1. Joint model

1.1 Introduction

When one dimension of a body becomes very large with respect to the others, it is often the case that continuum elements are replaced by structural elements (beam, plate or shell). When the dimension of one of the constituent of an heterogeneous structure becomes small with respect to the other, it is possible to replace the continuum elements by an interface or joint element (see Fig. 1). This situations occurs when modelling the thin mortar layer located between the various blocks or between blocks and masonry.

The isoparametric joint elements implemented within CASTEM 2000 follows the proposal of Beer [1]. For 2-D applications, four-node (JOI2) or six-node (JOI3) elements are available (see Fig. 1). These elements use the same type of nodal quantities as the continuum element and therefore they can be easily combined. However, while for continuum elements the relevant quantities at the Gauss integration points are the component of local stress tensor σ, the relevant quantities for the joint element are the components of the local stress vector $\boldsymbol{T}$ acting on the interface. A constitutive law for the shear component S and the normal component N of this vector should be provided. It depends on δ and γ, the opening and the sliding components of the displacement jump $[\boldsymbol{u}]$ across the joint (see Fig. 2). Two constitutive laws of this kind will be presented in the following sections.

Remark. Joint elements have been also widely used for studying situation involving contact with friction.

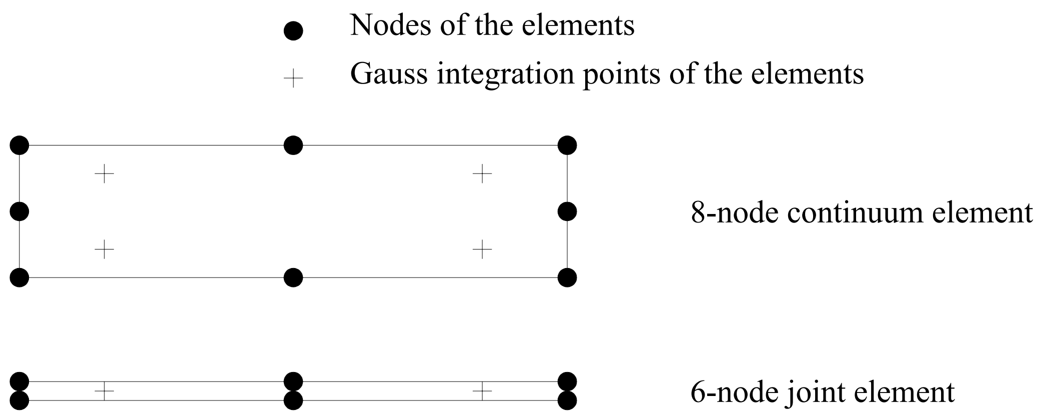

Figure 1 - Degeneration of a continuum element into a joint element.

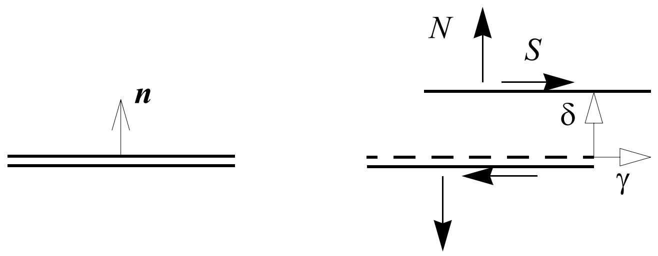

Figure 2 - Relevant quantities at the Gauss point of a joint element.

1.2 Elastic-perfectly plastic dilatant joint model

A simple elasto-plastic Coulomb friction law with a small dilatancy (a slight variant with cohesion of the constitutive law proposed by Snyman et Al. [2]) has been implemented in Castem 2000 by the first author.

The joint behaves elastically inside the domain (see Fig. 3) defined by $|S| < (N_t - N)\tan\phi$, i.e.

$$\begin{bmatrix} N \\ S \end{bmatrix} = \begin{bmatrix} k_n & 0 \\ 0 & k_s \end{bmatrix} \begin{bmatrix} \delta \\ \gamma \end{bmatrix} \tag{1}$$

A non-associated plastic flow rule defined by the angle μ is adopted when the stress vector lies on the edges of the domain. A particular treatment of the vertex is introduced in order to remove the uncertainty of the plastic flow direction at this point (see [2]).

The model is characterized by five constants:

- k_n the compression elastic modulus,
- k_s the shear elastic modulus,
- N_t the maximum normal stress (traction),
- ϕ the friction angle,
- μ the dilatancy angle.

The identification of the parameters of the joint model may be performed through two elementary tests on a mortar joint with the real thickness: a monotonic uniaxial tension normal to the joint and a pure monotonic shear. Note that the result of this identification is independent of the assumption (plane stress or plane strain) because the stress and strain fields are assumed to be homogeneous in the joint. For a continuous mortar joint of thickness e obeying to a Mazars damage law [3], defined by a Young modulus E, a Poisson coefficient ν and a damage threshold ε_{d0}, one gets

$$k_n = \frac{E}{e} \qquad k_s = \frac{E}{2(1+\nu)e} \qquad N_t = E\varepsilon_{d0} \qquad \tan\phi = \frac{1}{1+\nu} \tag{2}$$

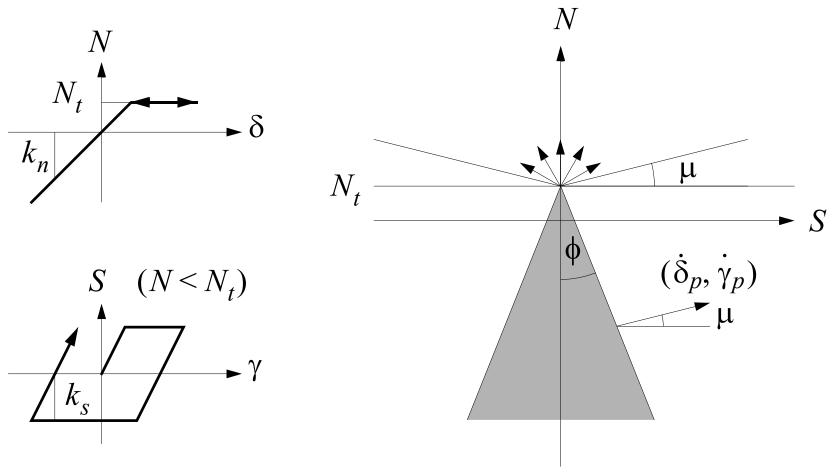

Figure 3 - The dilatant joint model.

Of course, the principle of this identification is imperfect since the responses of the two joints (the continuous and the discrete ones) still differ under any other loading history.

A small value of the dilatancy (say $\mu = 5°$) is usually introduced.

The incremental form $\Delta \boldsymbol{T} = \boldsymbol{F}(\Delta[\boldsymbol{u}])$ of the constitutive law is readily derived from [2]. Note that the consistent tangent operator of the law, namely $\partial(\Delta \boldsymbol{T})/\partial(\Delta[\boldsymbol{u}])$, is non-symmetric, unless in those unlikely situations where $\phi = \mu$.

References

[1] Beer, G. (1985) ‘An isoparametric joint/interface element for finite element analysis’, I.J.N.M.E., Vol. 21, pp. 585-600.

[2] Snyman, M.F., Bird, W.W., Martin, J.B. (1991) ‘A simple formulation of a dilatant joint element governed by Coulomb friction’, Engineering Computation, Vol. 8, pp. 215-229.

[3] Mazars, J., Pijaudier-Cabot, G. and Saouridis, C. (1991) ‘Size Effect and Continuous Damage in Cementitious Materials’. International Journal of Fracture, Vol. 51, pp. 159-73.