

Les performances des modèles hyperélastiques dédiés aux Caoutchoucs

*Modèles Cast3M : Mooney Rivlin, Gent, Biderman,
Arruda-Boyce, Hart-Smith, Ogden , Gornet
Desmorat*

Laurent GORNET

Maître de Conférences HDR

Plan

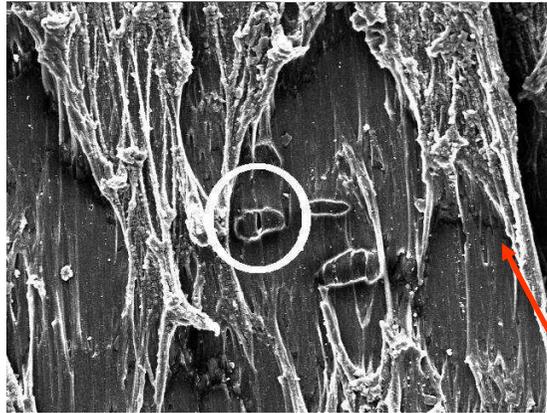
- **Contexte**
- **La Performance des modèles hyperélastiques**
 - De Mooney Rivlin à **G D**
- **Développement UMAT**
 - De la théorie à la programmation
- **Exemples de validation**
 - 2D, 3D, analytique et Abaqus
- **Conclusion**

Stratégie de recherche

- **Modélisation du comportement**
 - Essais mécaniques
 - Implantation numérique *ABAQUS, Cast3M*
- **Mécanismes microscopiques**
 - Observations au MEB des mécanismes
 - Approche micro-macro
- **Critère de fatigue**
 - Essais de fatigue
 - Elaboration de critères
- **Simulations numériques**

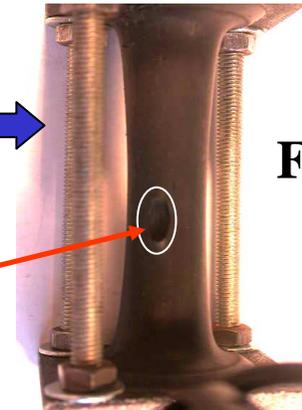
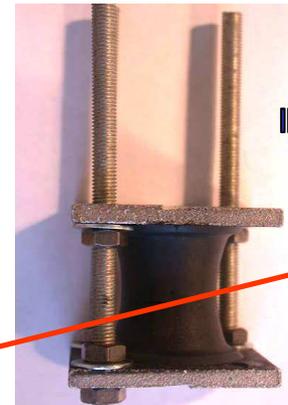
Durabilité des structures

Interaction modèle-expérience



100 μm

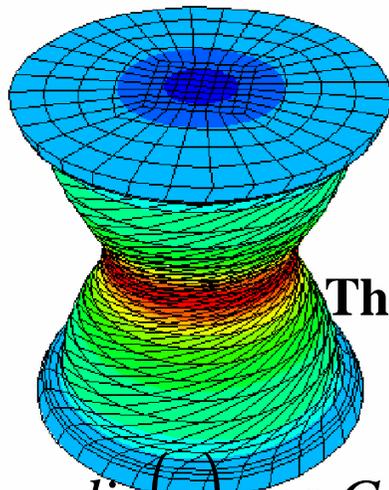
Rupture



Fatigue

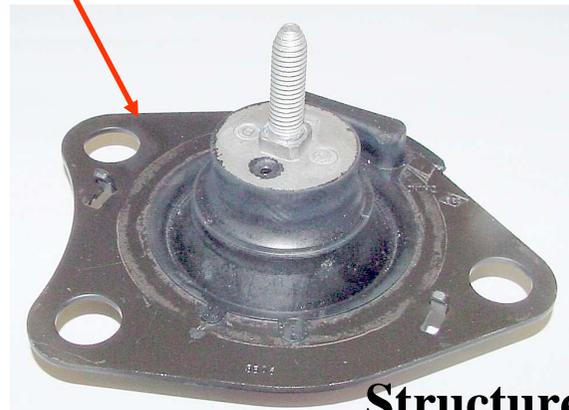


Caoutchouc : études multi-échelles

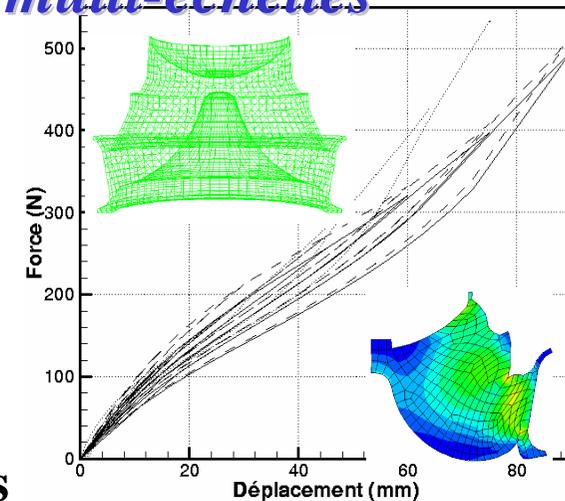


Thermique

$$r - \text{div}(\underline{q}) = \rho \cdot C_p \cdot \frac{\partial T}{\partial t}$$



Structures

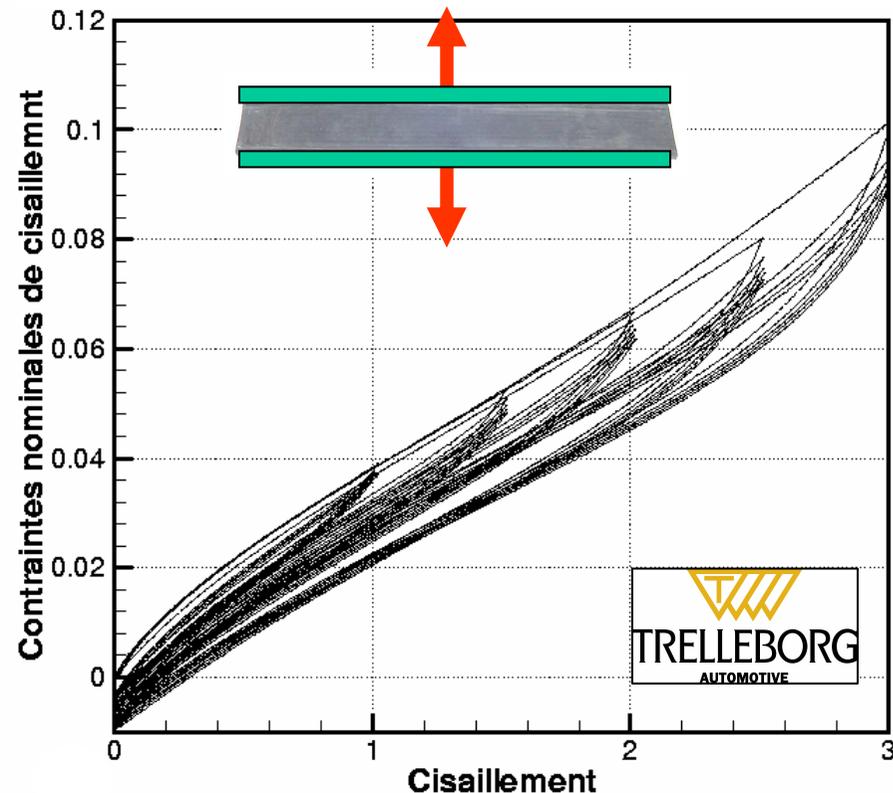
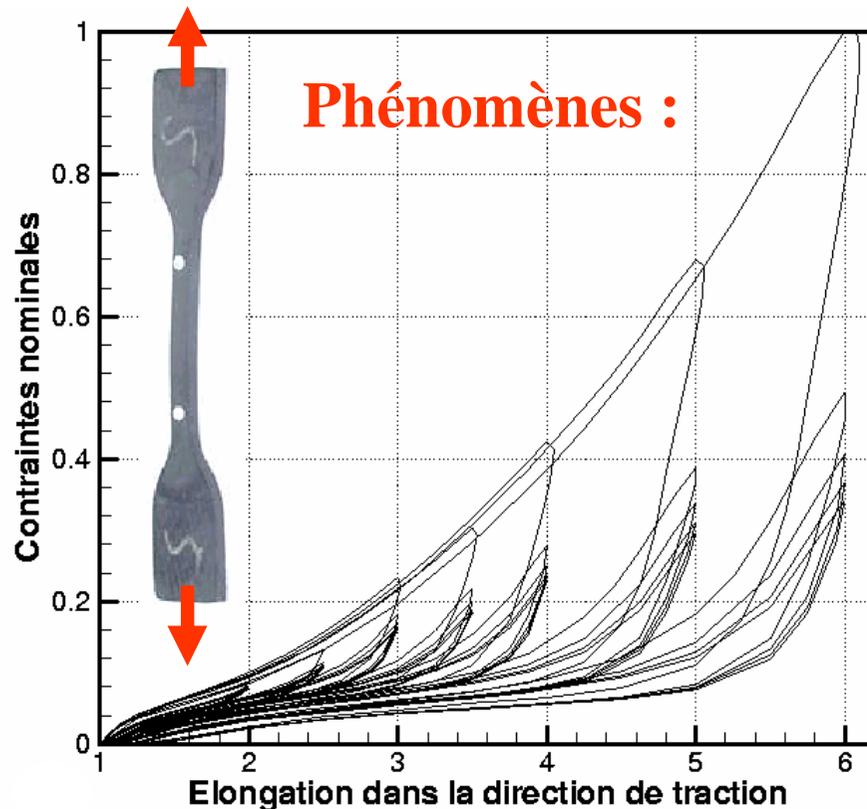


Plan

- **Contexte**
- **Performances des modèles hyperélastiques**
 - De Mooney Rivlin à GD
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 - De la théorie à la programmation
- **Exemples de validation**
 - 2D, 3D, Effet Mullins
- **Conclusion**

La physique du caoutchouc

Essais cyclés de traction et de glissement pur



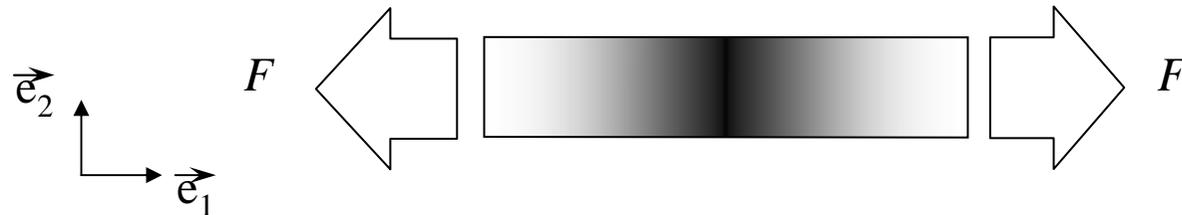
Effet Payne

Hystérésis

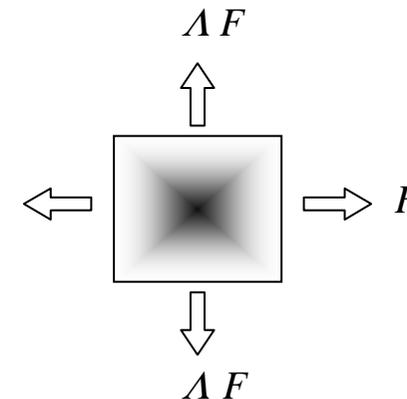
Effet Mullins

Essais Classiques

– Extension simple



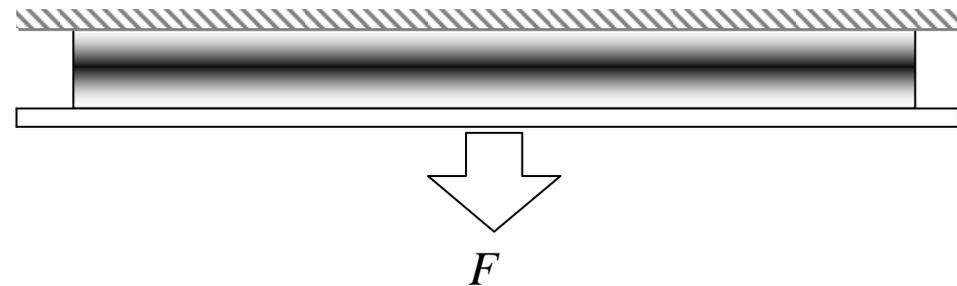
– Extension équilibiaxiale



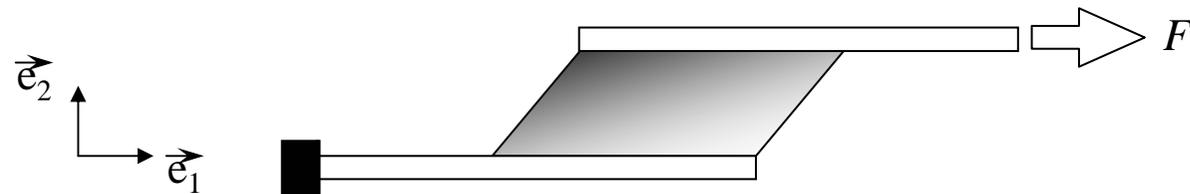
– Extension biaxiale

Essais Classiques

– Glissement pur



– Glissement simple



Algorithme Génétique
Marckmann 2002

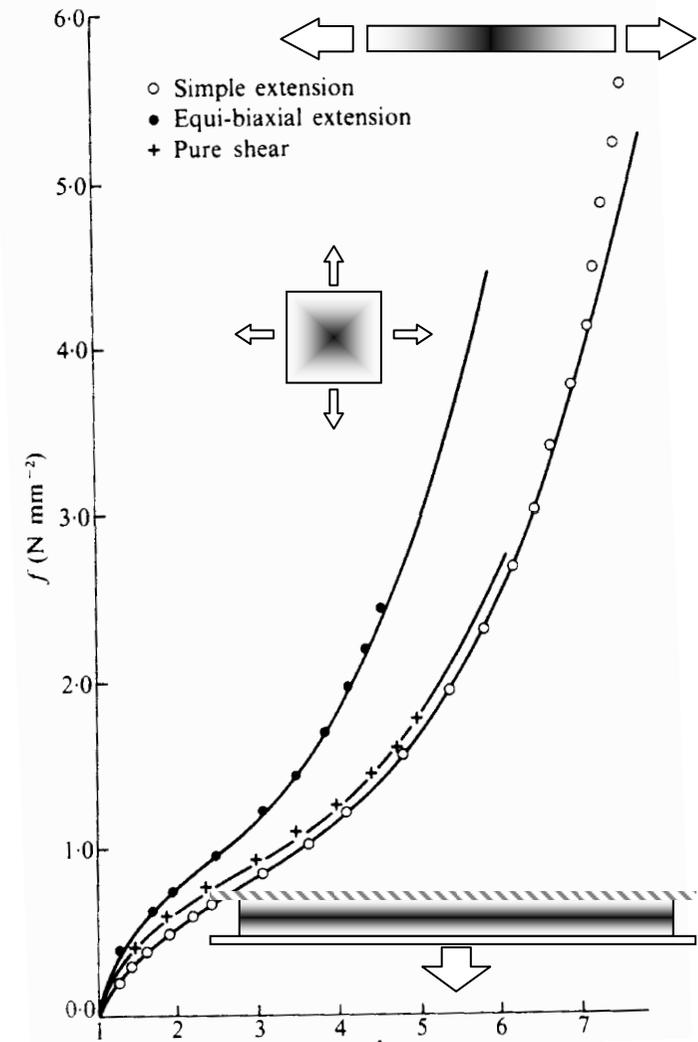
- **Treloar 1944**
 - Extension simple
 - Extension équibiaxiale
 - Compression simple
 - Glissement pur
 - Traction Equi-Biaxiale
- **Kawabata 1981**
 - Traction biaxiale

