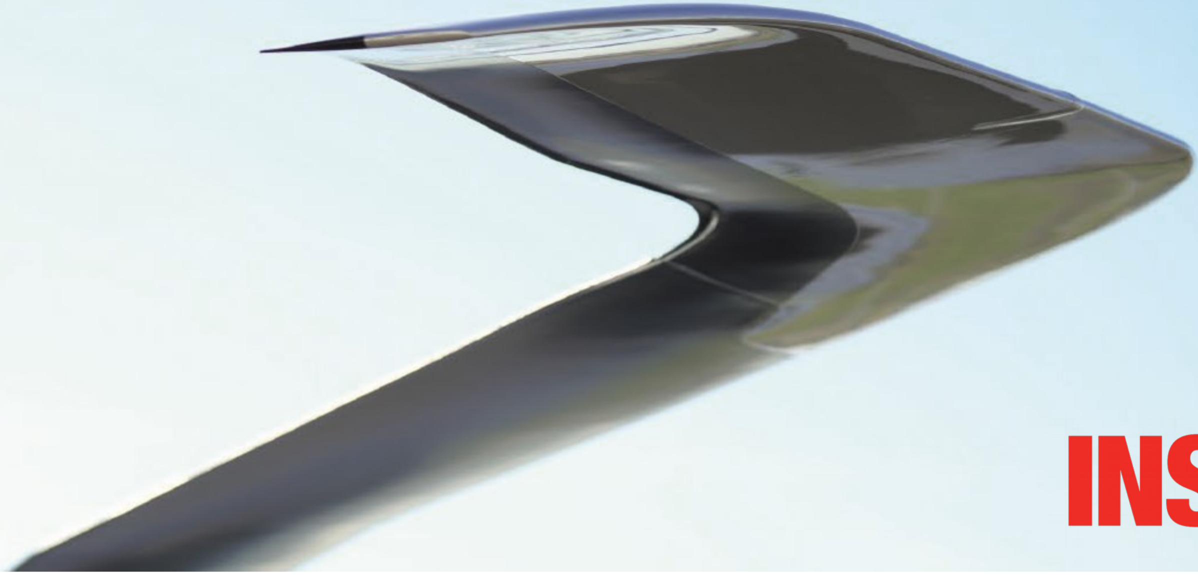
Club Cast3M 2020

November 27, 2020



HELICOPTERS

A SAM / X-FEM Coupling Approach for Numerical Simulation of Three-Dimensional Crack Propagation under Rolling Contact Fatigue

F. Meray[1,2], D. Nélias[1], A. Gravouil[1], T. Chaise[1], B. Descharrieres[2]

[1] Univ Lyon, INSA-Lyon, CNRS UMR5259, LaMCoS, F-69621, France

[2] Airbus Helicopters, Aéroport international Marseille Provence, Marignane, France

Context & Motivations

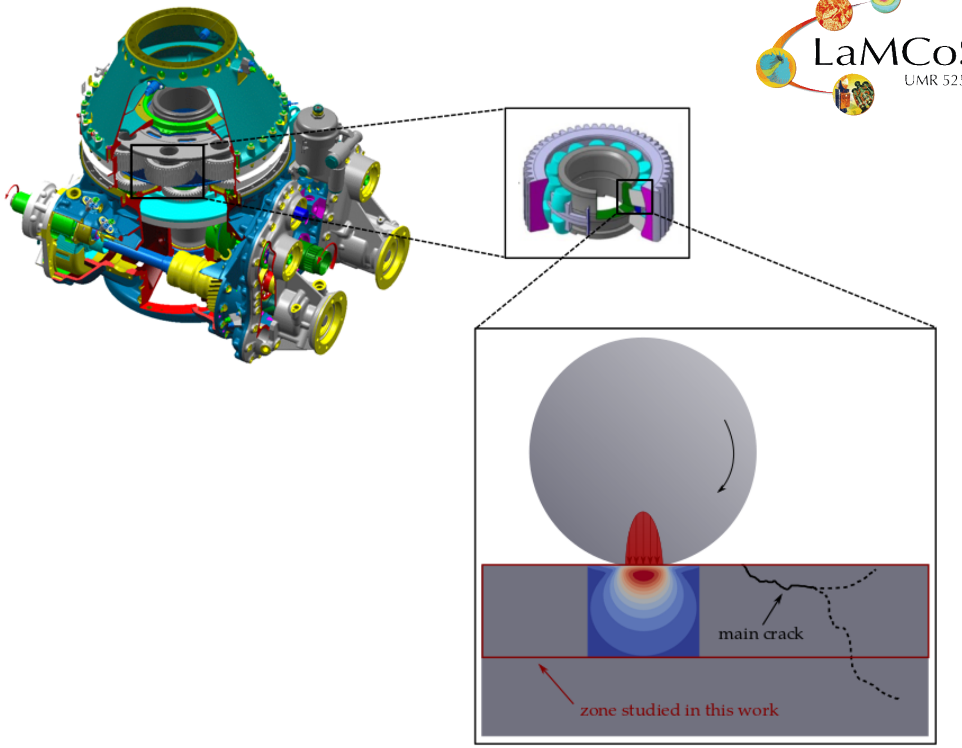
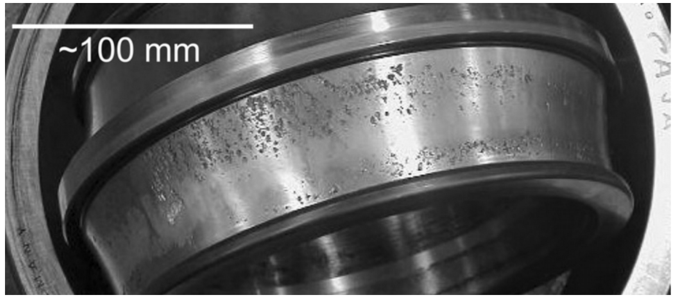


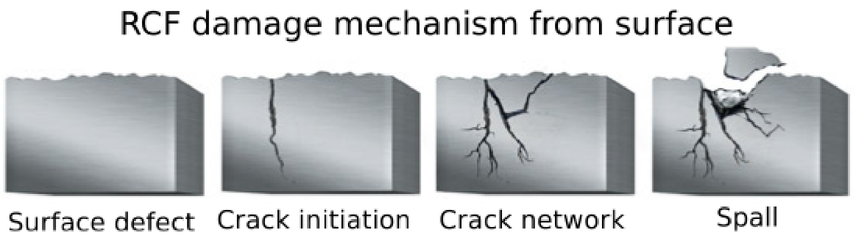
Illustration of rolling contact fatigue crack problem in planet gear

- **Work focus on the Rolling Contact Fatigue (RCF) crack growth** in mechanical components of helicopter MGB,
- **Due to the cyclic passages of rolling elements, cracks can initiate on bearing raceway and then propagate,**
- **Propagation scenarios for the main crack:**
  - Propagation towards the surface causing spalling phenomenon,
  - Deep subsurface propagation leading to complete bearing failure.



RCF of inner raceway of spherical roller bearing [OLV 05]

Spall initiated at surface [MAR 67]



Surface initiated damage from RCF [ZAR 12]

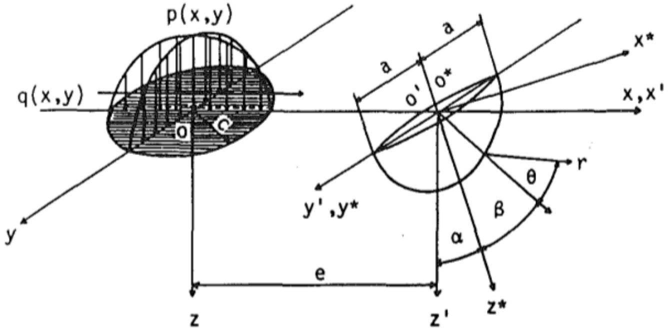
[MAR 67] Martin J. A., Eberhardt A. D. *Identification of potential failure nuclei in rolling contact fatigue.* Journal of Basic Engineering, vol. 89 (4), 1967, p. 932-942.
[OLV 05] Olver AV. *The Mechanism of Rolling Contact Fatigue: An Update*. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology, vol. 219 (5), 2005, p. 313-330.
[ZAR 12] Zaretsky E. *How to Determine Bearing System Life.* Rapport, 2012, Machinery Lubrication.

Context & Motivations

- **Main objectives:**
  - Modeling the complex 3D crack growth behavior induced by contact fatigue,
  - Predict and prevent the ruin of mechanical components such as bearings.

> ⚠ This work is focused only on the propagation stage, the crack initiation is not investigated here.

- **Actual limitations:**
  - Most of the studies are carried out with 2D models,
  - 3D simulations limited to simple configurations or/and very laborious + time consuming.
- **Proposed solution:**
  - Development of a 3D efficient numerical method based on the coupling of the Semi-Analytical Method (SAM) and the eXtented-Finite Element Method (X-FEM):
    - SAM: Resolution of the 3D contact problem,
    - X-FEM: Simulation of the 3D crack propagation with frictional contact between the crack faces.



Example of a 3D simple case studied by Kaneta [KAN 86] with the body force method

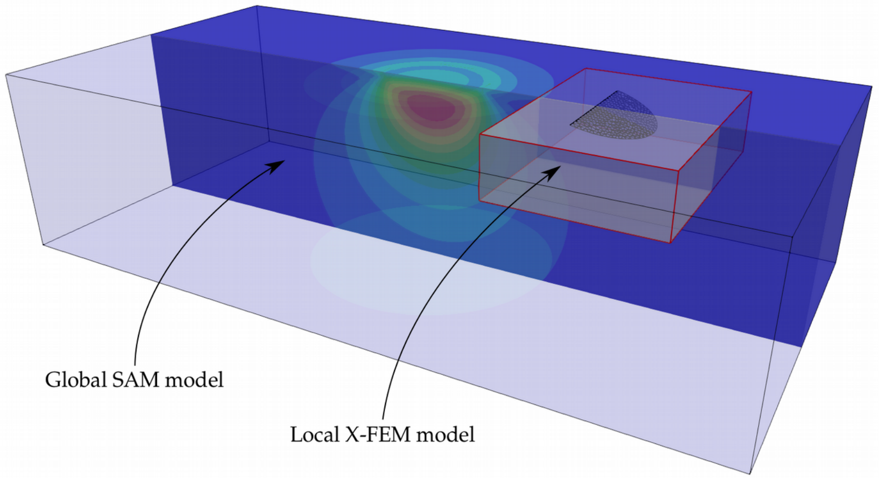


Illustration of the proposed SAM/X-FEM approach

[KAN 86] Kaneta M., Suetsugu M., Murakami Y. *Mechanism of Surface Crack Growth in Lubricated Rolling/Sliding Spherical Contact*. ASME. J. Appl. Mech, vol. 53 (2), 1986, p. 354–360.

