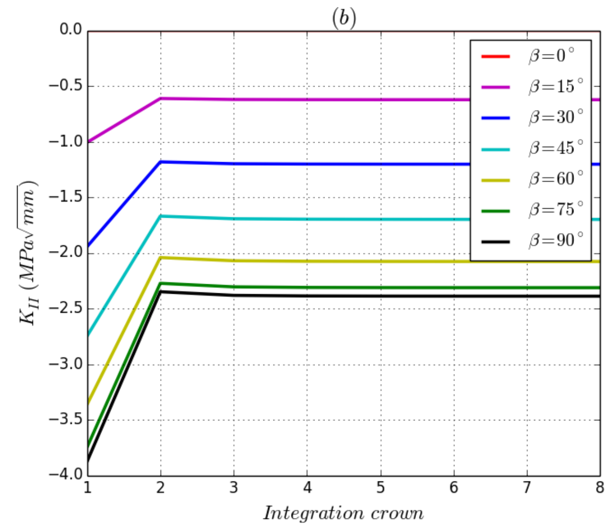
Numerical results for stress intensity factors with thermal load

Path independence verification of stress intensity factor for ΔT = 10 °C :

(a) Opening mode K_I,

(b) (b) Shear mode K_{II}

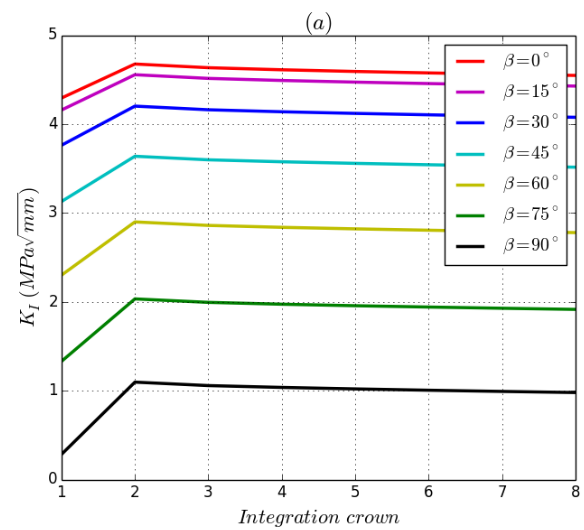


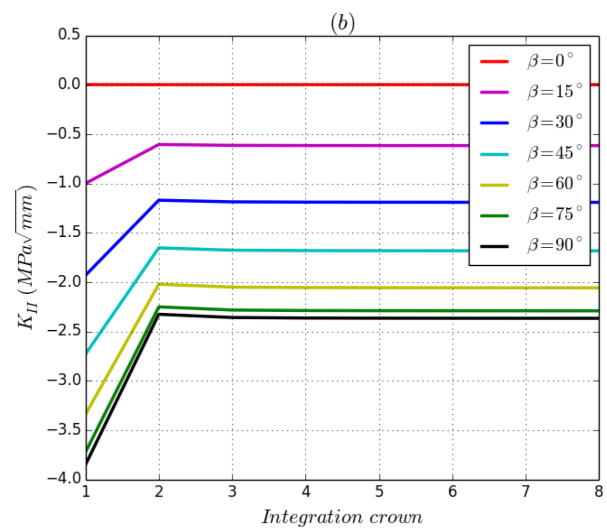


Path independence verification of stress intensity factor for ΔT = -10 °C :

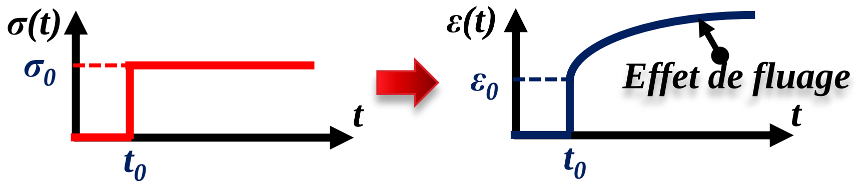
(a) Opening mode K_I,

(b) Shear mode K_{II}





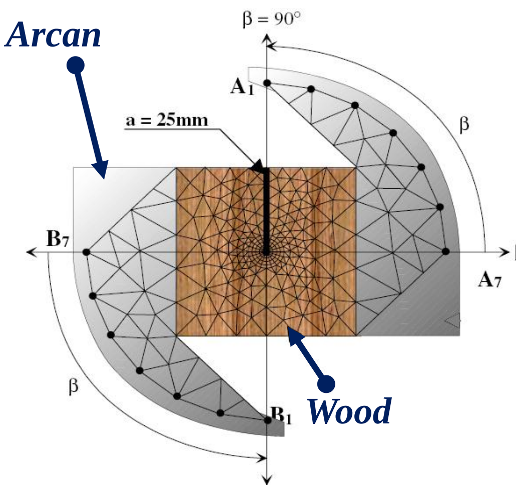
Expérience de fluage/formulation intégrale