Expériences 1944

- Caoutchouc naturel vulcanisé
 - Traction simple
 - Traction Equi-Biaxiale
 - Glissement pur

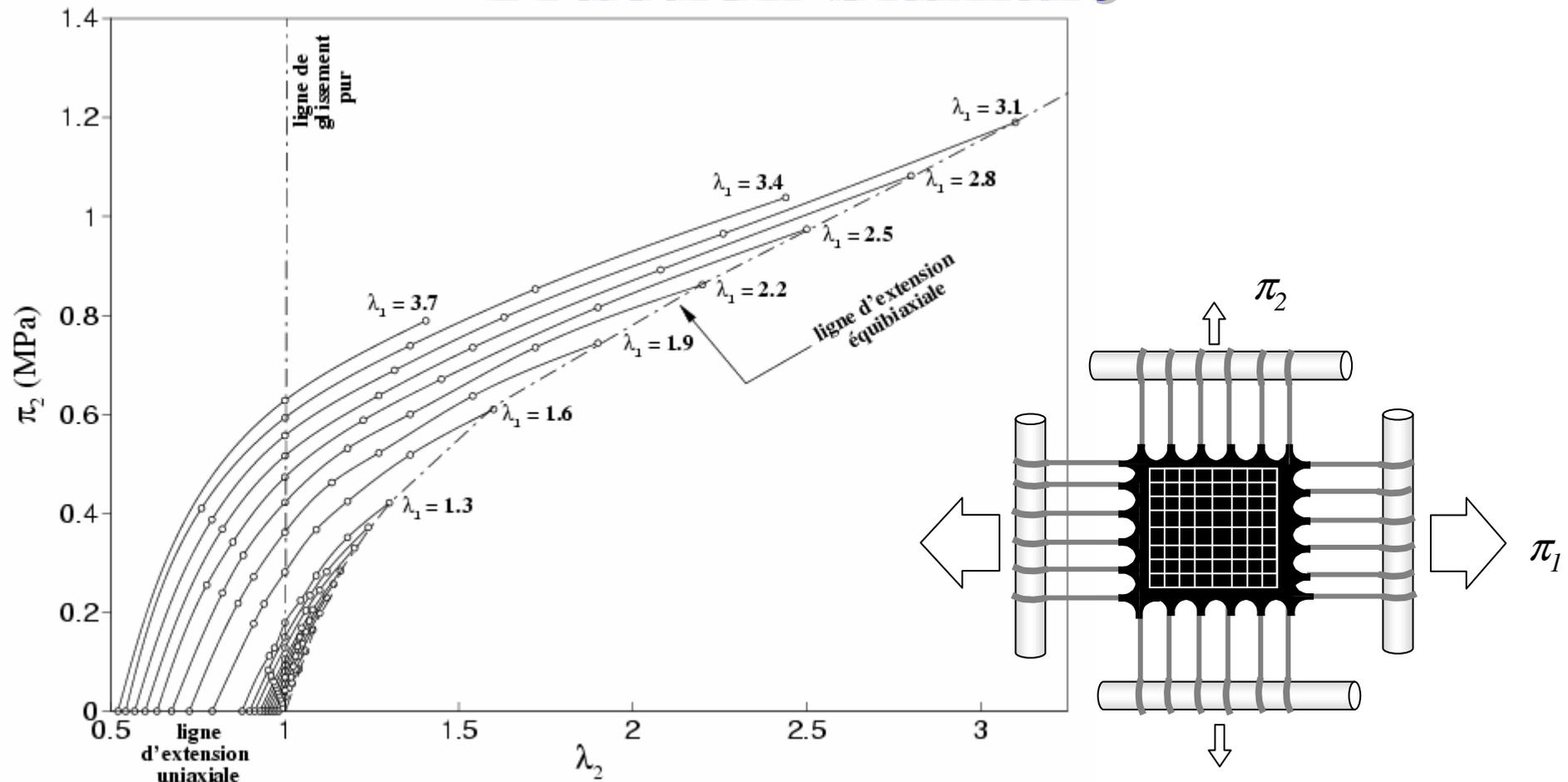
Simulations avec Ogden (1972)

6 constantes



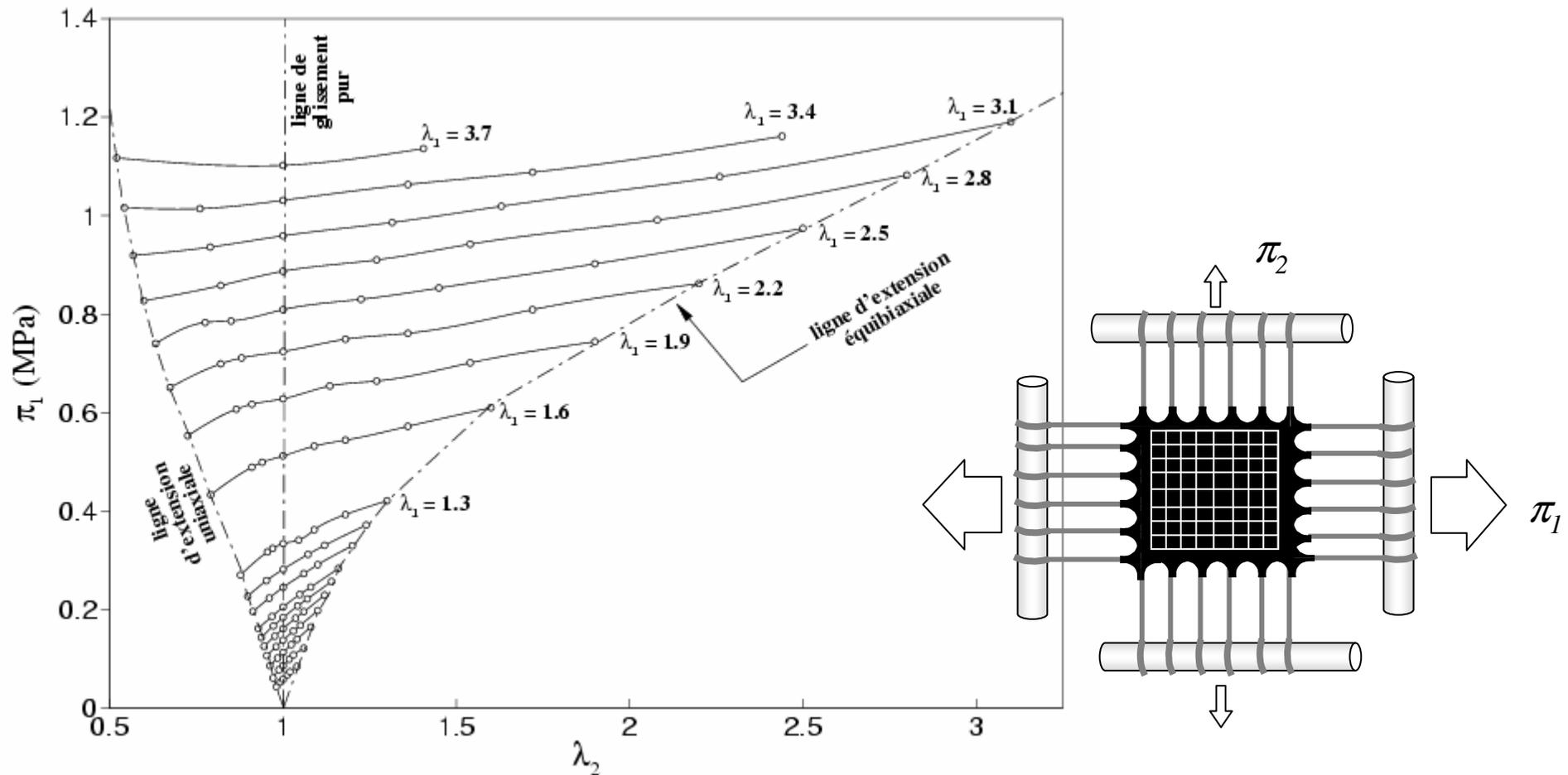
Kawabata 1981

Traction biaxiale



Kawabata 1981

Traction biaxiale



Treloar, Néohookien

Approche statistique (1940)

- Energie de déformation
- Statistique Gaussienne des chaines

$$W = C_{10}(I_1 - 3) \quad C_{10} = 0.5NkT$$

- N: nombre de chaines moléculaires par unité de volume
- K: constante de Boltzmann
- T: température absolue

Modèle hyperélastique phénoménologique

Caoutchouc 1940 - ?

- Mooney Rivlin, Mooney (1940)

$$W = C_1(\underline{I_1} - 3) + C_2(\underline{I_2} - 3)$$

2 constantes

Séries Polynômiales

Mooney Rivlin (1948)

$$W = \sum_{i=0, j=0}^{\infty} C_{ij} (I_1 - 3)^i (I_2 - 3)^j$$

- Biderman (1958) 4 constantes

$$W = \underline{C_{10}} (I_1 - 3) + \underline{C_{01}} (I_2 - 3) + \underline{C_{20}} (I_1 - 3)^2 + \underline{C_{30}} (I_1 - 3)^3.$$

- Haines-Wilson (1975) 6 constantes

$$W = \underline{C_{10}} (I_1 - 3) + \underline{C_{01}} (I_2 - 3) + \underline{C_{11}} (I_1 - 3)(I_2 - 3) \\ + \underline{C_{02}} (I_2 - 3)^2 + \underline{C_{20}} (I_1 - 3)^2 + \underline{C_{30}} (I_1 - 3)^3.$$

Modèles hyperélastique phénoménologiques

Caoutchouc 1940 - 1975

- Mooney Rivlin (1948)

N constantes

$$W = \sum_{i=0, j=0}^{\infty} \underline{C_{ij}} (I_1 - 3)^i (I_2 - 3)^j$$

- Gent Thomas (1958)

2 constantes

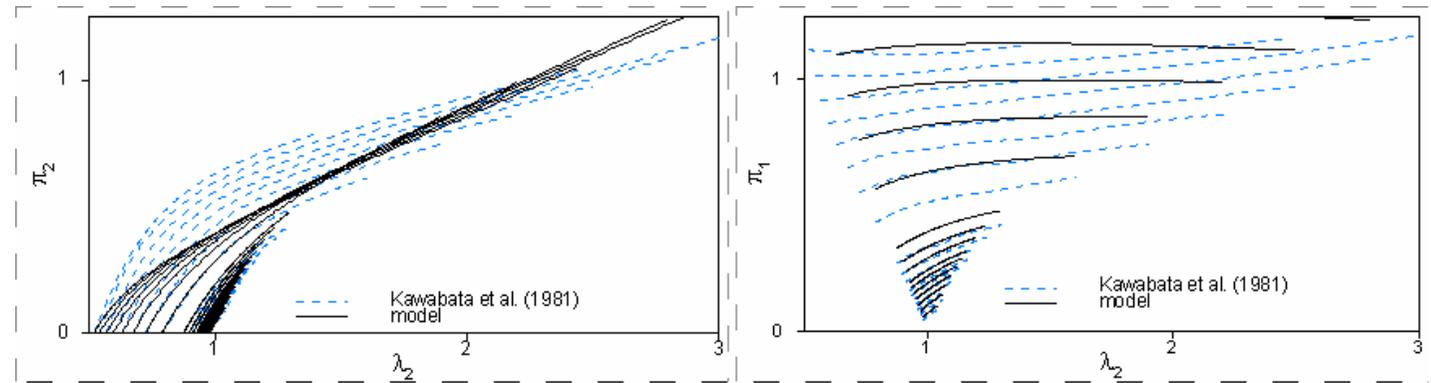
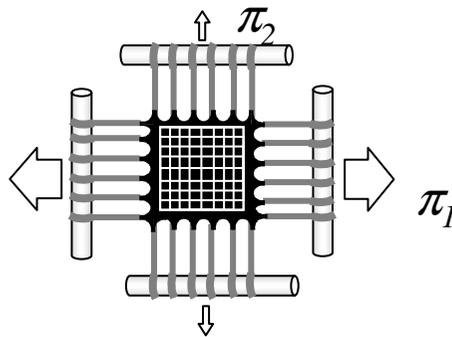
$$W = \underline{C_1} (I_1 - 3) + \underline{C_2} \ln(I_2 / 3)$$

- Hart-Smith (1967)

3 constantes

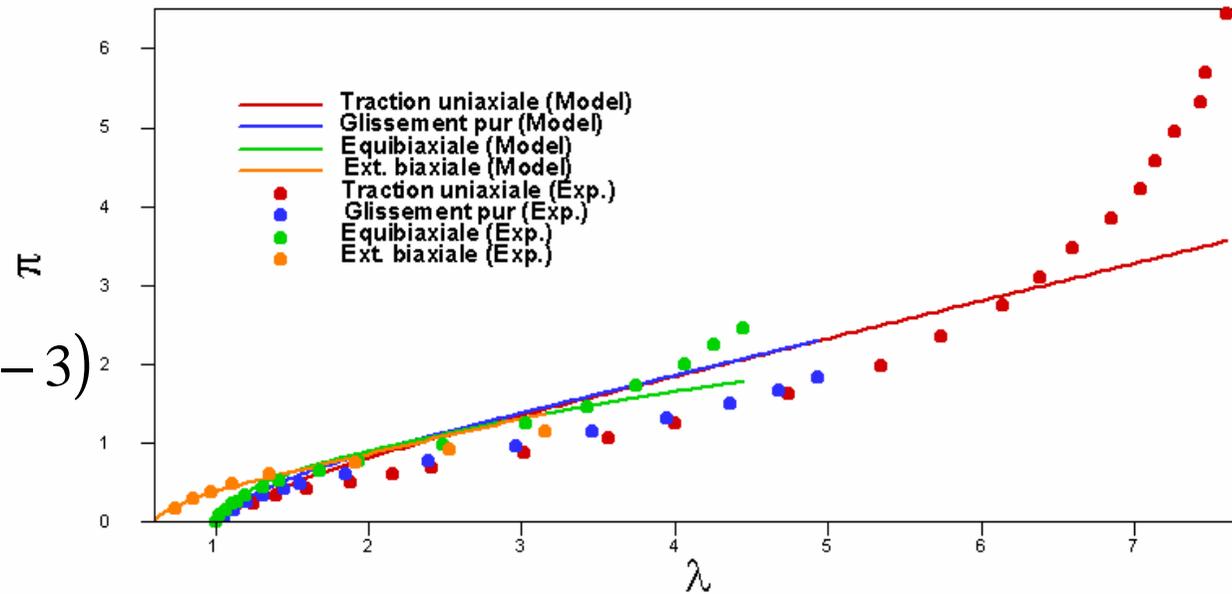
$$W = \underline{C_1} \int_0^{I_1-3} \exp(\underline{C_3} I_1'^2) dI_1' + \underline{C_2} \ln \frac{I_2}{3}$$