Outline

- **The SAM / X-FEM coupling approach**
- **Validation**
- **A Practical Example**
- **Conclusion and Perspectives**

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The SAM / X-FEM coupling approach

Semi Analytical Method (SAM)

- **Method dedicated to the resolution of 3D complex contact problem** and initially proposed by Bentall and Johnson [BEN 67] and Paul and Hashemi [PAU 81],
- **Based on the superposition of elementary analytical solutions** (Green's functions),
- **Used of Conjugate Gradient Method (CGM) and FFT techniques** [POL 99] to solving the set of equations/inequations which define the BCs at the contact interface,
- **Main assumptions:**
  - Half-space bodies,
  - Small strains and small displacements,
  - Regular mesh density along each direction.

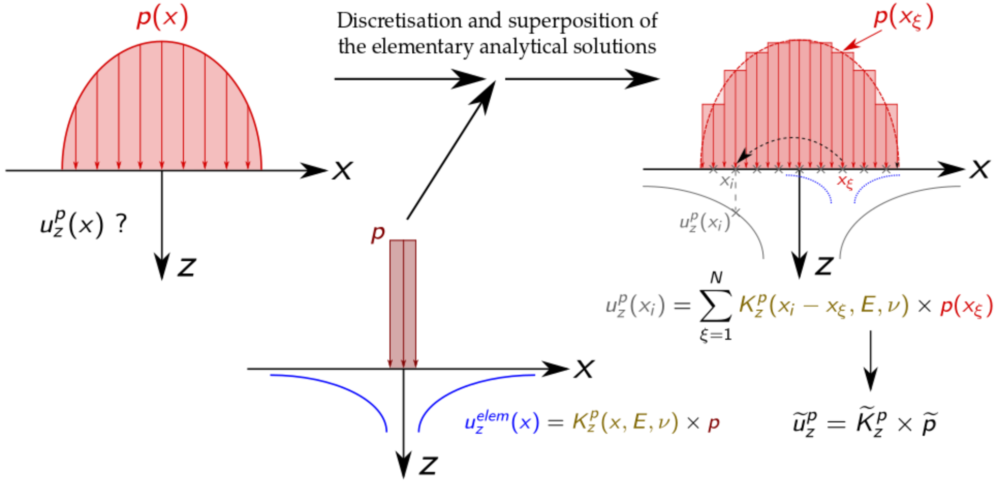
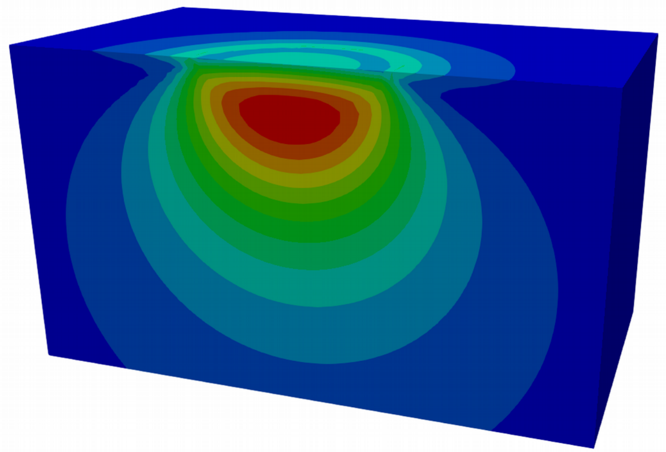


Illustration of SAM principle



Subsurface Von Mises Stresses extracted from SAM calculation

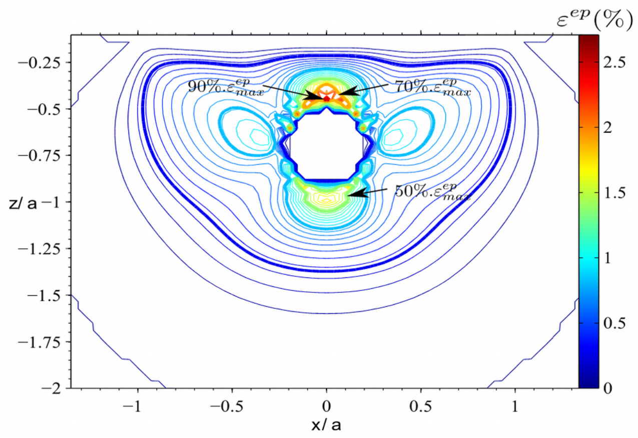
[BEN 67] Bentall R. H., Johnson K. L. *Slip in the Rolling Contact of Two Dissimilar Elastic Rollers*. International Journal of Mechanical Science, vol. 9 (6), 1967, p. 389-404.
[PAU 81] Paul B., Hashemi J. *Contact Pressure on Closely Conforming Elastic Bodies*. ASME Journal of Applied Mechanics, vol. 48 (3), 1981, p. 543-548.
[POL 99] Polonsky I. A., Keer L. M. *A Numerical Method for Solving Rough Contact Problems Based on the Multi-Level Multi-Summation and Conjugate Gradient Techniques*. Wear, vol. 231 (2), 1999, p. 206-219.

The SAM / X-FEM coupling approach
Semi Analytical Method (SAM)

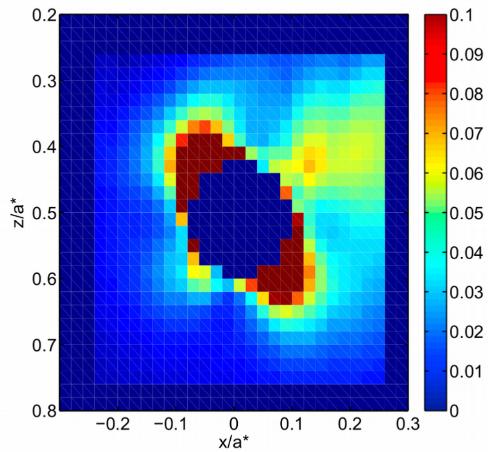
- **Main advantages:**
  - Shorter CPU time compared to FEM,
  - Better accuracy for a given mesh size,
  - Easier pre-processing.

- **Implemented in a code named ISAAC®** and developed by the group of Prof D. Nélias at the LaMCoS laboratory,

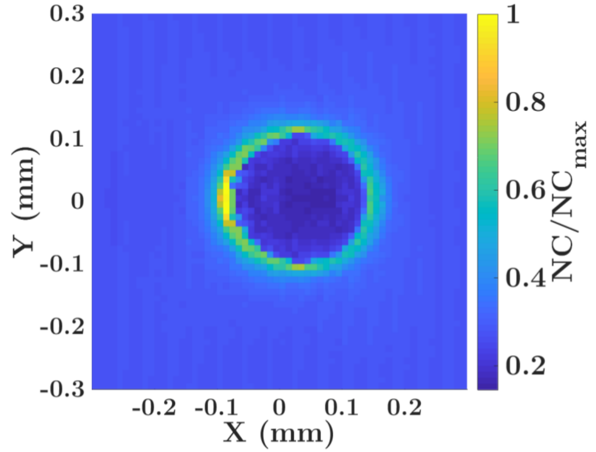
- **Capabilities of ISAAC code:**
  - Resolution of normal and tangential contact,
  - Computation of subsurface stresses,
  - Modeling of elastoplastic/viscoelastic behavior,
  - Taking into account heterogeneity/inclusion,
  - Calculation of surface wear,
  - Modeling of material properties gradient, of material anisotropy, of surface coating.



Plastic strain distribution around stiff heterogeneity [AMU 16]



Modeling of butterfly wing formation around a stiff inclusion [BEY 19]



Simulation of crack nucleation around a dent [BON 20]

[AMU 16] Amuzuga K. V., Chaise T., Duval A., Nelias D. *Fully Coupled Resolution of Heterogeneous Elastic–Plastic Contact Problem*. ASME. J. Tribol, 2016, vol. 138 (2): 021403.
[BEY 19] Beyer T., Sadeghi F., Chaise T., Leroux J., Nélias D. *A coupled damage model and a semi-analytical contact solver to simulate butterfly wing formation around nonmetallic inclusions*. International Journal of Fatigue, vol. 127, 2019, p. 445-460.
[BON 20] Bonetto A. *Étude de l'indentation et de la fatigue des contacts roulants*. PhD thesis, INSA de Lyon. 2020.

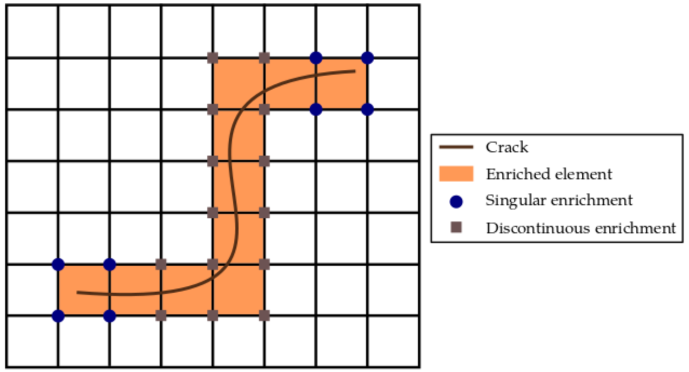
The SAM / X-FEM coupling approach

eXtended-Finite Element Method (X-FEM)

- **Method dedicated to the 2D/3D crack growth simulation** and initially introduced by Moës et al. [MOË 99],
- **Based on the addition of local enrichment functions** into the classical FE displacement approximation (local partition of unity):

$$\underline{u}(x) = \sum_{i \in N} N_i(x) u_i + \boxed{\sum_{i \in N^e} N_i(x) \Phi(x) u_i^e}$$

- **Usually coupled with the level set technique** to handle arbitrary 3D non planar cracks [MOË 02],
- **Main advantages:**
  - Crack / X-FEM mesh decoupling (easier pre-processing),
  - Better accuracy than FEM for a given mesh size,
  - Re-meshing not required during crack propagation.