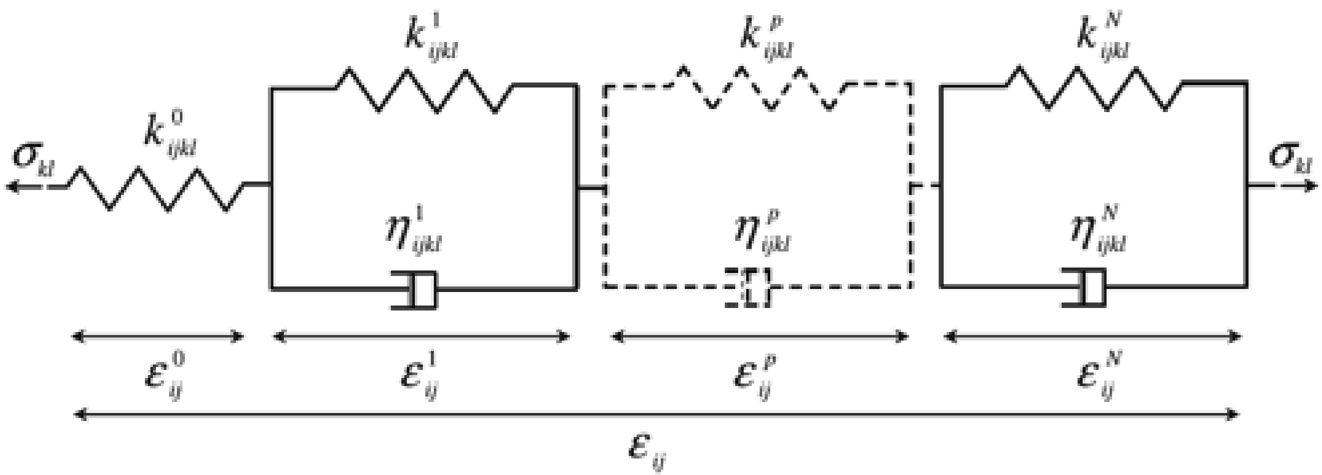
Intégrale de BOLTZMANN

$$\varepsilon(t) = J(t_0,t)\sigma(t_0) + \int_{t_0}^{t} J(\tau,t)\,\dot{\sigma}(\tau)\,d\tau$$

Tenseur de fluage



Modèle rhéologique de Kelvin-Voigt



Formulation de l'intégrale A pour le comportement viscoélastique

$$A_\theta^{(p)} = \int_V -\frac{1}{2}\left[{}^{(p)}\sigma^{v}_{ij,k}u_i^{(p)} - {}^{(p)}\sigma^{u}_{ij}v_{i,k}^{(p)} - \gamma\Delta T_{,j}\left(v_k^{(p)} - \Psi_k\right) - \gamma\Delta T\left(v_{i,k}^{(p)} - \Psi_{k,j}\right)\right]\theta_{k,j}\,dV$$

Terme classique A_1 — Terme A_2 : chargement thermique

Paramètres de rupture dans le cas viscoélastique

Facteur d'inténsité de contraintes

Mode I
$$ {}^{u}K_I^{(p)} = \frac{Aq^{(p)}\left({}^{v}K_I^{(p)}=1;\,{}^{v}K_{II}^{(p)}=2\right)}{C_1^{(p)}} $$
and

Mode II
$$ {}^{u}K_{II}^{(p)} = \frac{Aq^{(p)}\left({}^{v}K_I^{(p)}=0;\,{}^{v}K_{II}^{(p)}=1\right)}{C_2^{(p)}} $$

$C_1^{(p)}$, $C_2^{(p)}$ → Complaisances viscoélastiques

Taux de restitution d'énergie viscoélastique

$$ {}^{1}G\theta_v^{(p)} + {}^{2}G\theta_v^{(p)} = C_1^{(p)} \cdot \frac{\left({}^{u}K_I^{(p)}\right)^2}{8} + C_2^{(p)} \cdot \frac{\left({}^{u}K_{II}^{(p)}\right)^2}{8} \quad \text{with} $$

$$ {}^{1}G_v = \sum_p {}^{1}G\theta_v^{(p)} \text{ and } {}^{2}G_v = \sum_p {}^{2}G\theta_v^{(p)} \quad p \in \{0,1,\ldots N\} $$

Formulation incrémentale en fluage

Décomposition du tenseur de déformation

$$\Delta\varepsilon_{ij}(t_{n+1}) = \Psi_{ijkl} \cdot \Delta\sigma_{kl}(t_{n+1}) + \tilde{\varepsilon}_{ij}(t_n)$$