Mooney Rivlin, 1940

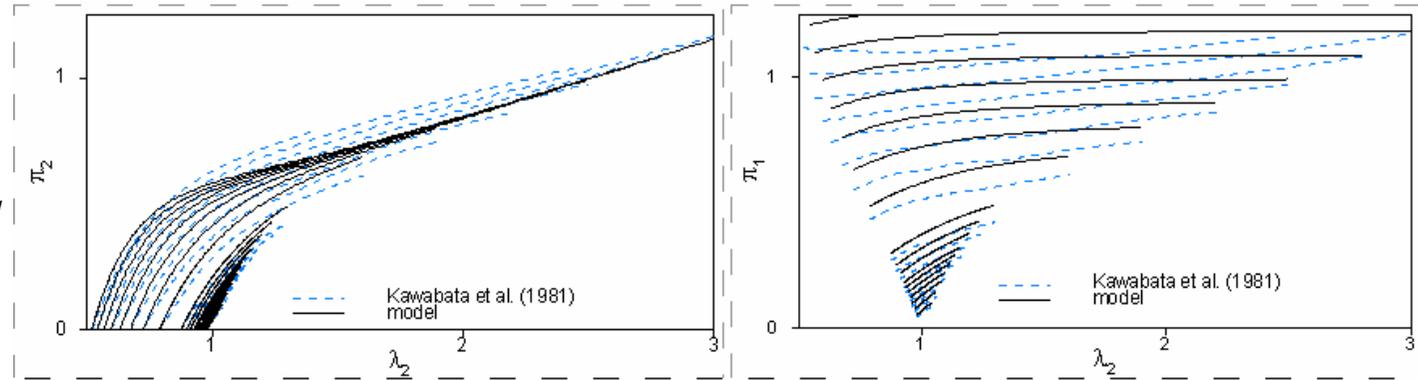
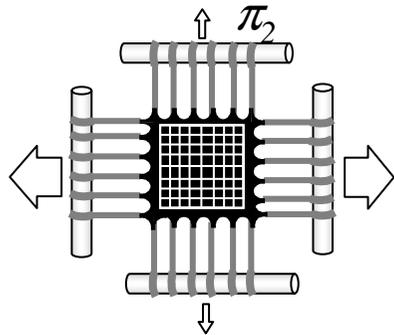


2 constantes

$$W = C_1(I_1 - 3) + C_2(I_2 - 3)$$

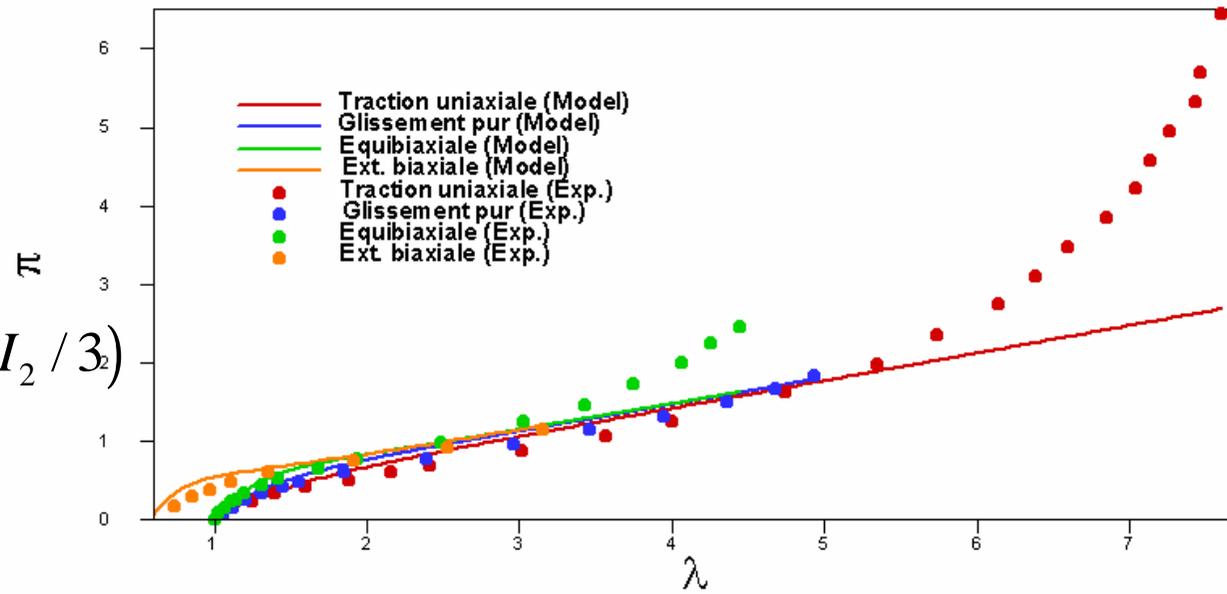


Gent Thomas, 1958

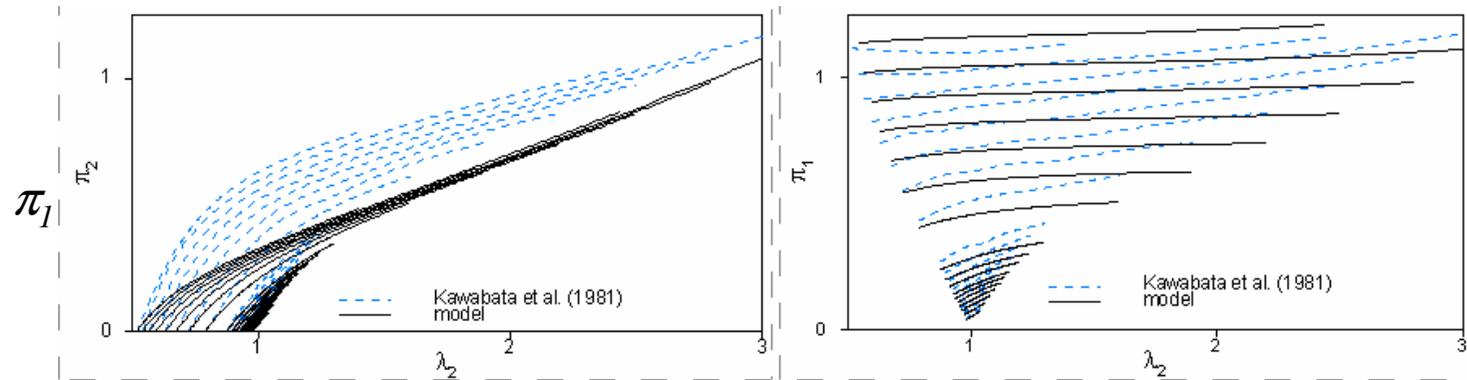
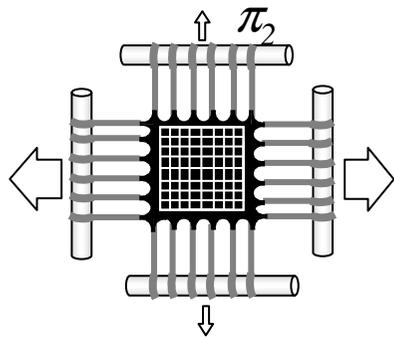


2 constantes

$$W = C_1 (I_1 - 3) + C_2 \ln(I_2 / 3)$$

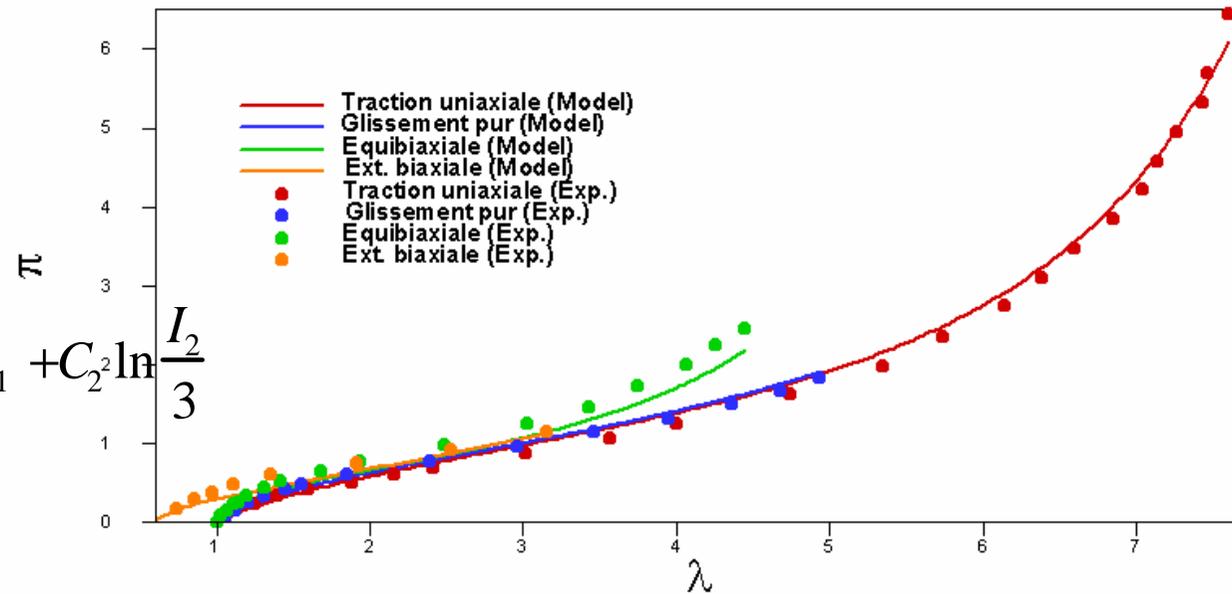


Hart Smith, 1966



3 constantes

$$W = C_1 \int_0^{I_1-3} \exp(C_3 I_1'^2) dI_1' + C_2 \ln \frac{I_2}{3}$$



Ogden 1972

phénoménologiques

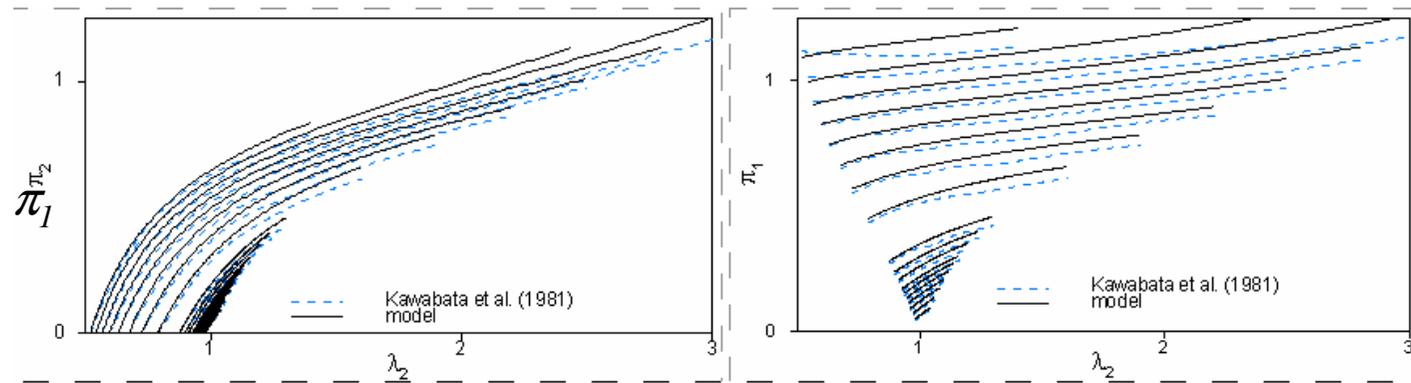
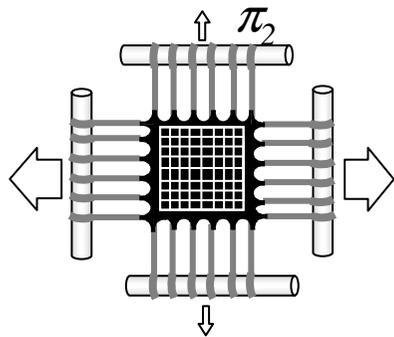
$$W = \sum_{i=1}^3 \frac{\mu_i}{\alpha_i} \lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3$$

6 constantes

Modèle en directions principales !

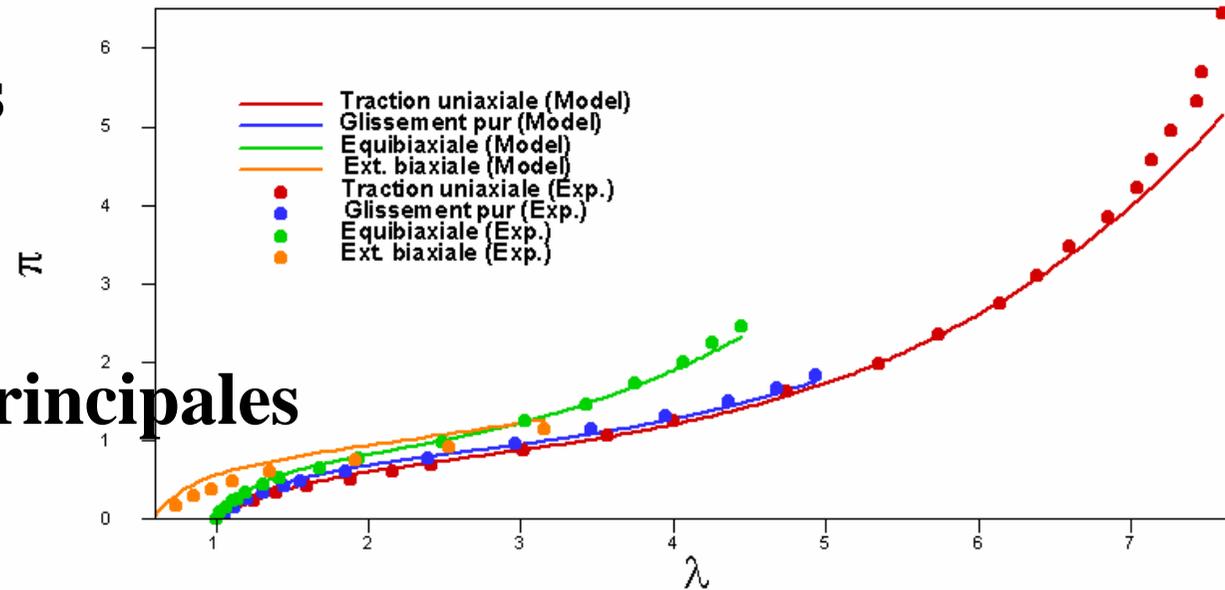
Bonne simulation des essais

Ogden, 1972

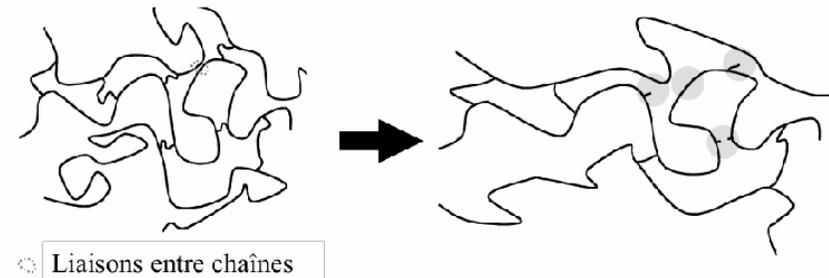


6 constantes

Directions principales



- Chaines macromoléculaires



- Modèles phénoménologiques macromoléculaires
- Statistique Gaussienne
- Statistique non Gaussienne

*Les Modèles statistiques justifient les modèles
phénoménologiques*

Modèles statistiques caoutchouc 1940 - 1975

Modèle Non Gaussien

- Treloar, Néo-Hookéen, *une chaine (1943)* 1 constante
- James et Guth, *Trois chaines (1947)* 2 constantes
- Arruda et Boyce, *Huit chaines (1993)* 2 constantes

Réseau Fantôme

Modèles en Invariant généralisé

- Heinrich et Kaliske, *Modèle tube (1997)* 3 constantes
- Kaliske et Heinrich, *Tube étendu (1999)* 4 constantes

Arruda Boyce

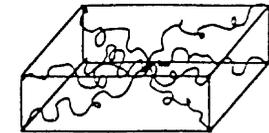
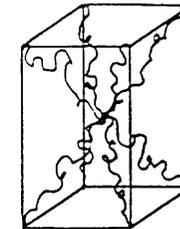
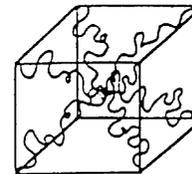
Huit Chaines 1993

- Statistique non Gaussienne

$$W = nk\theta.N \left[\frac{r_{chain}}{Nl} \beta + \ln \frac{\beta}{\sinh \beta} \right]$$

$$\beta = L^{-1} [r_{chain} / Nl]$$

$$r_{chain} = \frac{1}{\sqrt{3}} \sqrt{Nl} (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^{1/2}$$