Enrichment strategy of X-FEM

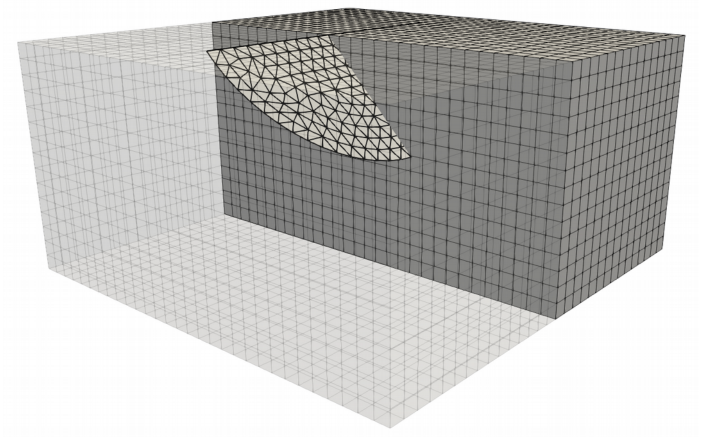


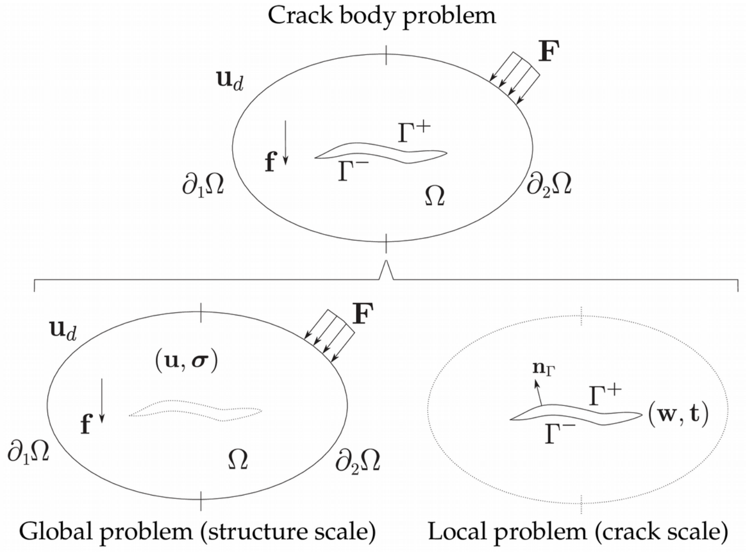
Illustration of X-FEM mesh independence

[MOË 99] Moës N., Dolbow J., Belytschko T. *A finite element method for crack growth without remeshing*. International Journal for Numerical Methods in Engineering, vol. 46, 1999, p. 131–150.
[MOË 02] Moës N., Gravouil A., Belytschko T. *Non-planar 3D crack growth by the extended finite element and level sets - Part I : Mechanical model*. International Journal for Numerical Methods in Engineering, vol.53, 2002, p. 2549–2568.

The SAM / X-FEM coupling approach

3D X-FEM frictional contact crack model

- **Model based on a two scales strategy (structure / crack),**



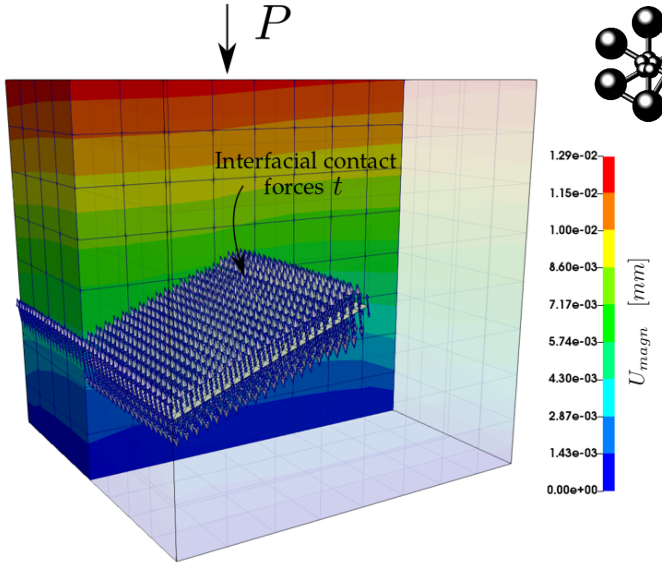
- **Introduction of a three-field weak formulation:**

$$0 = -\int_\Omega \sigma : \varepsilon(u^*) d\Omega + \int_\Omega f.u^* d\Omega + \int_{\partial_2\Omega} F.u^* dS + \int_\Gamma \lambda.u^* dS$$
$$+ \int_\Gamma (t - \lambda).w^* dS$$
$$+ \int_\Gamma (u - w).\lambda^* dS$$

Weak coupling between u et w

+ constitutive law in bulk
+ frictional contact law at the crack interface

- **Discretization of the three-field weak formulation within a X-FEM framework and resolution with the incremental LATIN method,**
- **Method developed by the group of Prof. A. Gravouil** at LaMCoS [PIE 10, GRA 11] and **implemented in the Cast3M code** by B. Trollé [TRO 14].



[PIE 10] Pierres E., Baietto M.-C., Gravouil A. *A two-scale extended finite element method for modeling 3D crack growth with interfacial contact.* Computer Methods in Applied Mechanics and Engineering, vol. 199, 2010, p. 1165-1177.
[GRA 11] Gravouil A., Pierres E., Baietto M.-C. *Stabilized global–local X-FEM for 3D non-planar frictional crack using relevant meshes.* Int. J. Numer. Meth. Engng., vol. 88, 2011, p. 1449-1475.
[TRO 14] Trollé B. *Simulation multi-échelles de la propagation des fissures de fatigue dans les rails*. PhD thesis, INSA de Lyon, 2014.

The SAM / X-FEM coupling approach
Main idea

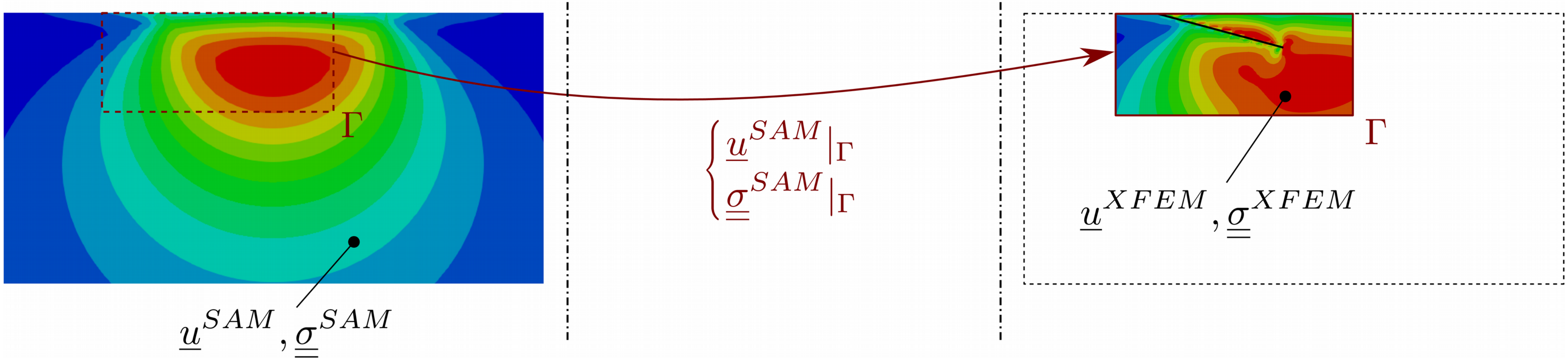
- **Hypothesis:**
  - Frictional crack problem has no significant influence on the contact problem.

- **Development of a two scales SAM / X-FEM strategy based on a submodeling technique [COR 99] (or top-down approach):**

1 - Global Analysis:
3D contact problem solving (SAM)

2 - Data Transfer:
Definition of the BCs for the local analysis on Γ

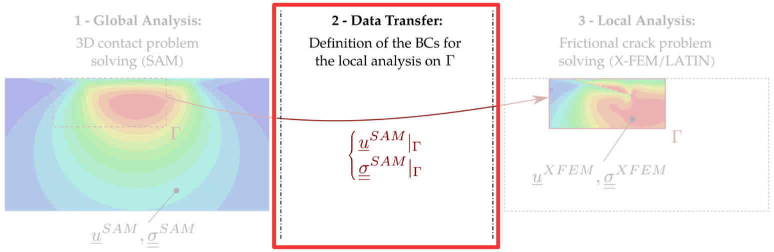
3 - Local Analysis:
Frictional crack problem solving (X-FEM/LATIN)



[COR 99] Cormier N. G., Smallwood B. S., Sinclair G. B., Meda G. *Aggressive submodelling of stress concentrations*. Int J Numer Methods Eng, vol. 46 (6), 1999, p. 889-909.

The SAM / X-FEM coupling approach

Data Transfer



- **2 quantities can be transferred from global model to local model surface boundary Γ:**
  - Displacement field,
  - Stress/surface traction field.

- **In the SAM / X-FEM framework, only the stress field is transferred** because cracks cause a stiffness change,

- **Procedure to define the force boundary conditions for the local analysis:**

1) Linear interpolation of global model stresses on Γ_e:

$$\sigma_{ij}^{\Gamma_e}(\underline{x}) = [N(\underline{x})]\,\sigma_{ij}^{SAM}$$

2) Calculation of surface traction vector on Γ_e:

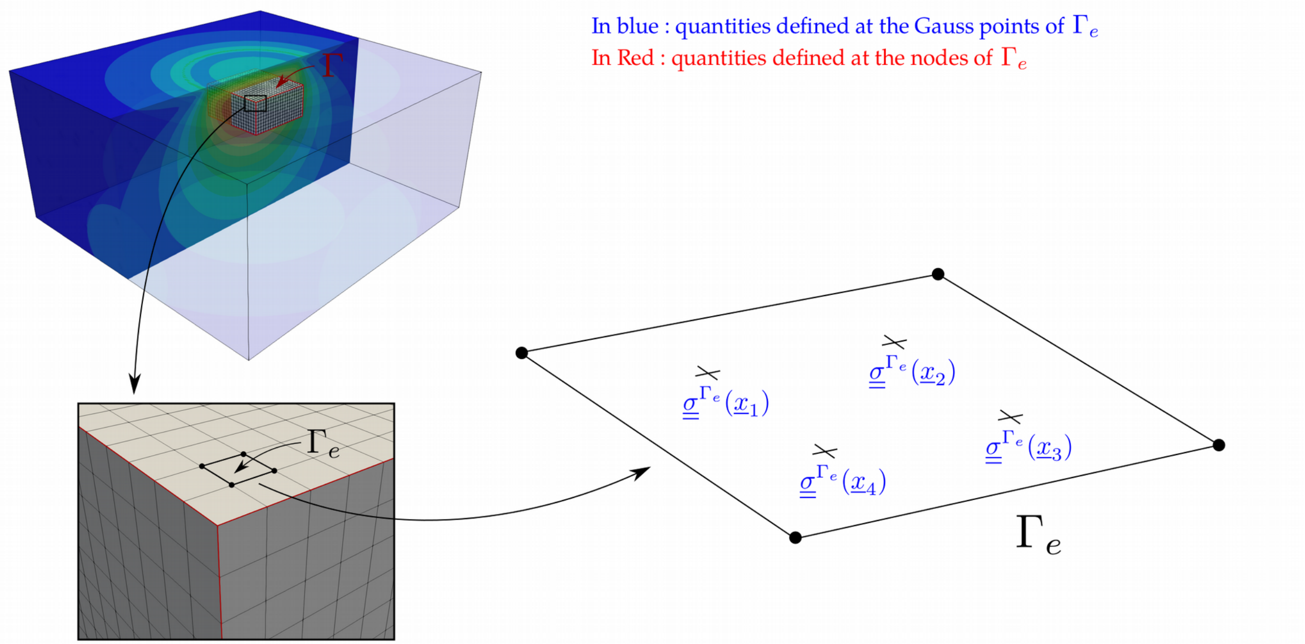
$$\underline{t}^{\Gamma_e}(\underline{x}) = \underline{\underline{\sigma}}^{\Gamma_e}(\underline{x})\,.\,\underline{n}^{\Gamma_e}(\underline{x})$$