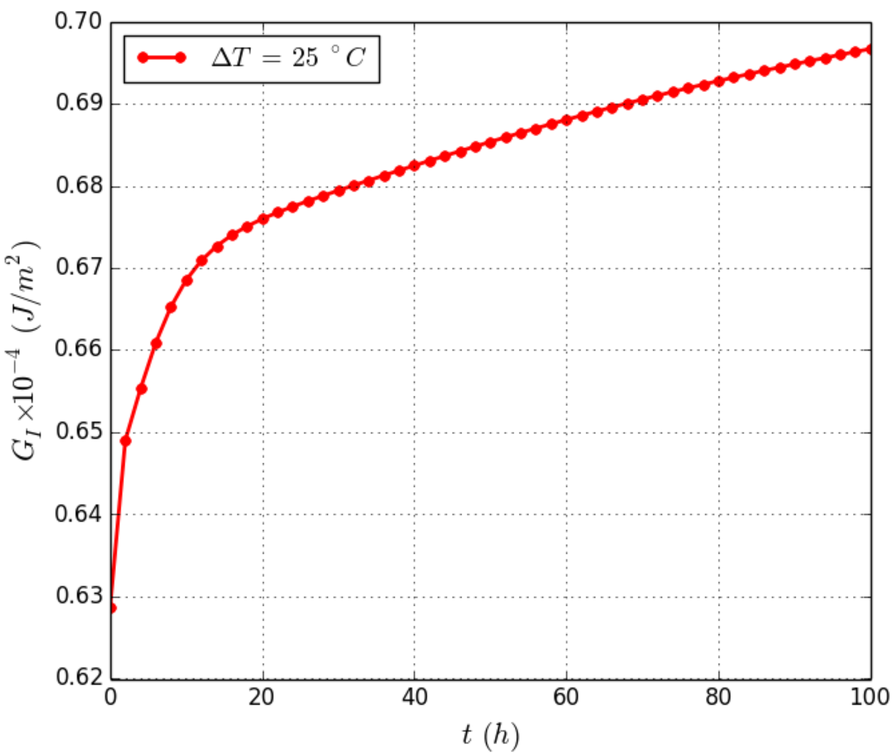
Ψ_{ijkl} → Matrice des matérieux

$\tilde{\varepsilon}_{ij}(t_n)$ → Histoire du chargement

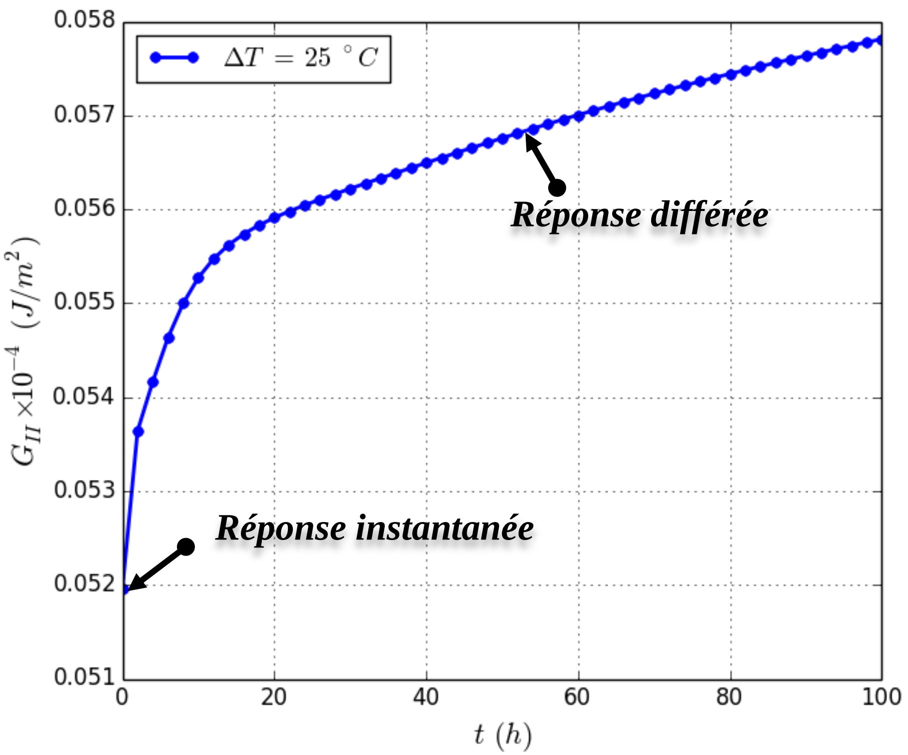
Equation d'équilibre

$$K_T^p \cdot \left\{\Delta u^p\right\}(t_n) = \left\{\Delta F_{ext}^p\right\}(t_n) + \left\{\tilde{F}^p\right\}(t_{n-1})$$

Numerical results for stress intensity factors



Opening mode



Shear mode

1. Improve the analytical formulation of T and A integrales
   a. Temperature variation effect
   b. Pressure on crack lips
   c. Crack growth process
2. Generalization for orthotropic material
3. Generalization for viscoelastic material
3. Implementation in FE software
   a. Accurate results
   b. Integration domain independency

A. Moisture variation and mechanosorptive law

B. Viscoelastic crack growth using mixed mode process zone

C. Reliability assessment (uncertainties)

Fissuration en milieux isotrope et orthotrope via les intégrales invariantes: prise en compte des effets environnementaux

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[2]CNRS, Institut Pascal, AUBIERE, France
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