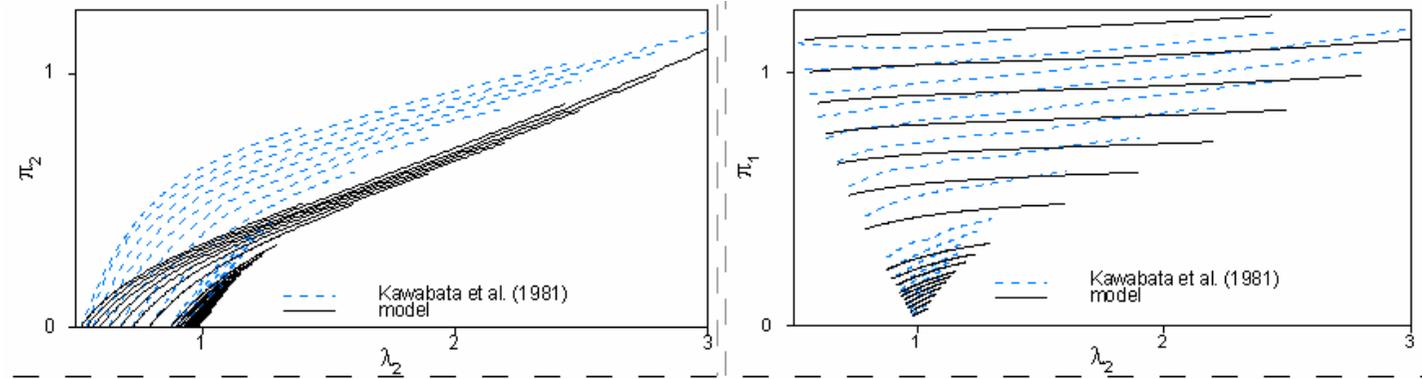
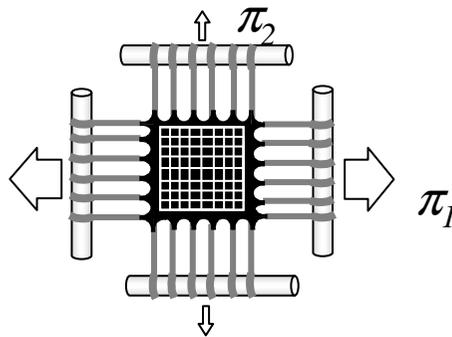


$$L(\beta) = \coth(\beta) - \frac{1}{\beta}$$

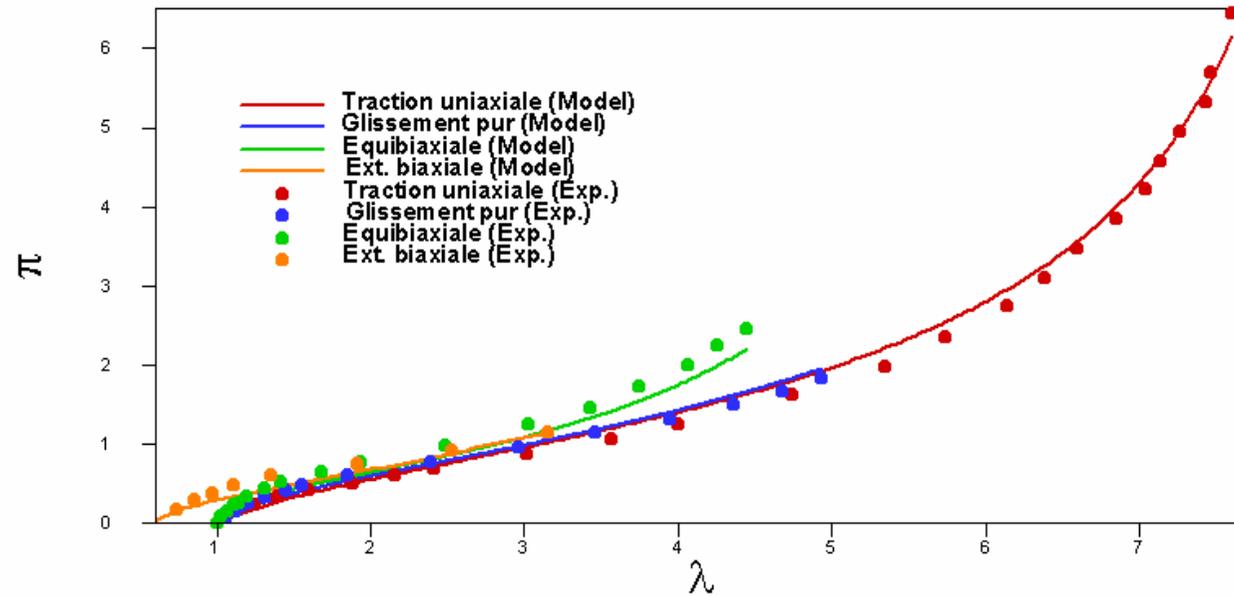
$$W = \mu \sum_{i=1}^5 \frac{C_i}{\lambda_m^{2i-2}} \left(\bar{I}_1^i - 3^i \right)$$

Arruda et Boyce, 1993

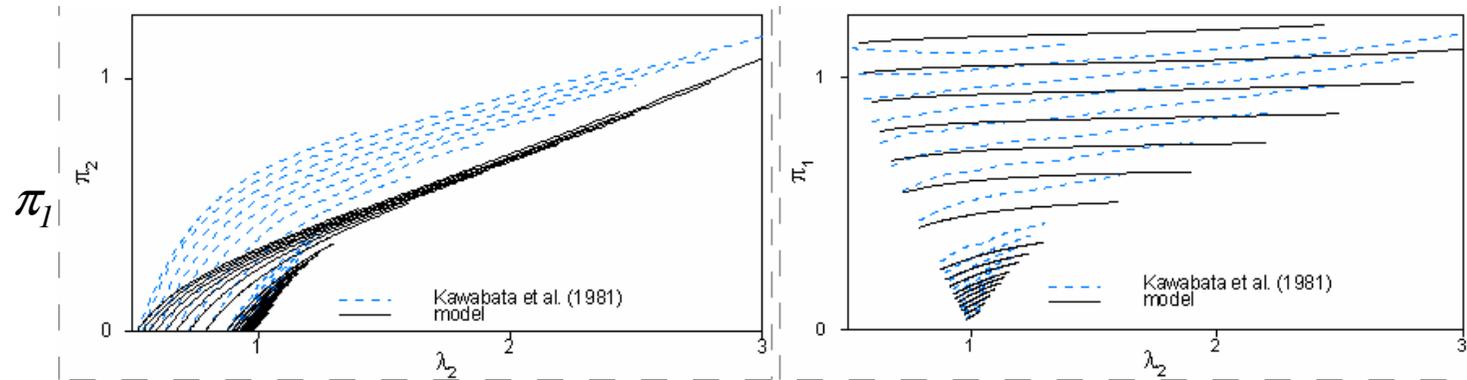
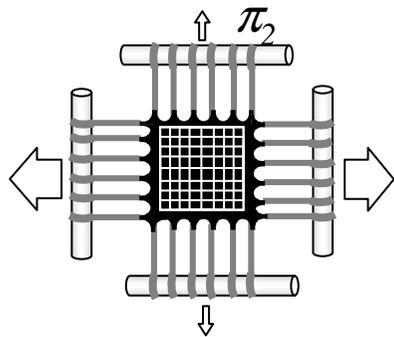
Huit Chaines



2 constantes

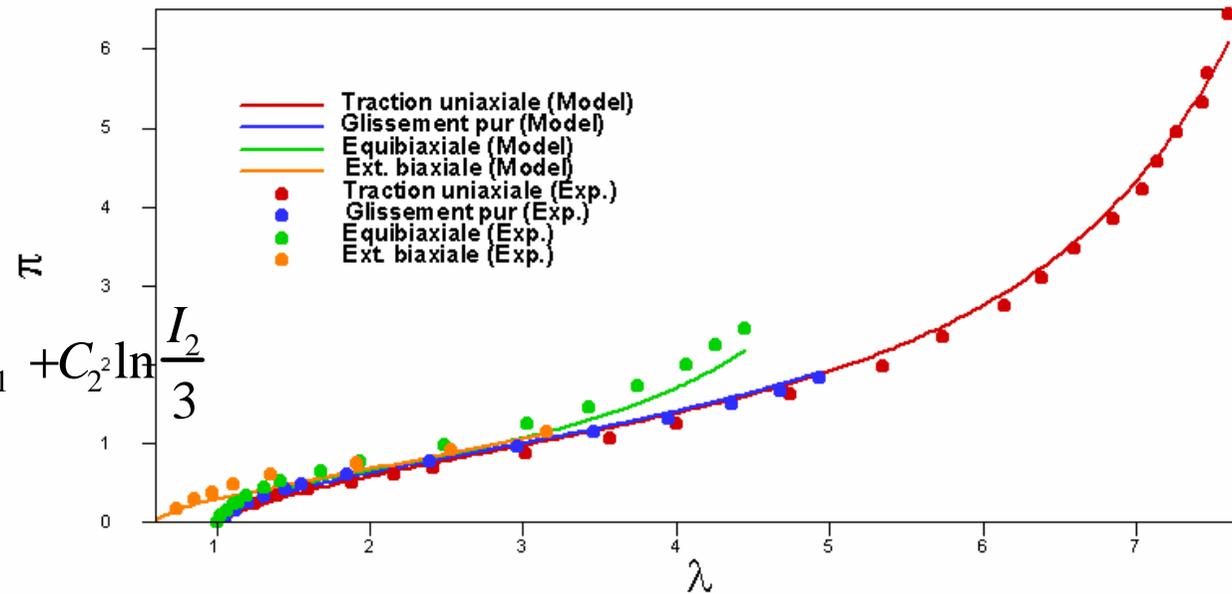


Hart Smith, 1966



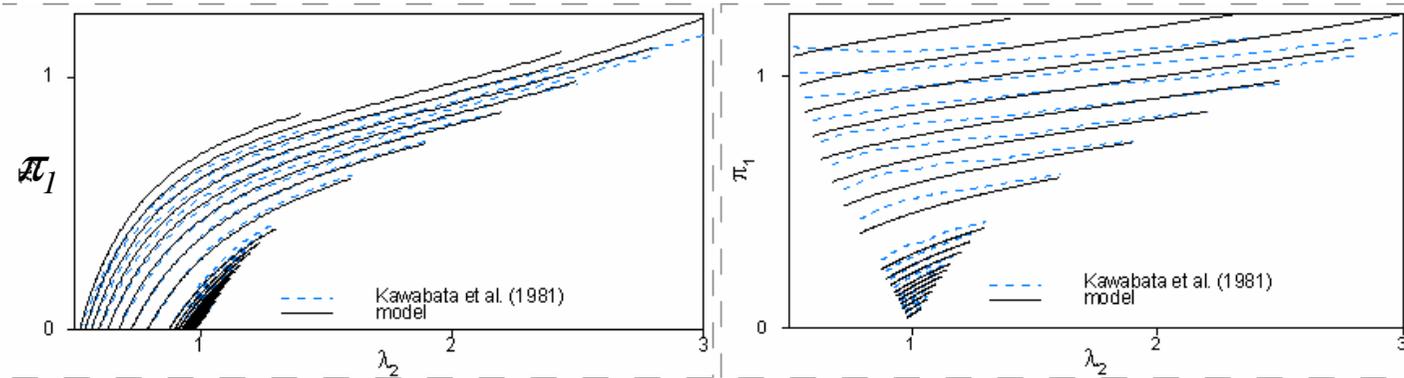
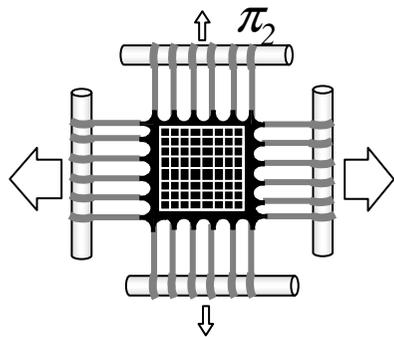
3 constantes

$$W = C_1 \int_0^{I_1-3} \exp(C_3 I_1'^2) dI_1' + C_2 \ln \frac{I_2}{3}$$

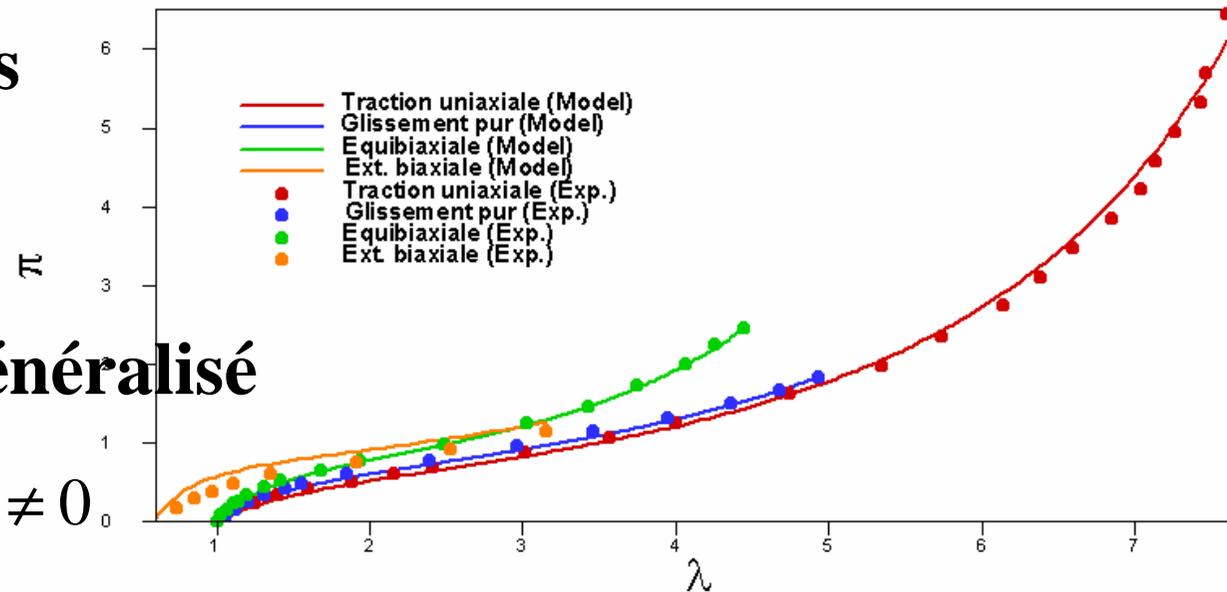


Extended Tube, 1999

Kaliske Henrich



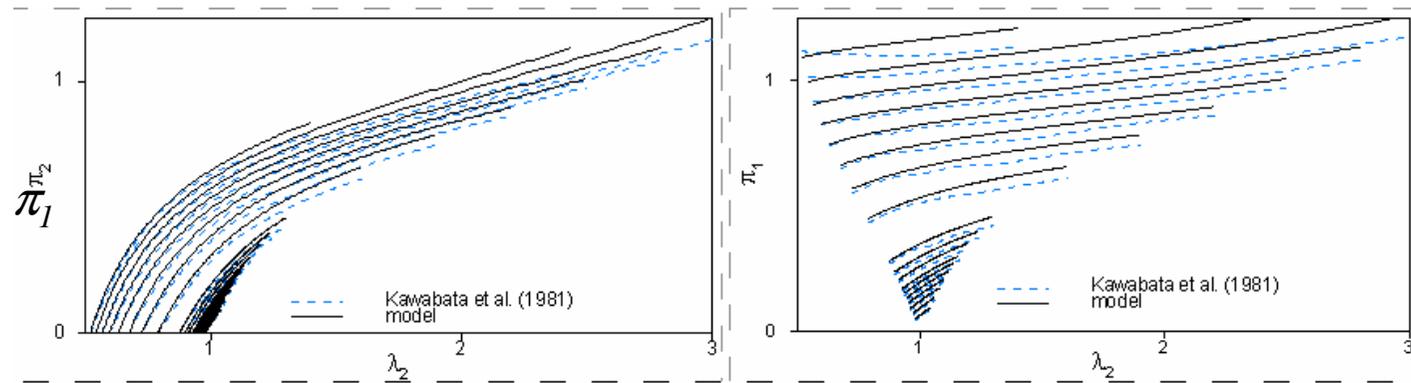
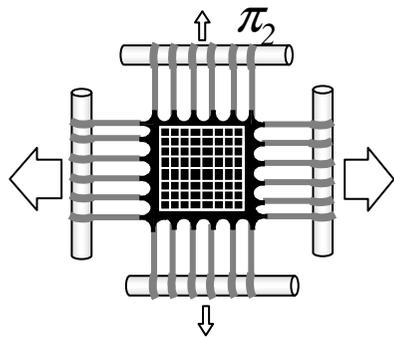
4 constantes