3) Integration of surface traction vector on Γ_e:

$$\{F^{\Gamma_e}\} = \iint_{\Gamma_e} \left[N^{\Gamma_e}(\underline{x})\right]^T \{t^{\Gamma_e}(\underline{x})\}\, d\Gamma$$

4) Summation of each elementary nodal force vector:

$$\{F^{\Gamma}\} = \sum \{F^{\Gamma_e}\}$$



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The SAM / X-FEM coupling approach

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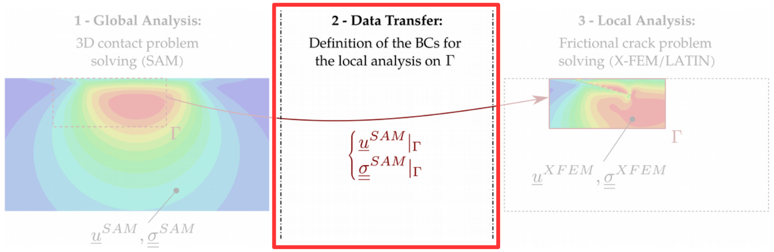

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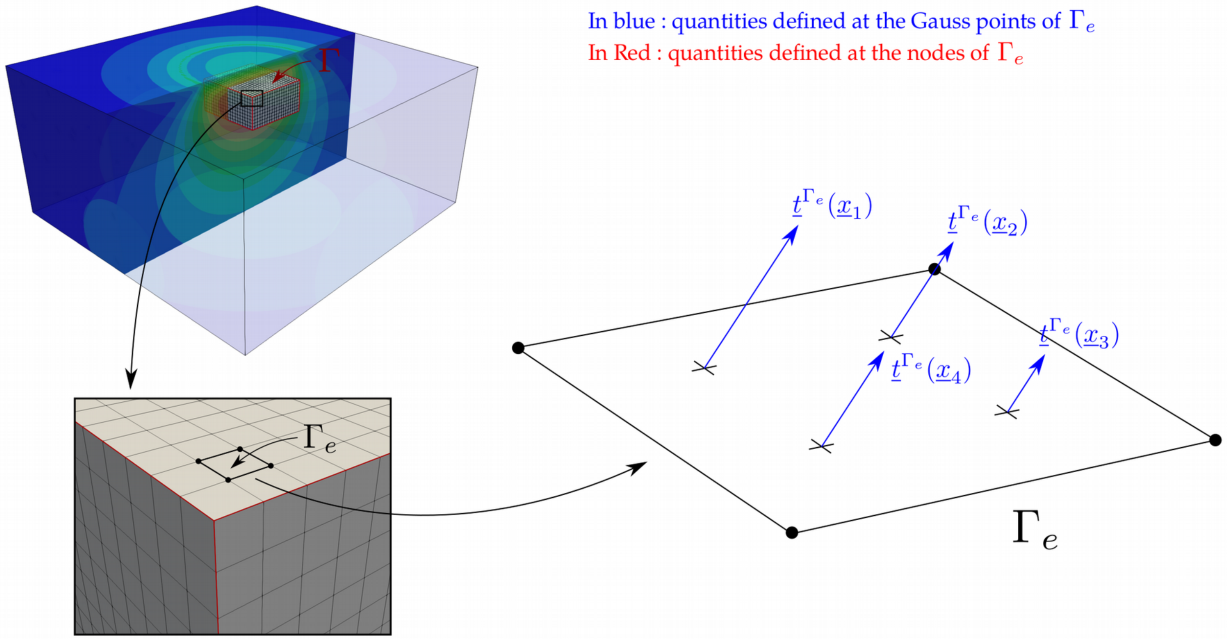

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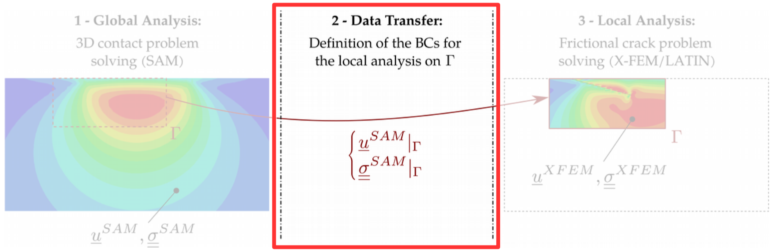

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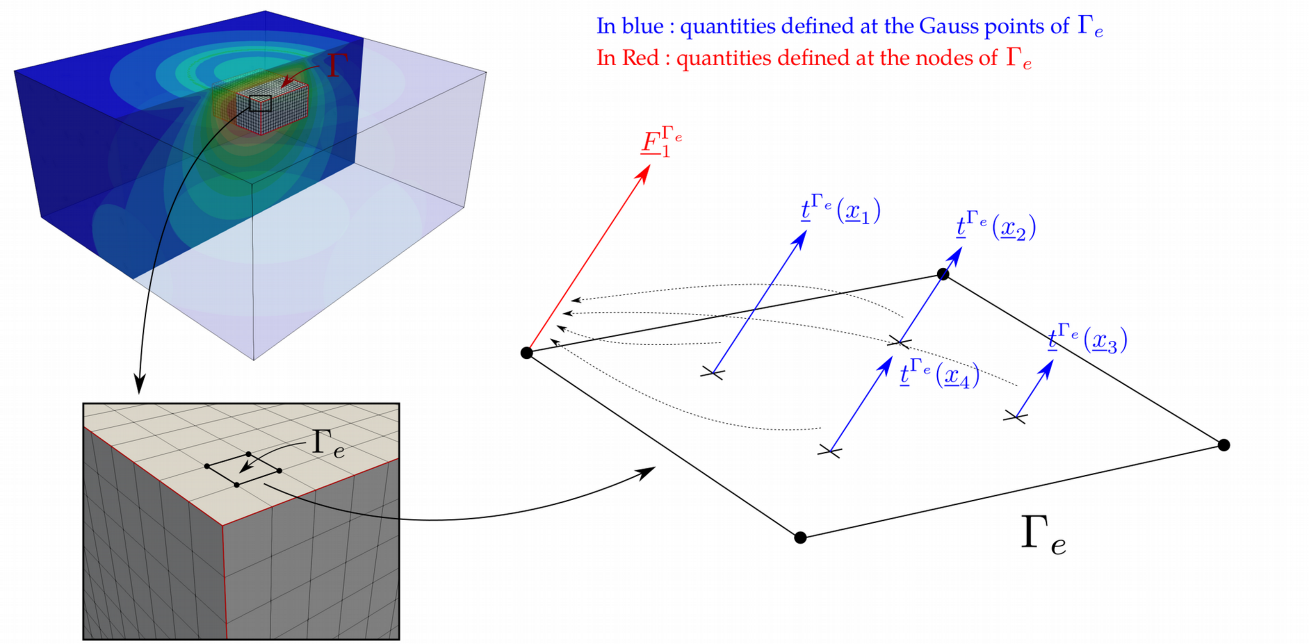
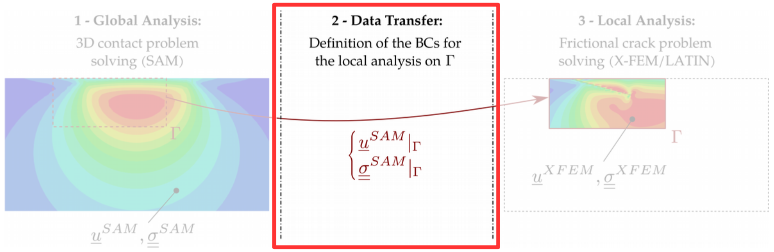


The SAM / X-FEM coupling approach

Data Transfer



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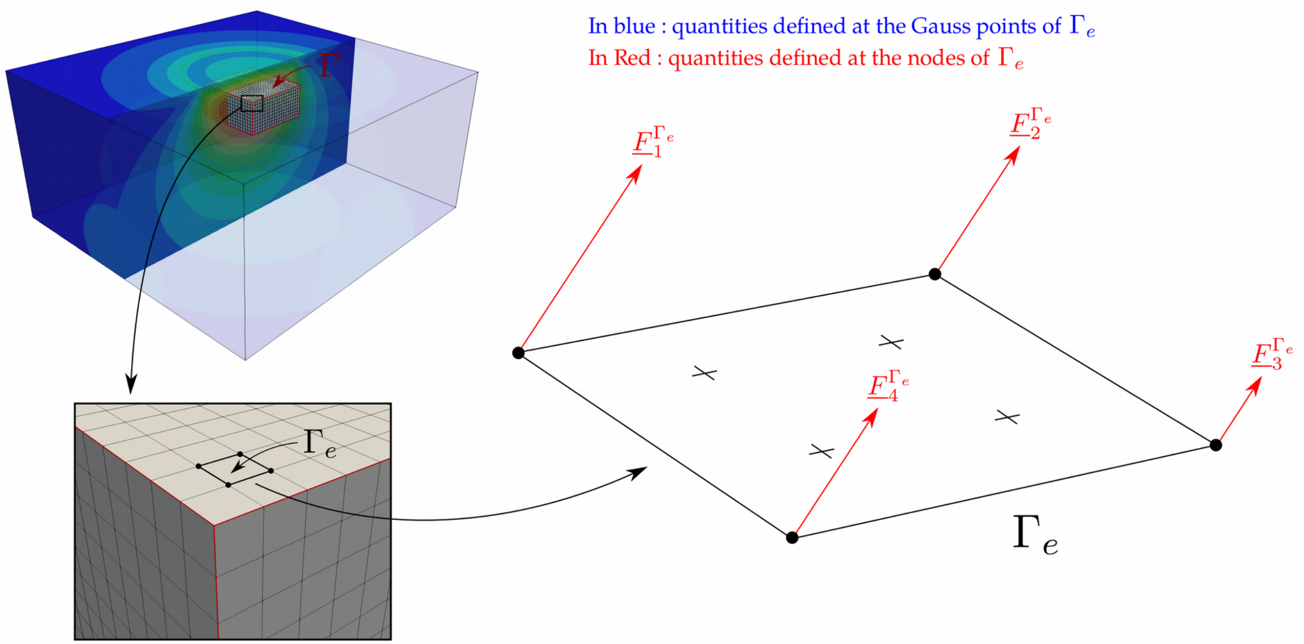
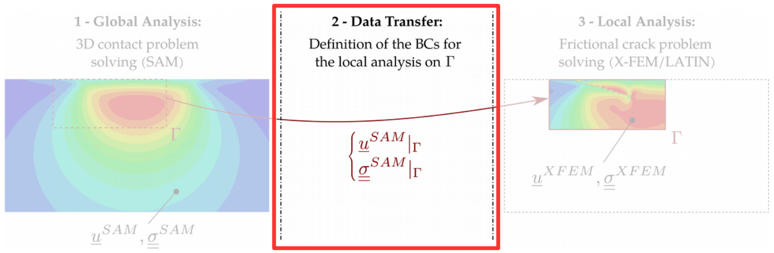


The SAM / X-FEM coupling approach
Data Transfer



- **Pure force boundary conditions exhibit rigid body motion issues** caused by unbalanced nodal forces:

$$\sum \underline{F}^{\Gamma} \neq 0$$

  - Root cause: interpolation step.

- **Introduction of an operator P to balance the nodal force vector F$^{\Gamma}$:**

$$F_p = PF^{\Gamma} = (I - R(R^T R)^{-1} R^T) F^{\Gamma} \Longrightarrow R^T F_p = 0$$

  - P enforces the orthogonality condition between the balanced force vector F_p and the rigid body modes R.

- **Initial self-balanced stresses (induced by surface hardening techniques) can be taken into account** by adding this field to the global model stresses:

$$\underline{\underline{\sigma}}^{SAM} + \underline{\underline{\sigma}}^{initial}$$

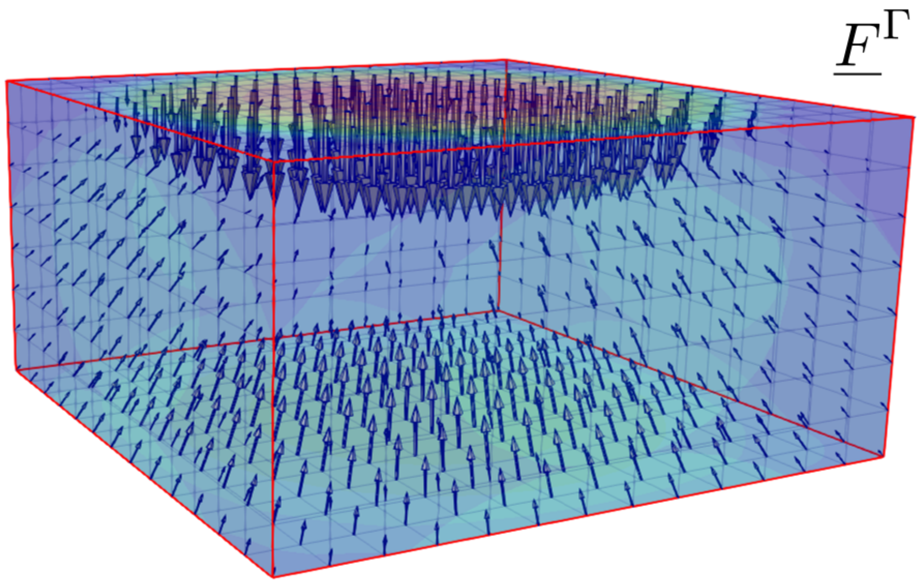
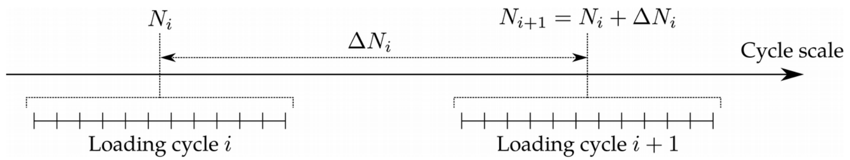


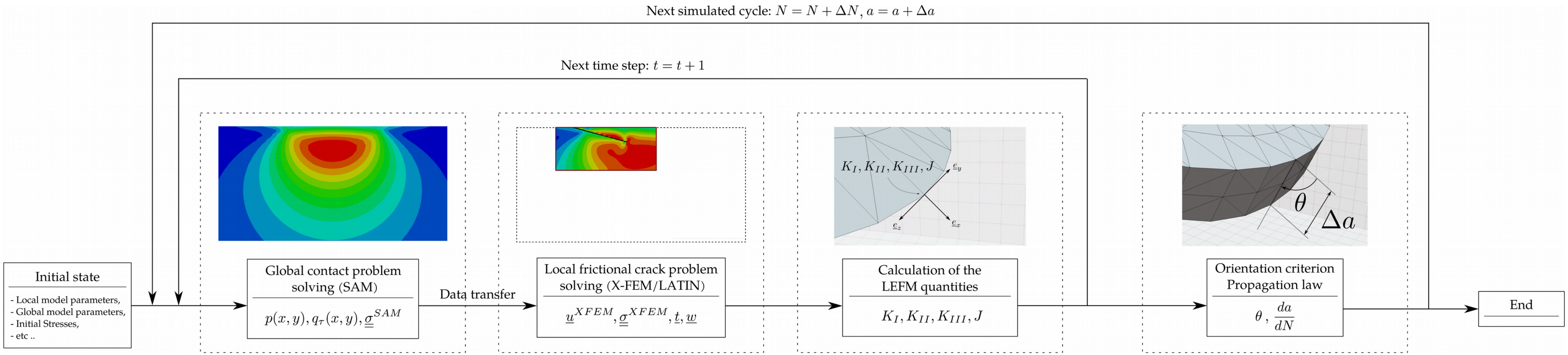
Illustration of nodal forces applied on local model surface boundary Γ

The SAM / X-FEM coupling approach

Application to crack propagation under rolling contact loading



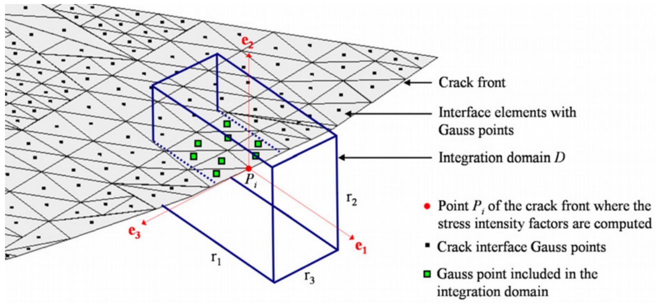
- **Algorithm for RCF crack growth simulation with the SAM / X-FEM method:**



- **Stress Intensity Factors (SIFs) calculation – 3D interaction integral [PIE 11, PRA 19]:**

$$I_h = -\int_D \left(\sigma^h_{kl}\varepsilon^{aux}_{kl}\delta_{ij} - \sigma^h_{kj}u^{aux}_{k,i} - \sigma^{aux}_{kj}u^h_{k,i}\right) q_{i,j}\, dV - \int_{\Gamma_C^+ \cup \Gamma_C^-} t^h_k u^{aux}_{k,1} q_1\, dS$$

Term accounting for contact and friction on crack faces



[PIE 11] Pierres E., Baietto M.-C., Gravouil A. *Experimental and numerical analysis of fretting crack formation based on 3D X-FEM frictional contact fatigue crack model*. Comptes Rendus Mécanique, Elsevier Masson, 2011, vol. 339 (7-8), p. 532-551.
[PRA 19] Prabel B. *Synthèse sur la méthode G-θ et son implémentation dans Cast3M*. Note Technique DEN, CEA, 2019.

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Outline

- The SAM / X-FEM coupling approach
- **Validation**
- A Practical Example
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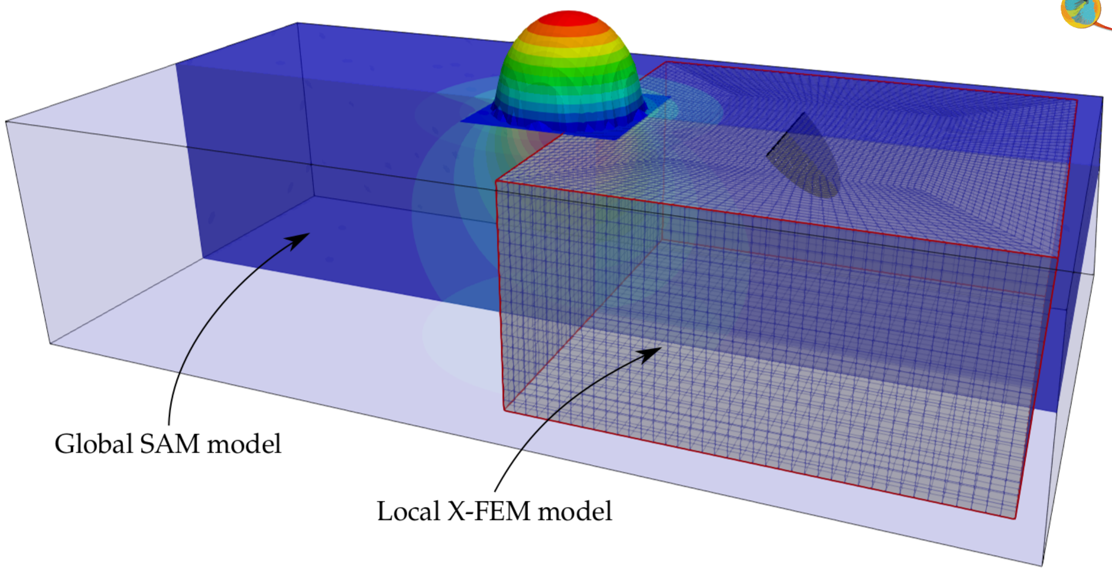
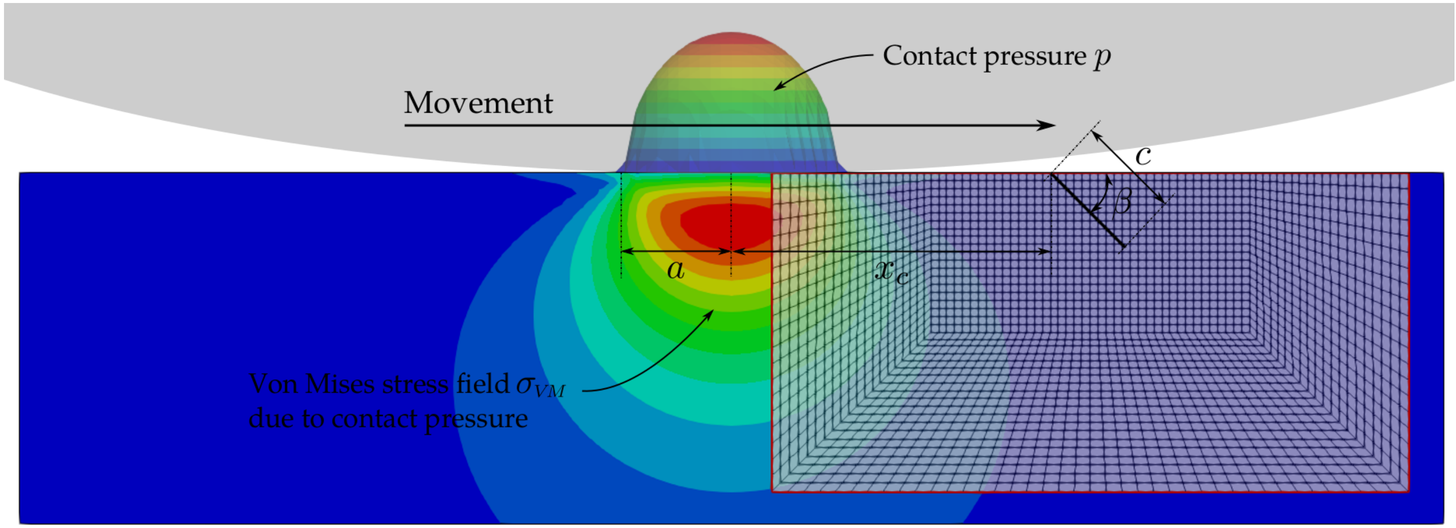
Validation