Invariant généralisé

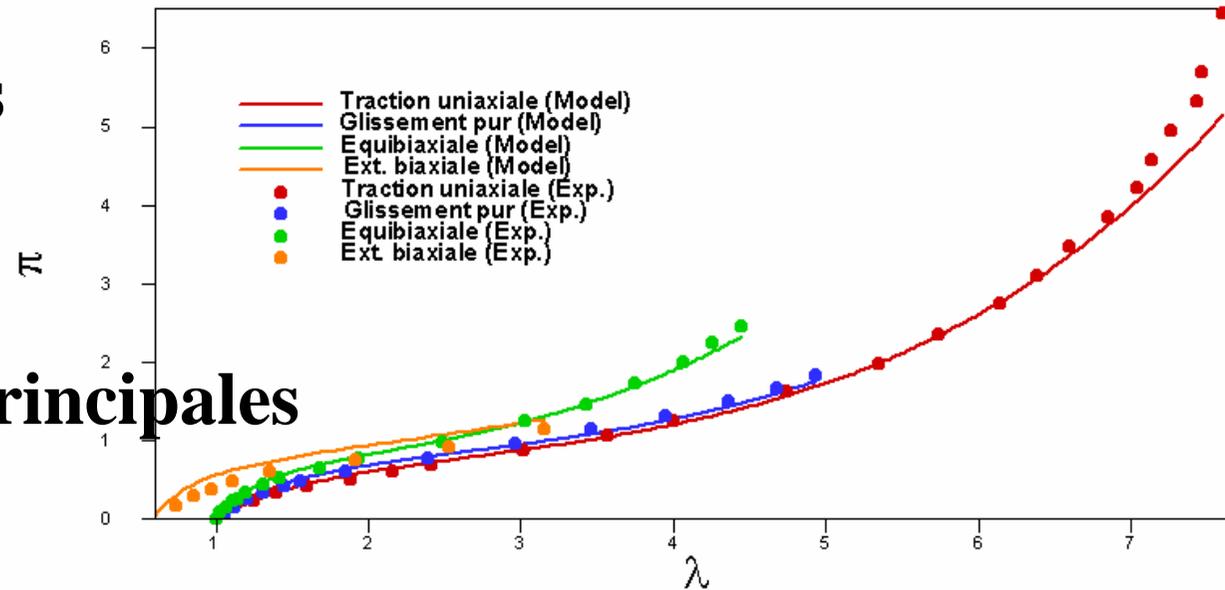
$$e_\beta = \frac{\beta}{B^2 - I} \text{ si } \beta \neq 0$$

Ogden, 1972



6 constantes

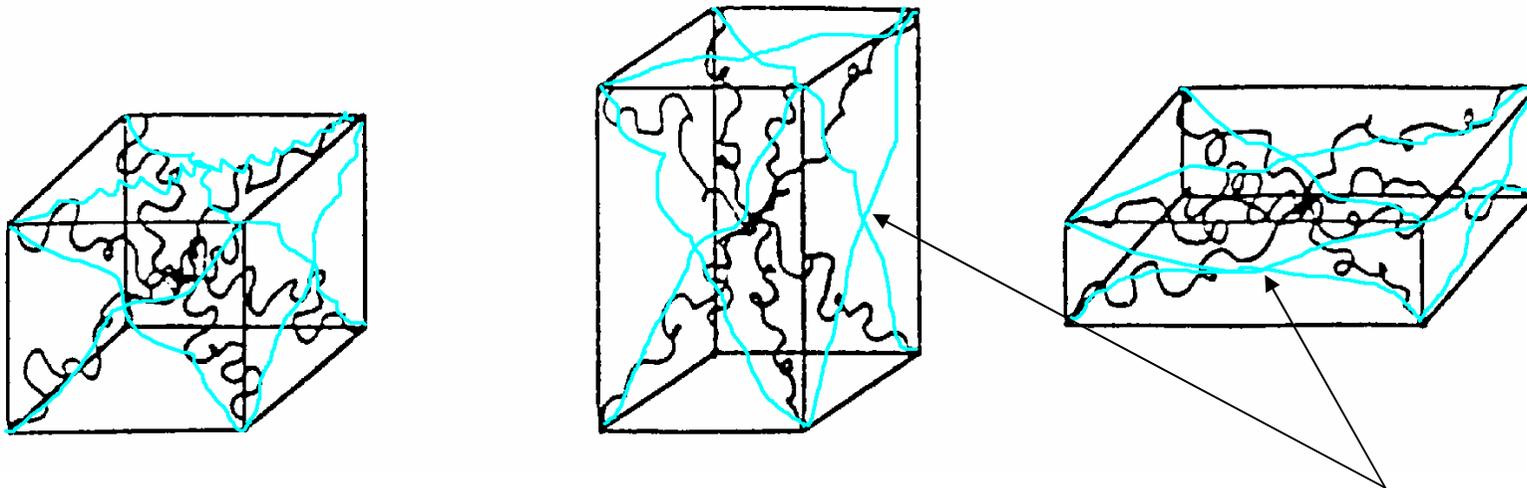
Directions principales



- **Néo-Hook** : déformations $< 50\%$
- **Mooney Rivlin** : déformations $\sim 100\%$
- **Biderman** : exemple de séries de Rivlin
- **Hart-Smith** : déformations $> 500\%$
- **Arruda Boyce** : déformations $> 500\%$
- **Ogden** : Bonne corrélation avec les essais
 - Modèle en Directions Principales !
- **Création du Modèle Grande Déformation**
 - *Objectif : Bonne corrélation avec les essais*
 - Modèle en Invariants*

Modèle Grande Déformation

- **Huit chaînes** modèle statistique non Gaussien $W_1(I_1)$



- **Réseau Fantôme** : énergie interne du modèle $W_2(I_2)$

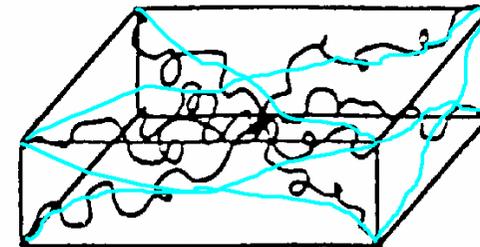
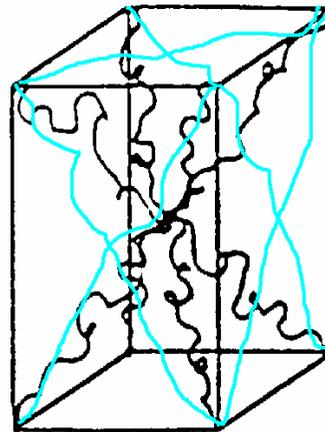
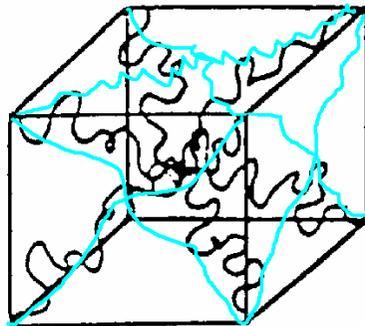
James, Guth 1949, Boggs 1952, Eichinger 1981

Energie libre : $F = e(v, T) + W(I_1)$

Statistique - phénoménologique

$$W = \underline{h_1} \int_0^{I_1-3} \exp(\underline{h_3} I_1'^2) dI_1' + \underline{3h_2} \int_0^{I_2} \frac{dI_2'}{\sqrt{I_2'}}$$

3 constantes

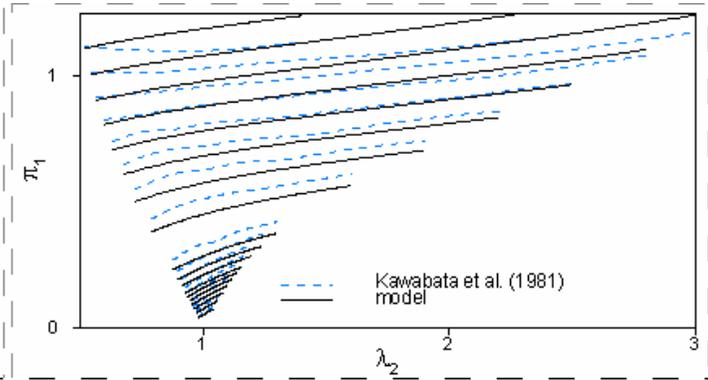
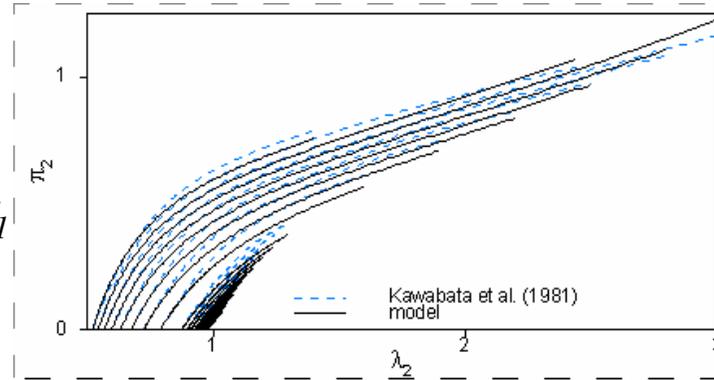
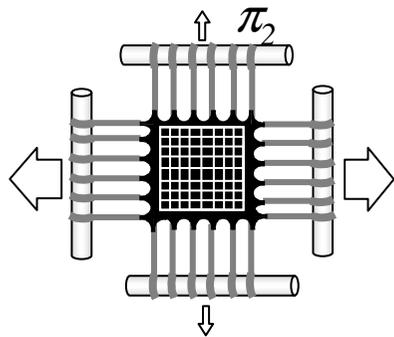


Energie libre

• **Huit chaines** confinées par un **Réseau Fantôme**

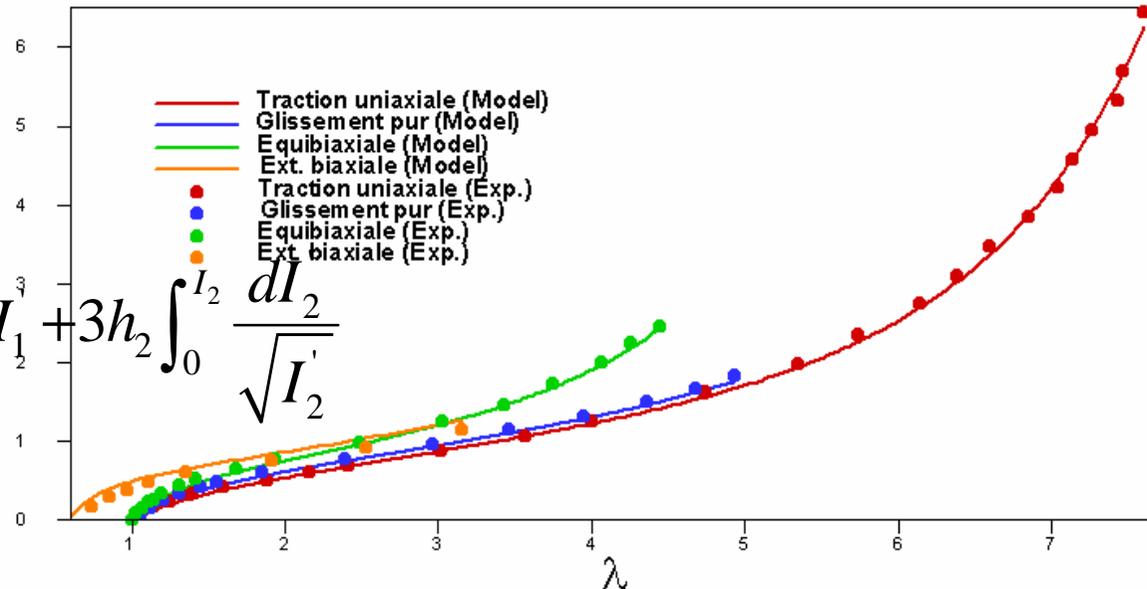
$h_1=0.142236$ $h_2=1.5854659E-02$ $h_3=3.4946541E-04$

Gornet Desmorat, 2009



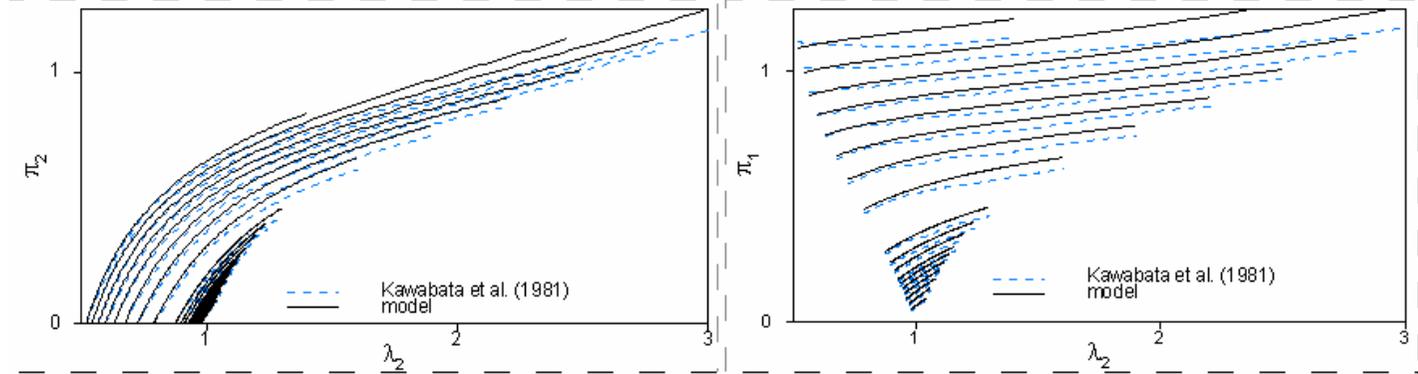
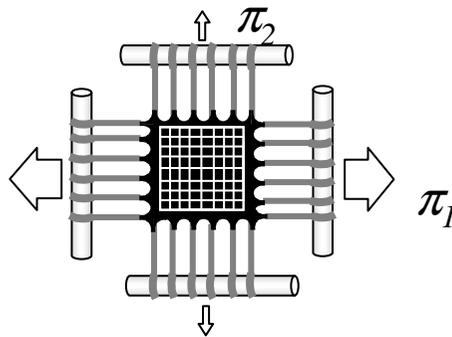
3 constantes

$$W = h_1 \int_0^{I_1-3} \exp(h_3 I_1'^2) dI_1' + 3h_2 \int_0^{I_2} \frac{dI_2}{\sqrt{I_2'}}$$