Comparison with Kaneta‘s results [KAN 86]

- **Model specifications:**
  - Semi-circular crack under spherical contact,
  - No contact between the crack faces,
  - Crack angle:
    - β = 45°
  - Contact radius / Crack radius:
    - a/c = 1.0
  - 2 contact friction coefficients:
    - f = -0.1
    - f = 0.1
  - Local model dimensions:
    - $L_x = L_y = 6c$
    - $L_z = L_x / 2$





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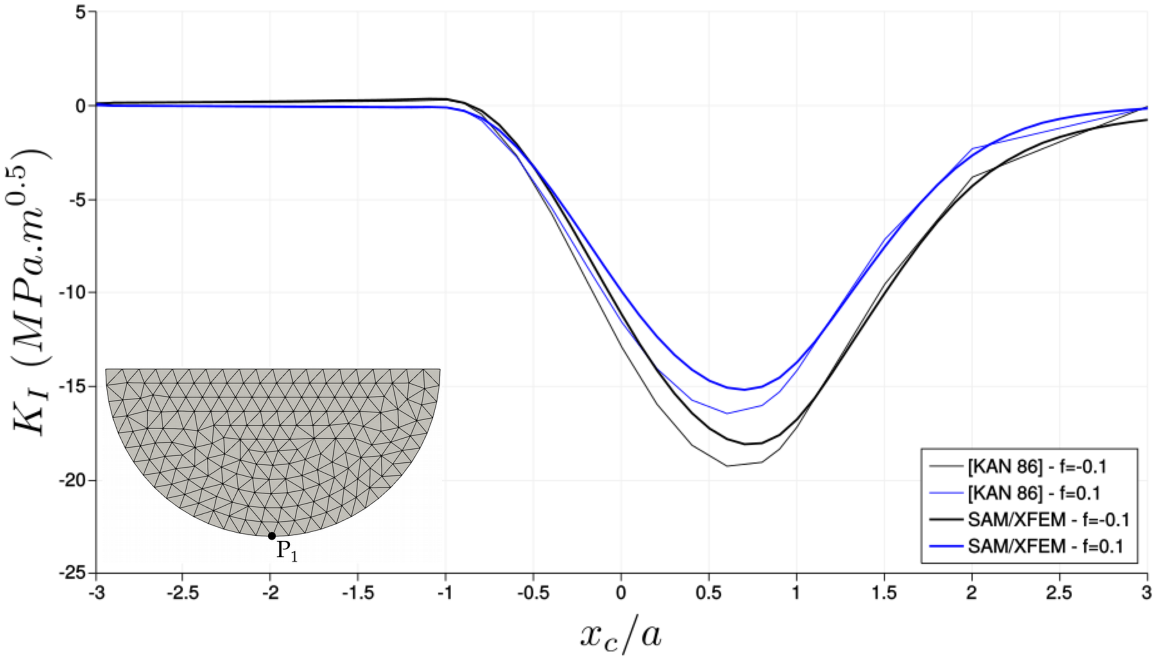
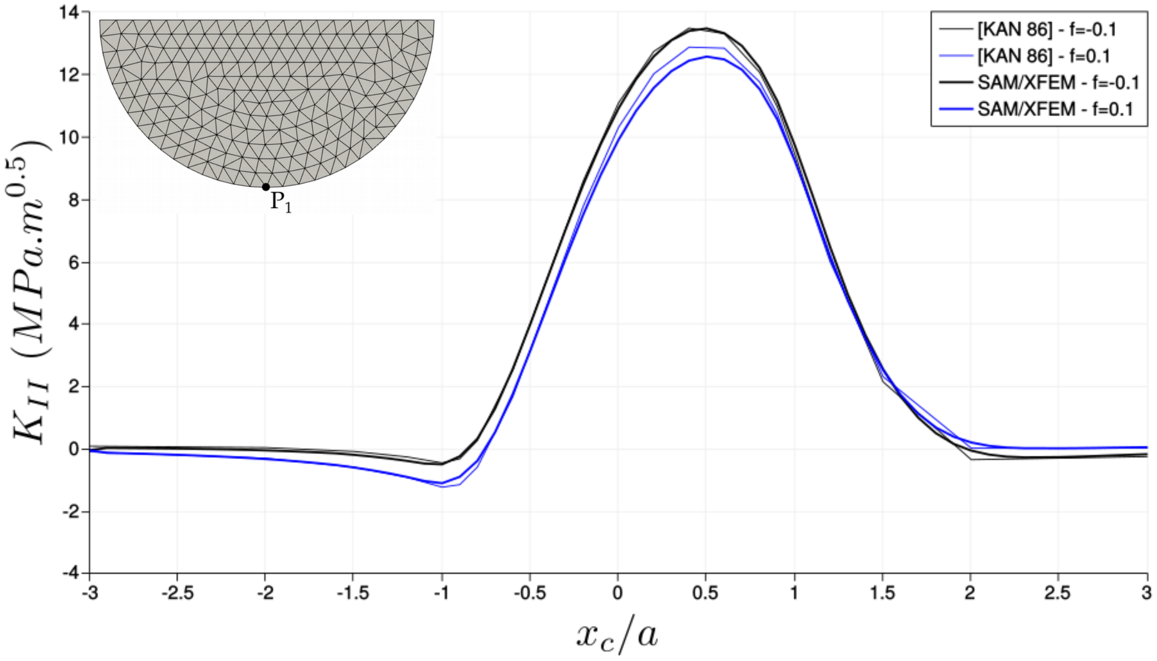
Validation

Comparison with Kaneta's results [KAN 86]

- **Results:**
  - SIFs variations at the P_1 point – a/c = 1.0

	f = -0.1		f = 0.1	
	ΔK_I ($MPa.m^{0.5}$)	ΔK_{II} ($MPa.m^{0.5}$)	ΔK_I ($MPa.m^{0.5}$)	ΔK_{II} ($MPa.m^{0.5}$)
SAM/X-FEM	18.45	13.97	15.25	13.67
[KAN 86]	19.57	13.93	16.45	14.10
Difference (%)	-5.72	+0.29	-7.29	-3.05





→ The results given by the developed SAM / X-FEM method show good agreement with the reference [KAN 86].

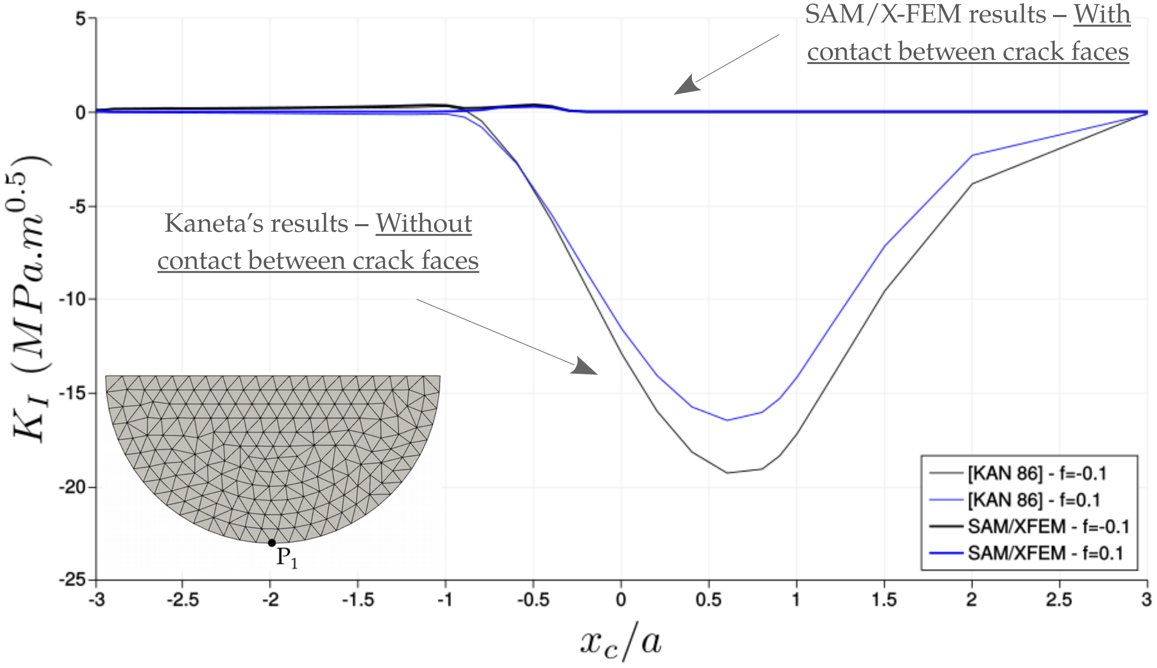
[KAN 86] Kaneta M., Suetsugu M., Murakami Y. *Mechanism of Surface Crack Growth in Lubricated Rolling/Sliding Spherical Contact*. ASME. J. Appl. Mech, vol. 53 (2), 1986, p. 354–360.