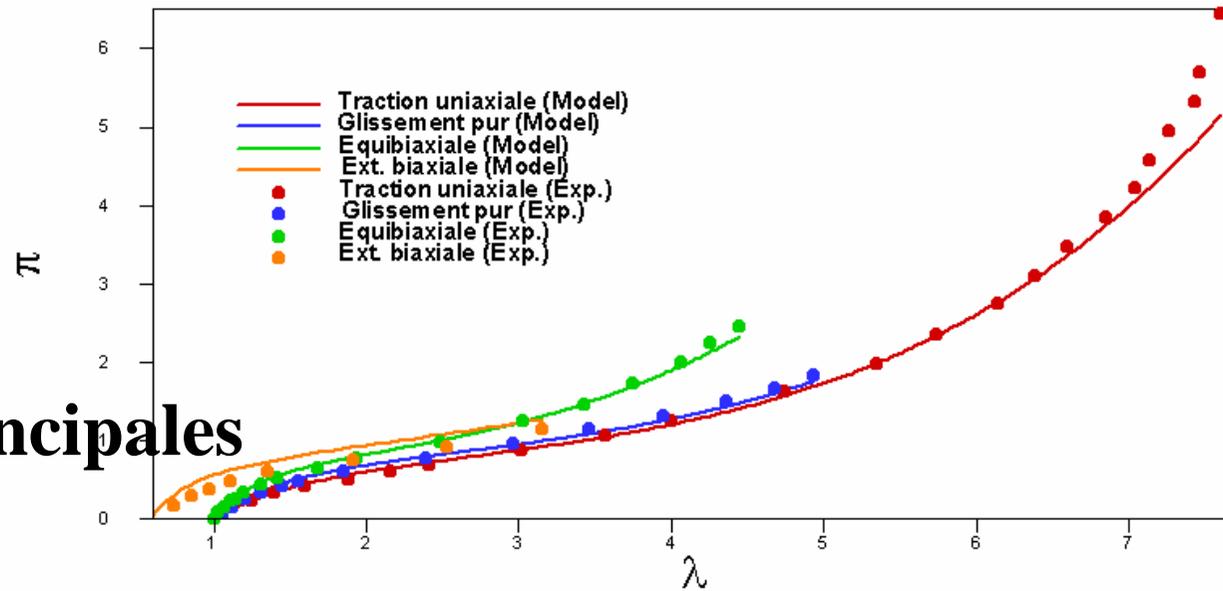


$h_1=0.142236$ $h_2=1.5854659E-02$ $h_3=3.4946541E-04$

Ogden, 1972



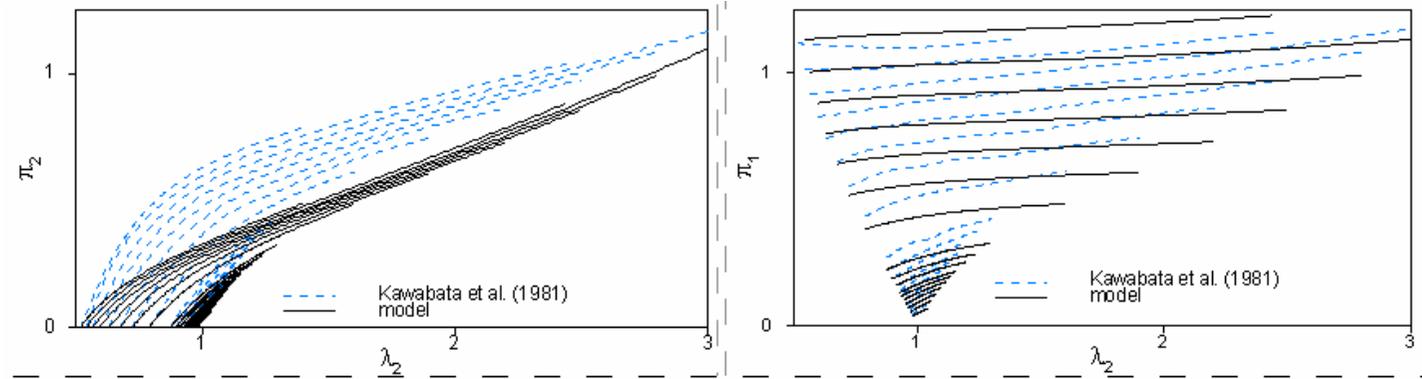
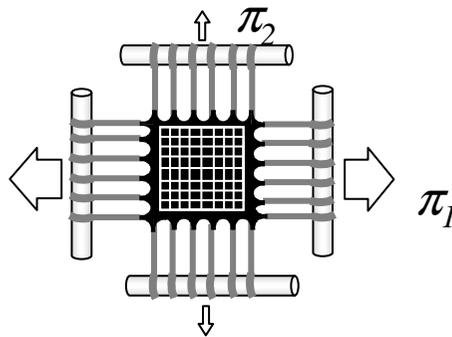
6 constantes



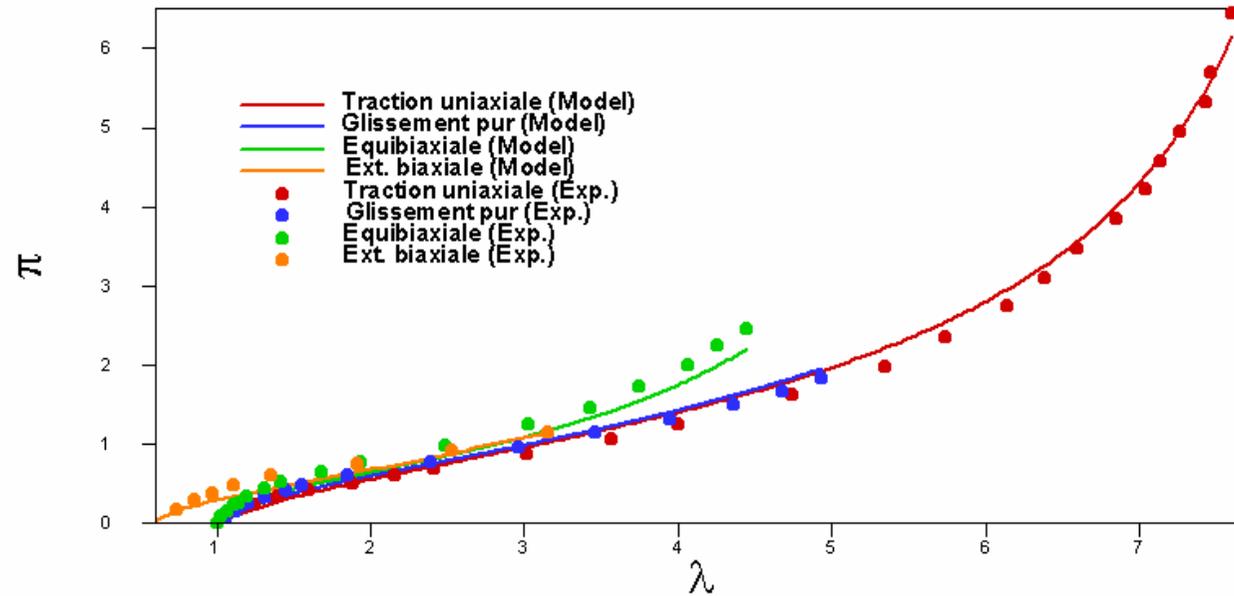
Directions principales

Arruda et Boyce, 1993

Huit Chaines



2 constantes



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Contraintes planes

$$\begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix}$$

- Formulation PK2 et Cauchy Green Droit

$$\bar{\bar{S}} = 2 \left(\left[\frac{\partial W}{\partial I_1} + I_1 \frac{\partial W}{\partial I_2} \right] \bar{\bar{I}} - \frac{\partial W}{\partial I_2} \bar{\bar{C}} \right) - p \bar{\bar{C}}^{-1}$$

- Incompressibilité

$$C_{33} = \frac{1}{(C_{11} C_{11} - C_{12}^2)}$$

Contraintes planes

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{12} & \sigma_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix}$$

- Formulation Cauchy et Green Gauche

$$[\sigma] = -p[I] + 2 \cdot \left[\frac{\partial W}{\partial I_1} + I_1 \frac{\partial W}{\partial I_2} \right] [B] - 2 \frac{\partial W}{\partial I_2} [B]^2$$

- Incompressibilité

Tridimensionnels

$$\bar{\sigma}_{iso} = 2J^{-5/3} \left[\frac{\partial W_{iso}}{\partial \bar{I}_1} + \bar{I}_1 \frac{\partial W_{iso}}{\partial \bar{I}_2} \right] \bar{B} - 2J^{-7/3} \frac{\partial W_{iso}}{\partial I_2} \bar{B}^2$$

$$\bar{\sigma}_{vol} = \frac{\partial W_{vol}}{\partial J} \bar{1} \quad W_{vol} = \frac{K}{2} (J - 1)^2$$

Implémentation d'un modèle

- Loi de comportement (mooney.eso)
 - Eso2For : mooney.f, mooney.o
- Modification de la UMAT (umat.eso)
 - Eso2For : umat.f, umat.o
- Fabrication de l'exécutable Cast3M
- Exemples :
 - Solutions analytiques, comparaisons ! ?

ABAQUS-UMAT

Mooney Rivlin, fichiers umat.eso, mooney.eso

*SUBROUTINE MOONEY (STRESS,
STATEV, DDSdde, SSE, SPD, SCD,RPL,
DDSDDT, DRPLDE, DRPLDT, STRAN,
DSTRAN, TIME, DTIME,TEMP, DTEMP,
PREDEF, DPRED,CMNAME, NDI, NSHR,
NTENS, NSTATV, PROPS, NPROPS,
COORDS,DROT, PNEWDT, CELENT,
DFGRD0, DFGRD1,NOEL, NPT, LAYER,
KSPT, KSTEP, KINC)*

Détection de la formulation EF, NDI: T. Charras 2008

Operateur MODE

Mooney Rivlin

LCMAT = MOTS 'YOUN' 'NU '
'C10' 'C20' 'D ' ;

MO = MODE SU 'MECANIQUE' 'ELASTIQUE'
'ISOTROPE' 'NON_LINEAIRE' 'UTILISATEUR '
'NUME_LOI' 31 'C_MATERIAU' LCMAT ;

$$W = C_{10} (\bar{I}_1 - 3) + C_{20} (\bar{I}_2 - 3) + \frac{1}{D} (J - 1)^2$$

Operateur MATE

Mooney Rivlin

MA = MATE MO 'YOUN' YU 'NU ' XNU
'C1' CoeC1 'C2' CoeC2 'D ' CoeD ;

En formulation incompressible:

« Contraintes Planes » 'D ' n'est pas utilisé !

$$W = C_1 (\bar{I}_1 - 3) + C_2 (\bar{I}_2 - 3) + \frac{1}{D} (J - 1)^2$$

$$\bar{I}_1 = J^{-2/3} I_1 \quad \bar{I}_2 = J^{-4/3} I_2 \quad J = \det(\bar{\bar{F}})$$

Fonctions de pénalisation

La bibliographie

$$W = \bar{W}(\bar{I}_1, \bar{I}_2) + U(J)$$

$$\lim_{J \rightarrow 0} U = +\infty$$

$$\lim_{J \rightarrow +\infty} U = +\infty$$

$$\lim_{J \rightarrow 0} \frac{\partial U}{\partial J} = -\infty$$

$$\lim_{J \rightarrow +\infty} \frac{\partial U}{\partial J} = +\infty$$

Référence	
Simo (1988); van den Bogert et de Borst (1990)	$U(J) = \frac{K}{2}(J - 1)^2$
Valanis et Landel (1967)	$U(J) = K \frac{(\ln J)^2}{2}$
Simo et Taylor (1982); van den Bogert et de Borst (1990)	$U(J) = K \frac{(J - 1)^2 + (\ln J)^2}{4}$
Ogden (1972); Kaliske et Rothert (1997b)	$U(J) = K\theta^2(\theta \ln J + J^{-\theta} - 1)$
Liu <i>et al.</i> (1992, 1994)	$U(J) = K(J \ln J - J + 1)$
Simo et Pister (1984)	$U(J) = \frac{1}{2}\lambda(\ln J)^2 - \mu \ln J$
Doll et Schweizerhof (2000)	$U(J) = K \frac{\exp(J - 1) - \ln J - 1}{2}$
Doll et Schweizerhof (2000)	$U(J) = K(J - 1) \ln \frac{J}{2}$
Kaliske et Rothert (1999)	$U(J) = \frac{K}{2} \left(\frac{J^2 - 1}{2} - \ln J \right)$

Procédure PASAPAS

```
TAB1 = TABLE;  
TAB1.'VARIABLES_INTERNES' = TABLE ;  
TAB1 . GRANDES_DEFORMATIONS = VRAI ;  
TAB1 . MODELE = MO;  
TAB1 . CARACTERISTIQUES = MA ;  
TAB1 . CHARGEMENT = CH1 ;  
TAB1 . TEMPS_CALCULES = PR1;  
TAB1 . 'TEMPS_SAUVES' = PR2;  
PASAPAS TAB1 ;
```

Les modèles Cast3M

- **Formulation Incompressible :**
 - Contraintes Planes
- **Formulation Quasi-incompressible :**
 - Déformations Planes, Axisymétrique
 - Tridimensionnel
- **Modèles disponibles :** *Format ABAQUS*
 - Mooney-Rivlin (Néo-Hook), Biderman, Gent-Thomas
 - Hart-Smith, Arruda Boyce, Gornet Desmorat
- **Exemples en incompressible :**
 - Traction, Bitraction, Cisaillement simple
- **Exemples en quasi-incompressible :**
 - Traction 3D et traction Déformations Planes

Plan

- **Contexte**
- **La Performance des modèles hyperélastiques**
 - Mooney Rivlin...
- **Développement UMAT**
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- **Exemples de validation**
 - 2D, 3D, analytique et Abaqus
- **Conclusion**

Traction

Solution analytique Mooney Rivlin



$$[F] = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1/\sqrt{\lambda} & 0 \\ 0 & 0 & 1/\sqrt{\lambda} \end{bmatrix} \quad [B] = F F^T = \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1/\lambda \end{bmatrix}$$

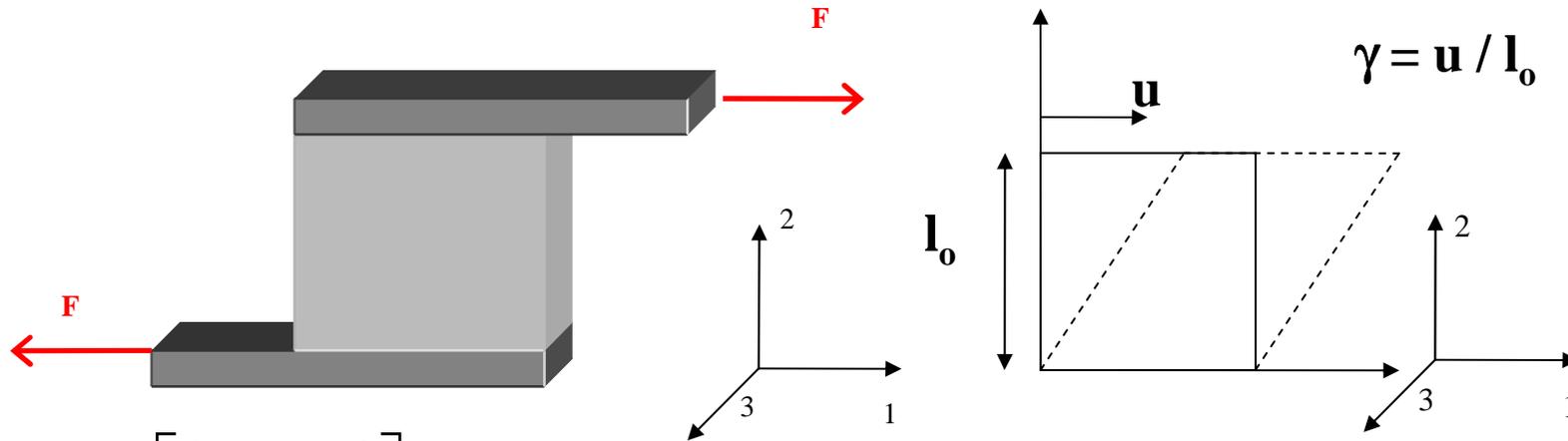
$$[\sigma] = \left(\frac{\partial W}{\partial I_1} + I_1 \frac{\partial W}{\partial I_2} \right) [B] - 2 \frac{\partial W}{\partial I_2} [B]^2 - p[I]$$

$$\begin{cases} I_1 = \text{tr}[B] = \lambda^2 + 2/\lambda \\ I_2 = \frac{1}{2} \cdot \left\{ (\text{tr}[B])^2 - \text{tr}[B]^2 \right\} = \frac{1}{\lambda^2} + 2 \cdot \lambda \\ I_3 = \det[B] = 1 \end{cases} \quad \sigma_{33} = \sigma_{22} = 0$$

$$\sigma = 2 \left[\left(\lambda^2 - \frac{1}{\lambda} \right) C_1 + \left(\lambda - \frac{1}{\lambda^2} \right) C_2 \right]$$

Glissement simple

Solution analytique 1/2



$$[F] = \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B] = F^T F = \begin{bmatrix} 1 + \gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} I_1 = \text{tr}[B] = 3 + \gamma^2 \\ I_2 = \frac{1}{2} \cdot \{(\text{tr}[B])^2 - \text{tr}[B]^2\} = 3 + \gamma^2 \\ I_3 = \det[B] = 1 \end{cases}$$

Glissement simple

Solution analytique Mooney Rivlin 2/2

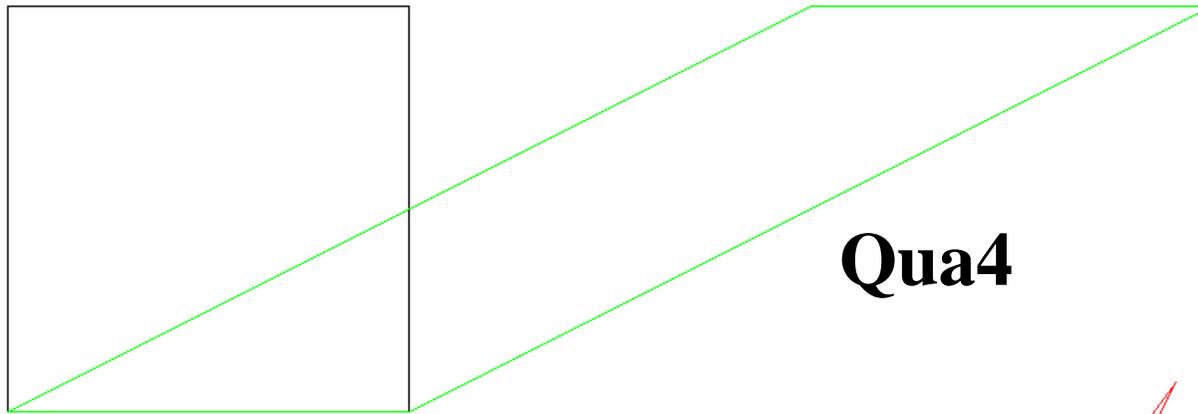
$$[\sigma] = \left(\frac{\partial W}{\partial I_1} + I_1 \frac{\partial W}{\partial I_2} \right) [B] - 2 \frac{\partial W}{\partial I_2} [B]^2 - p[I]$$

$$\sigma_{33} = 0 \longrightarrow p = 2.[C_1 + (2 + \gamma^2).C_2]$$

$$\sigma_{11} = 2 C_1 \gamma^2 \quad \sigma_{22} = -2 C_1 \gamma^2 \quad \sigma_{12} = 2.(C_1 + C_2).\gamma$$

Un élément linéaire

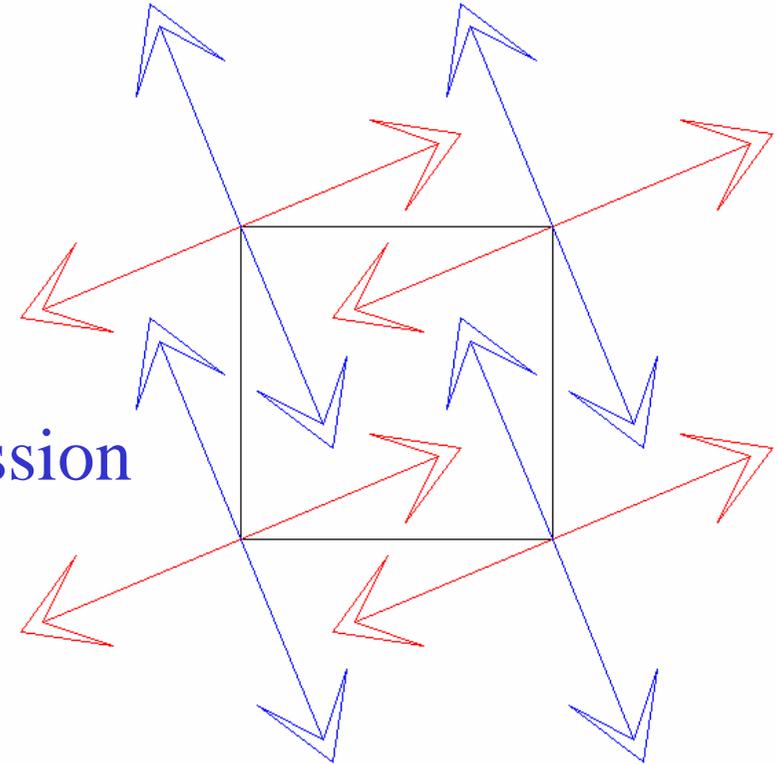
Solution analytique EF



Qua4

Traction

Compression

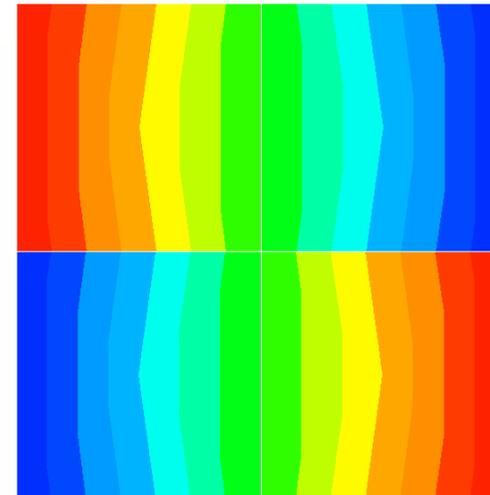
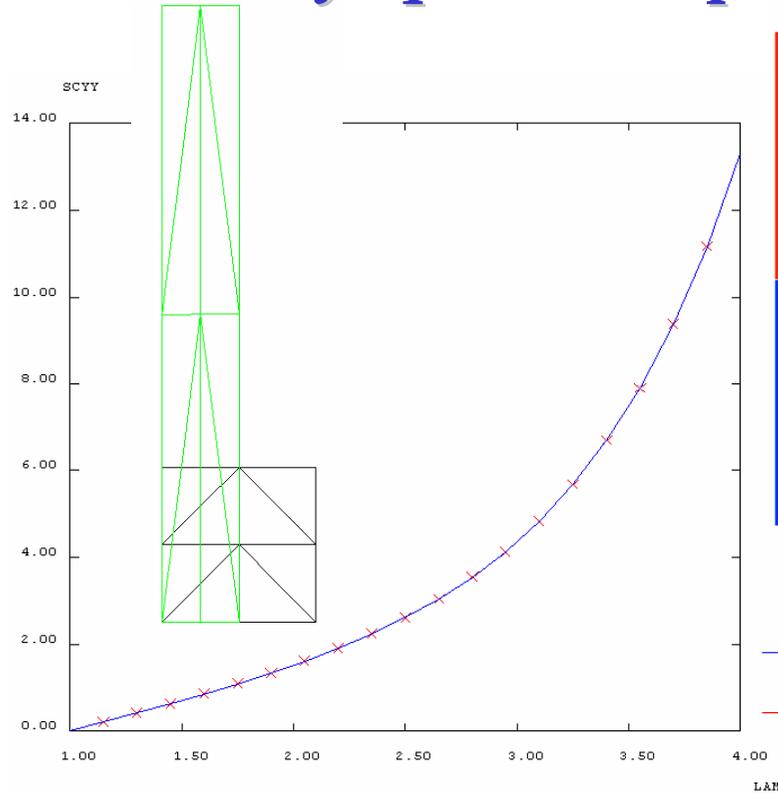
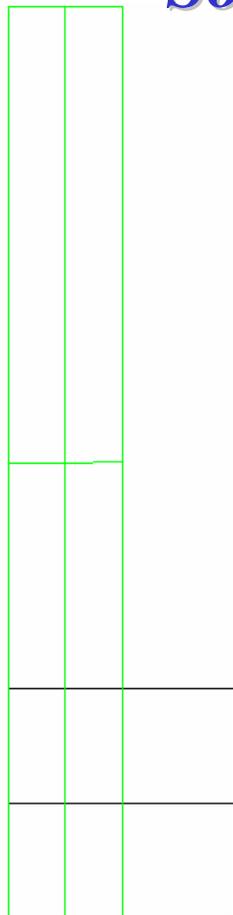


$$[\sigma] = \begin{bmatrix} \sigma & \\ & -\sigma \end{bmatrix}$$

$$\sigma = (C_1 - C_2)\gamma^2 + (C_1 + C_2)|\gamma|\sqrt{\gamma^2 + 4}$$

Traction

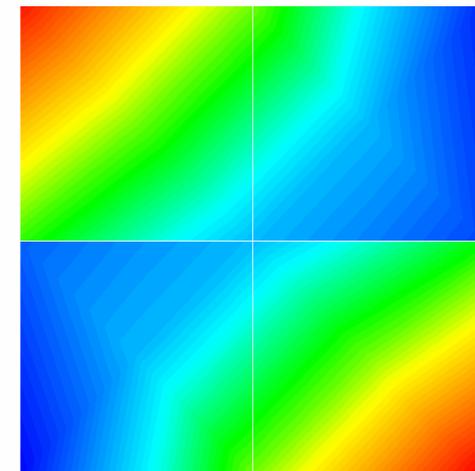
Solution analytique incompressible Biderman



Qua4



Qua8



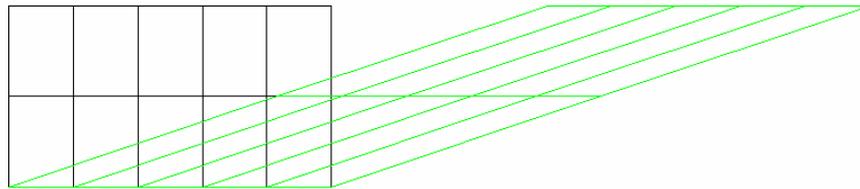
Cauchy

Qua4, Qua8, Tri3, Tri6

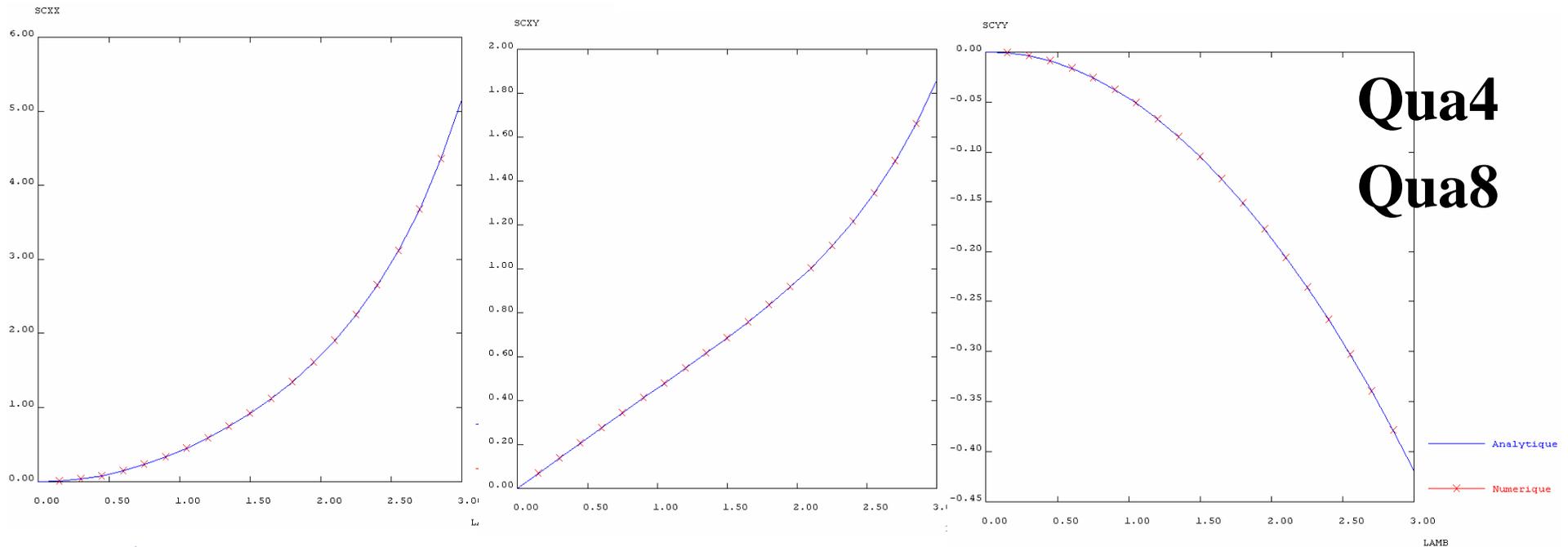
Glissement simple

Solution analytique incompressible Biderman

Cauchy

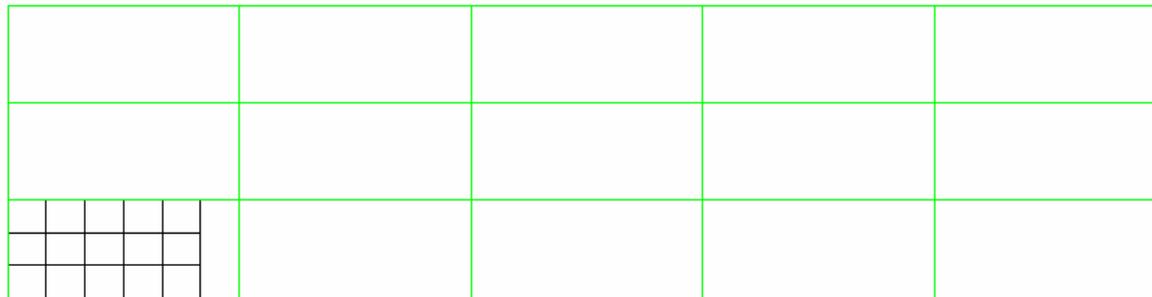


$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3.$$

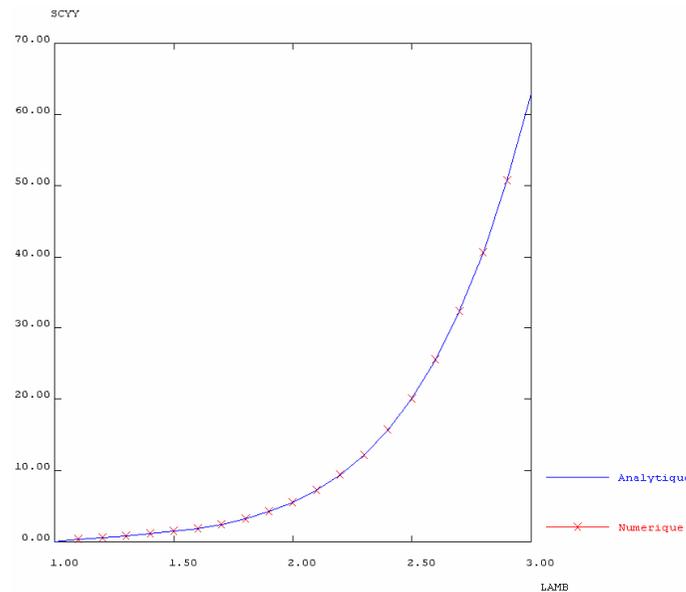
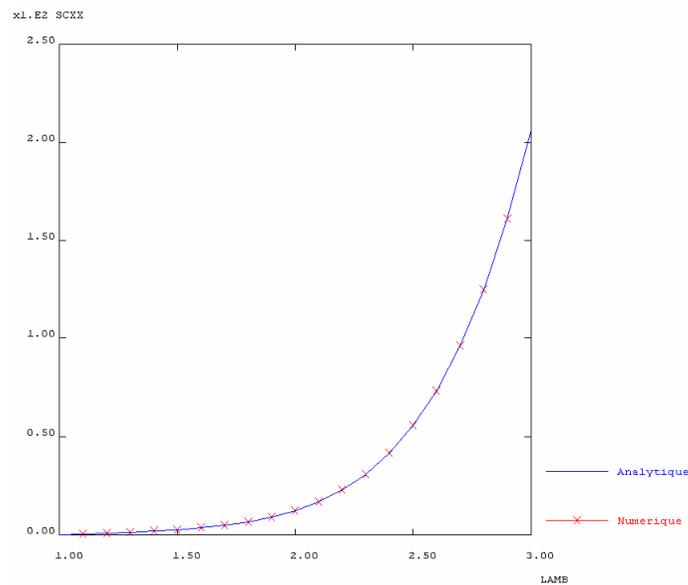