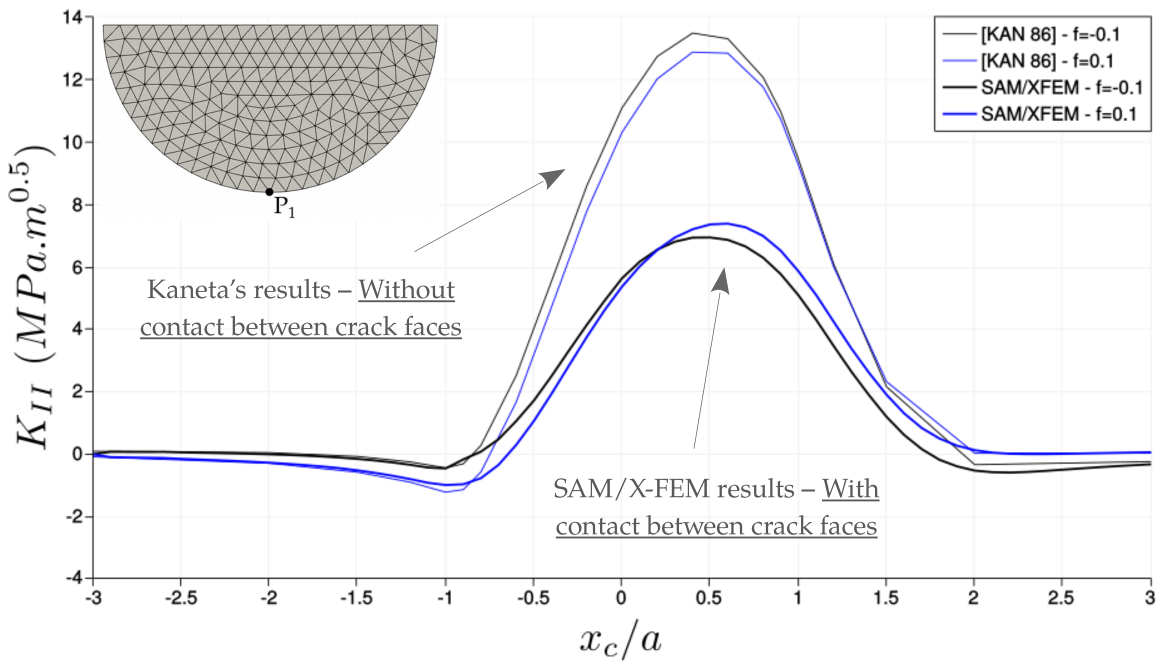
Validation

Comparison with Kaneta's results [KAN 86]

- **Consideration of the interfacial contact between the crack faces:**
  - SIFs variations at the P_1 point – a/c = 1.0 – f_c = 0

	f = -0.1	f = 0.1
	ΔK_{II} ($MPa.m^{0.5}$)	ΔK_{II} ($MPa.m^{0.5}$)
SAM/X-FEM	7.53	8.38
[KAN 86]	13.93	14.10
Difference (%)	-45.94	-40,57





→ Consideration of the interfacial contact is essential for the simulation of crack propagation under rolling contact loading.

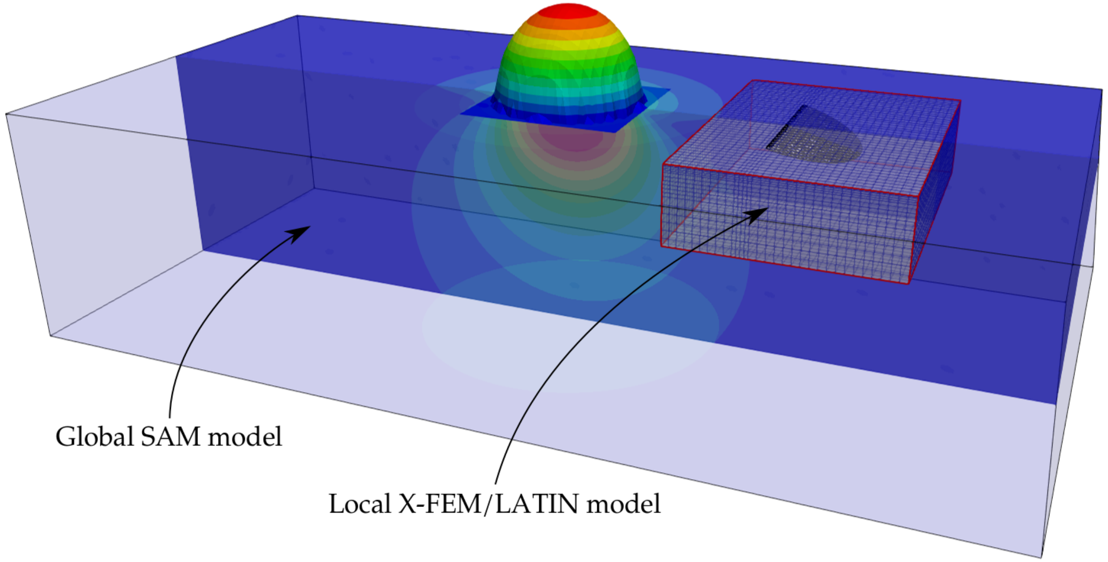
[KAN 86] Kaneta M., Suetsugu M., Murakami Y. *Mechanism of Surface Crack Growth in Lubricated Rolling/Sliding Spherical Contact*. ASME. J. Appl. Mech, vol. 53 (2), 1986, p. 354–360.

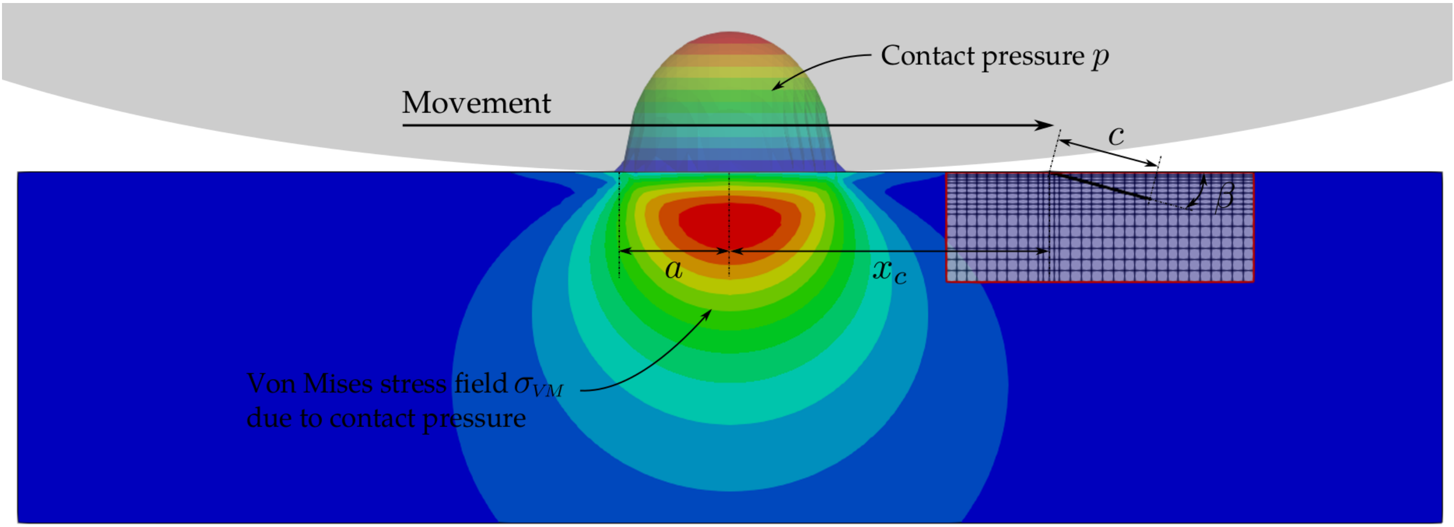
Outline

- The SAM / X-FEM coupling approach
- Validation
- A Practical Example
- Conclusion and Perspectives

A Practical Example
Crack growth under rolling contact loading

- **Model specifications:**
  - Semi-circular crack under spherical contact,
  - Frictional contact between the crack faces,
  - Contact parameters:
    - $P_0 = 1468.5$ Mpa,
    - a = 0.2 mm
    - f = 0
  - Crack parameters:
    - $\beta = 15°$
    - a/c = 1.0
    - $f_c = 0$
  - Local model dimensions:
    - $L_x = L_y = 2c$
    - $L_z = 4c.\sin(\beta)$

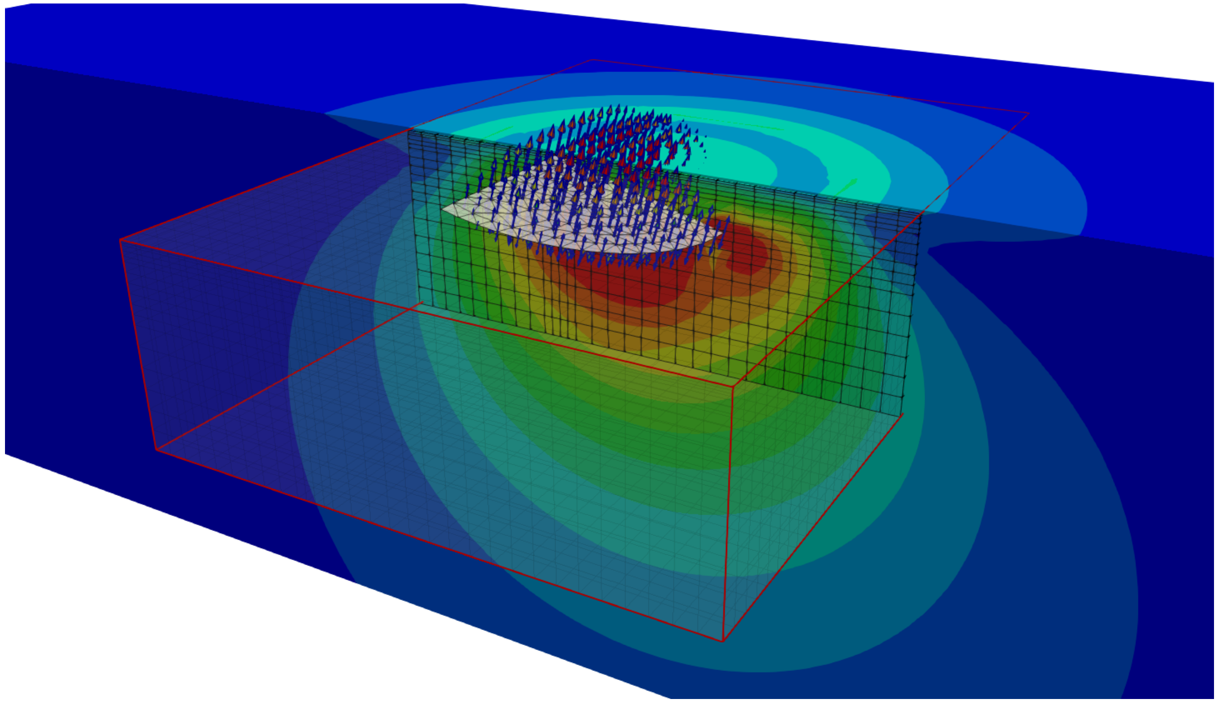


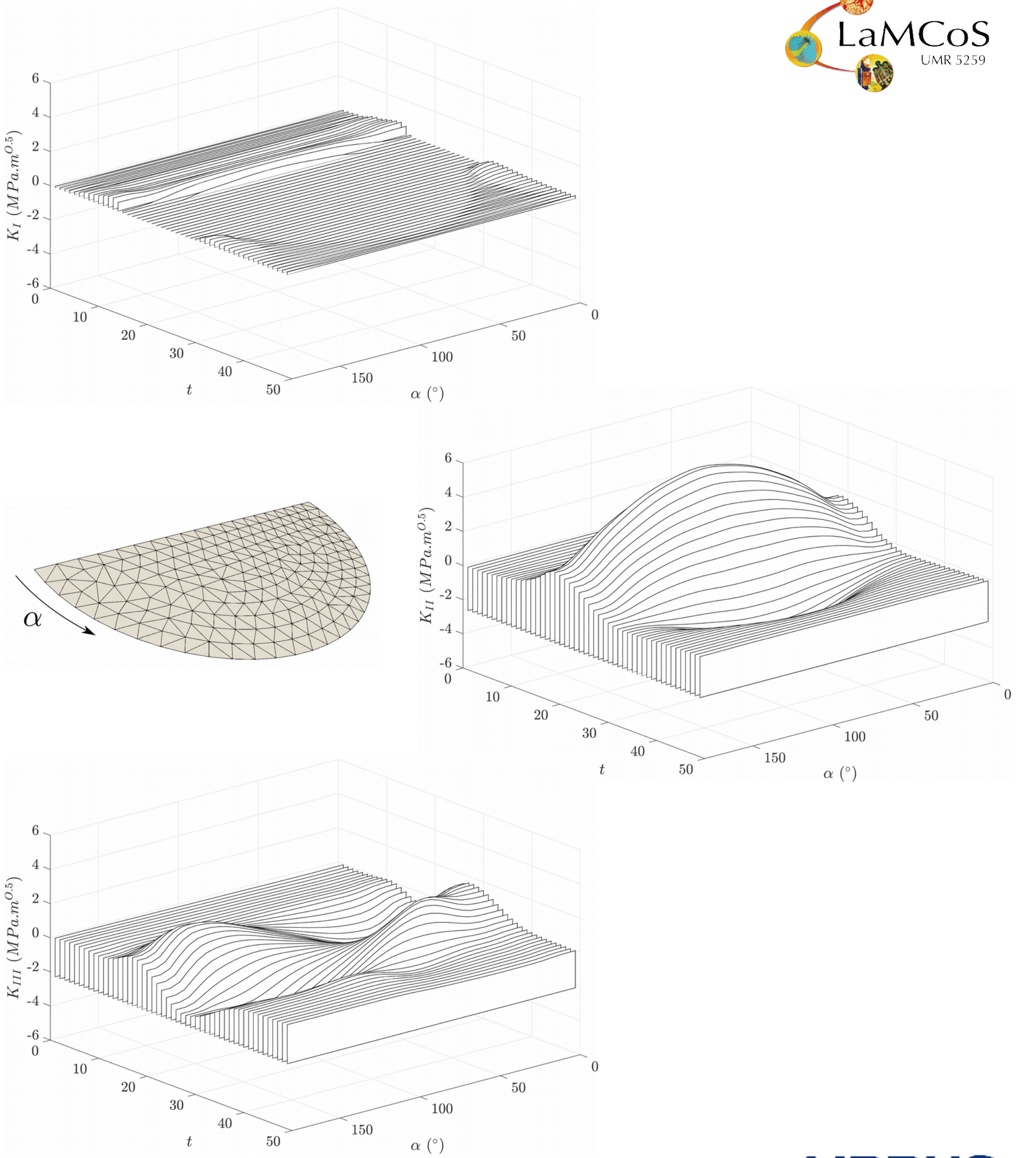


A Practical Example

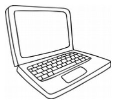
Crack growth under rolling contact loading

- **Results for the 1st loading cycle:**





CPU time: 40 min on personal laptop (49 time steps)

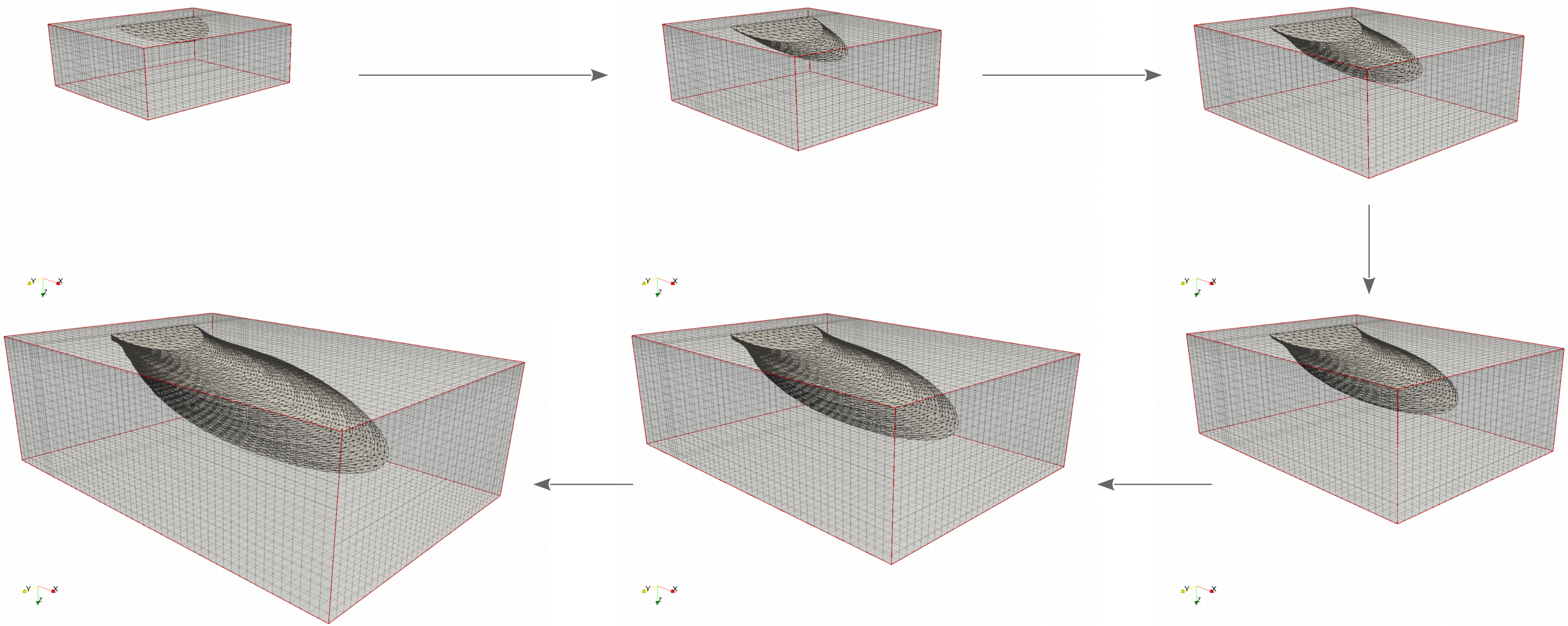


→ *RAM:* 31Go

→ *Processor:* Intel® Core™ i7-7820HQ CPU @ 2.90GHz × 8

A Practical Example
Crack growth under rolling contact loading

- **Mode II crack propagation over several cycle jumps:**



Outline

- The SAM / X-FEM coupling approach
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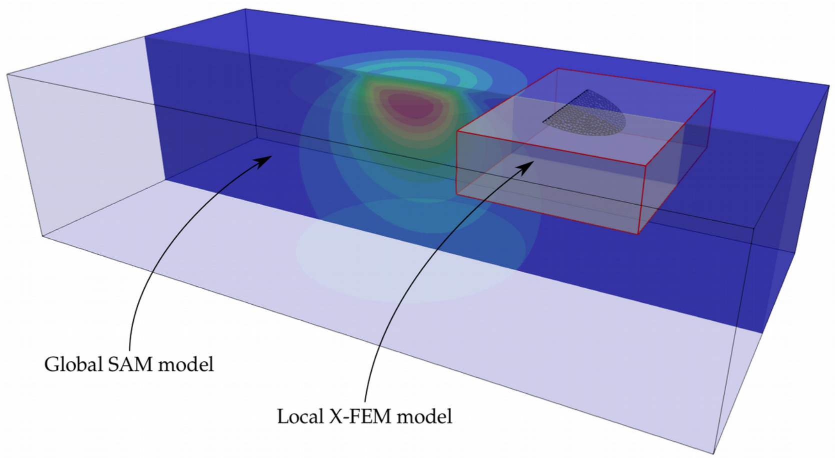
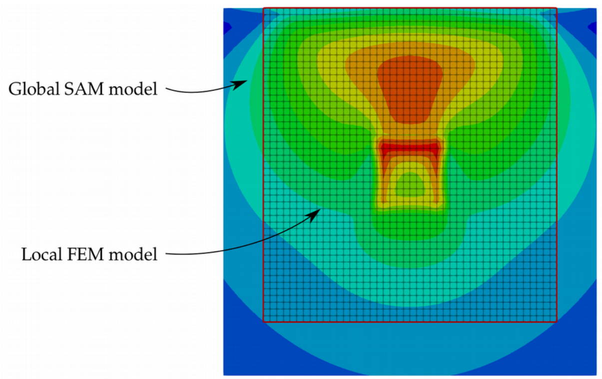
Conclusion & Perspectives

- **Key points:**
  - Development of a novel method based on a SAM / X-FEM coupling,
  - Capabilities of the developed approach:
    - 3D modeling of non-planar frictional crack propagation under contact loading,
    - Robust and accurate,
    - CPU time savings,
    - Easy pre-processing.
  - Validation with results extracted from literature.



=> Great potential of the SAM / X-FEM method to understand the complex crack growth behavior under contact loading.

- **Other potential applications:**
  - Crack propagation under fretting condition,
  - Local plasticity,
  - Heterogeneity problem,
  - …



SAM / FEM coupling applied to heterogeneity problem

Thank you for your attention