Traction biaxiale

Solution analytique incompressible Biderman



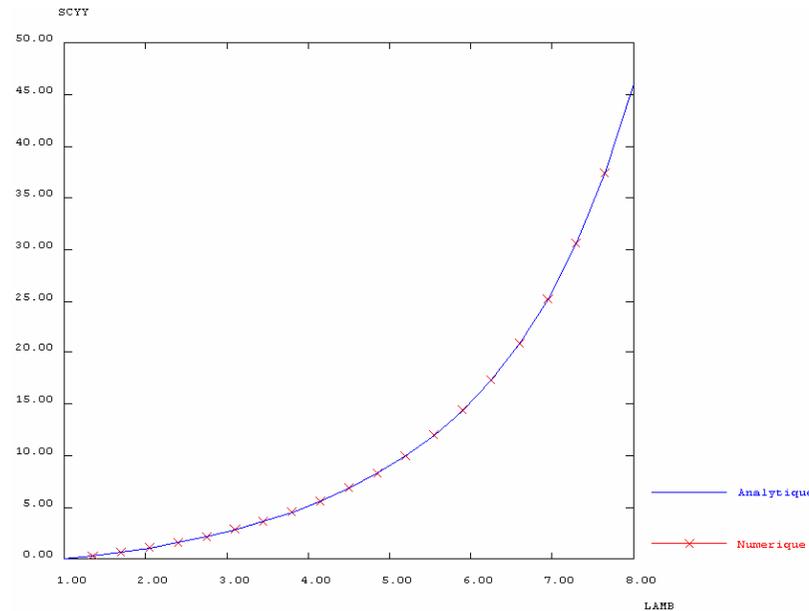
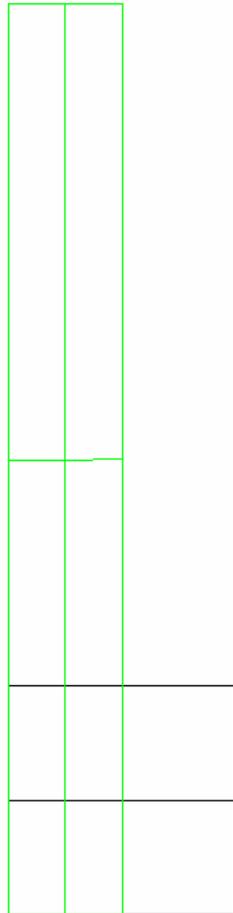
Cauchy



Qua4
Qua8

Traction

Solution analytique incompressible Huit Chaines

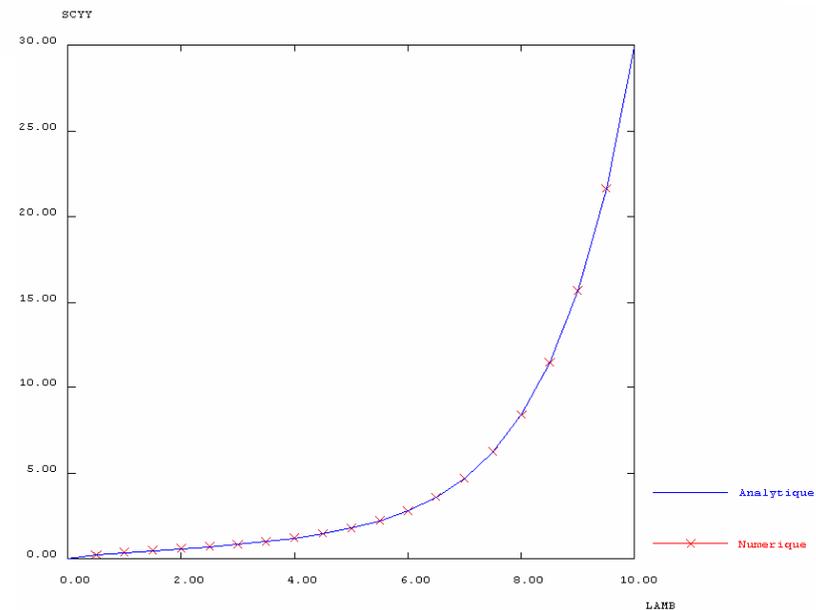
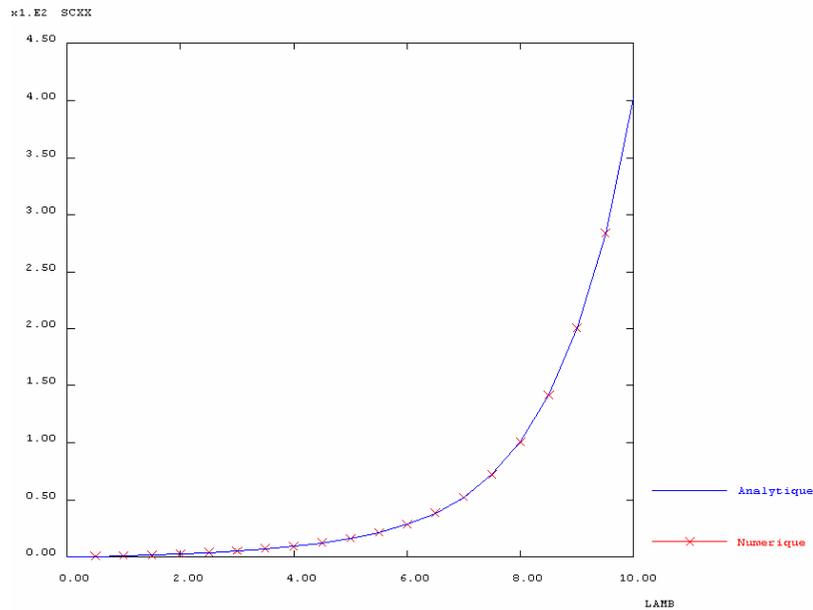


Cauchy

Traction biaxiale

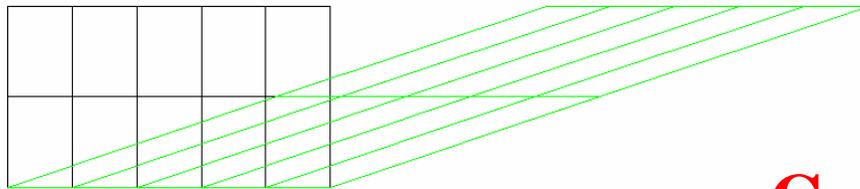
Solution analytique incompressible Huit Chaines

Cauchy

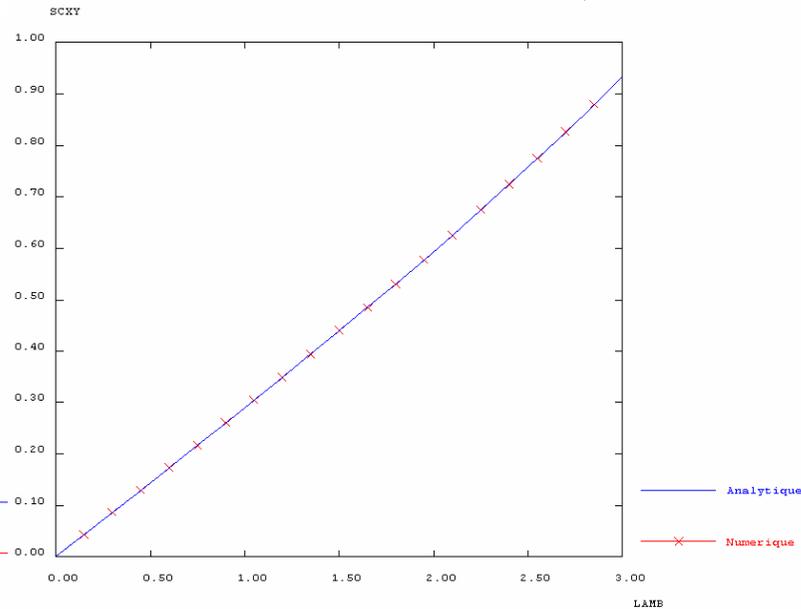
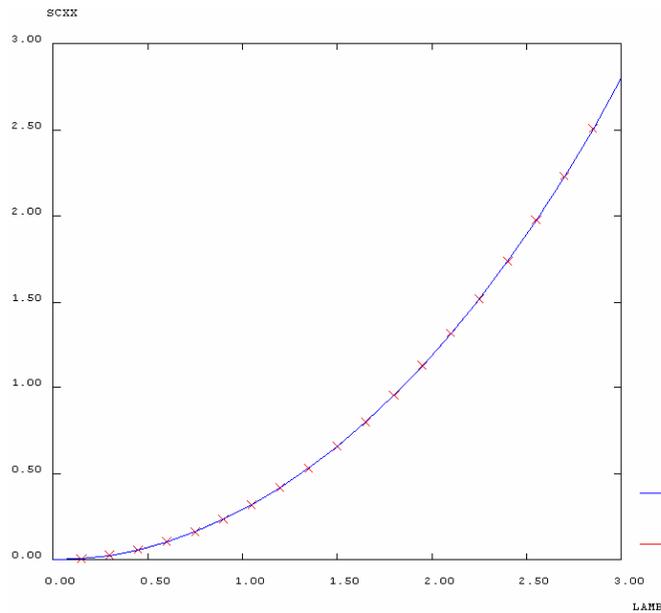
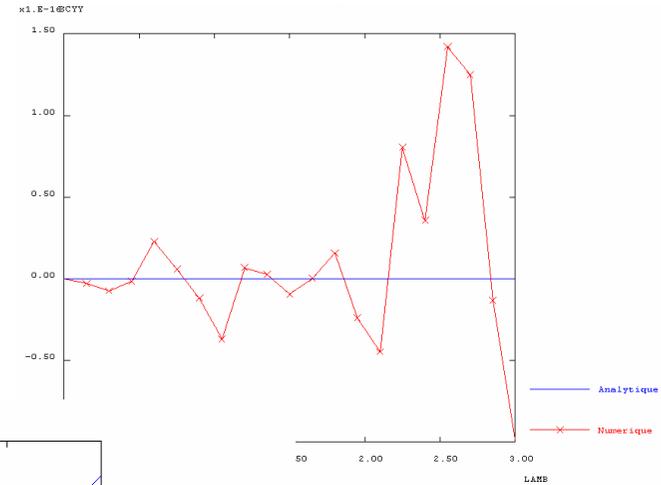


Glissement simple

Solution analytique incompressible Huit Chaines



Cauchy



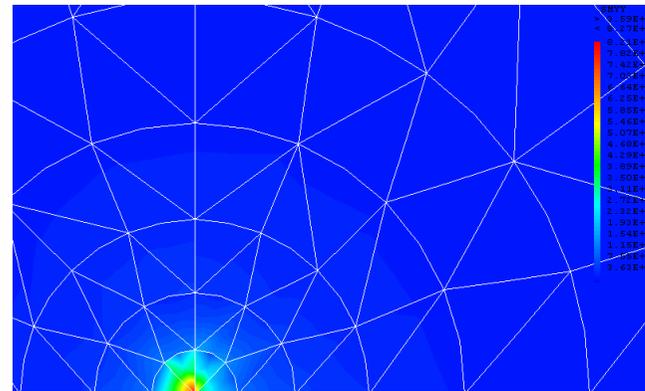
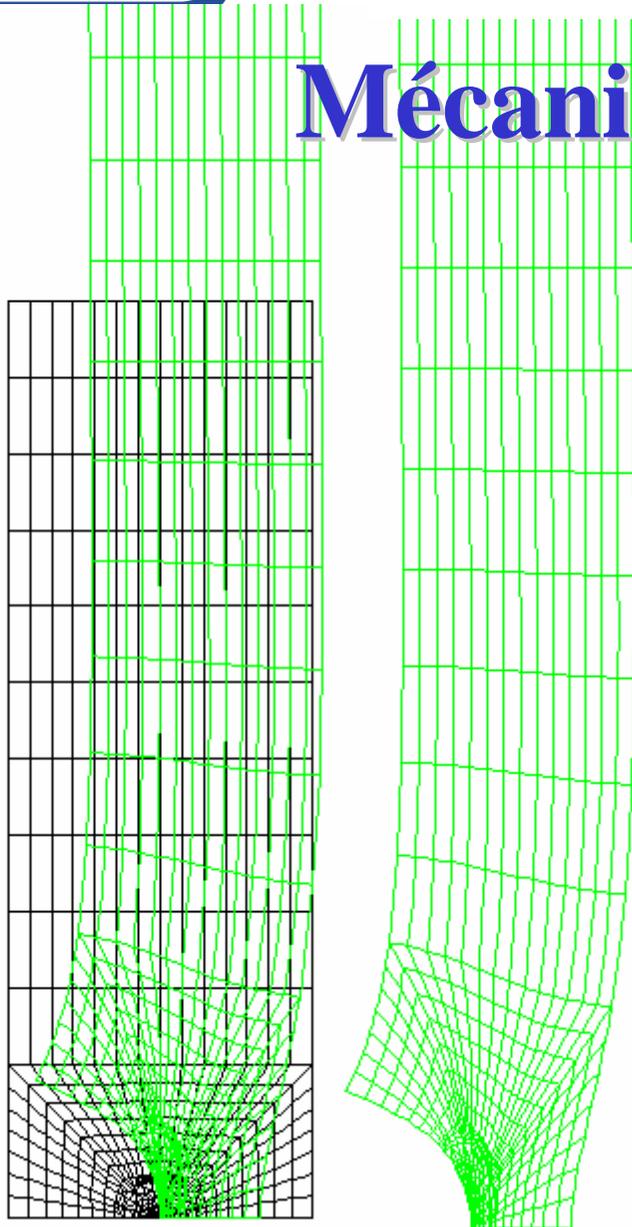
Mécanique de la Rupture

Mooney Rivlin

- Rivlin 1953, Thomas 1960

$$G = 2 k(\lambda) U a$$

$$k(\lambda) = \frac{\pi}{\sqrt{\lambda}}$$



G_Theta

Option

PASAPAS

Plan

- **Contexte**
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 - 2D, 3D, Effet Mullins
- **Conclusion**

Mooney Rivlin

Quasi incompressible 3D

- Densité d'énergie

$$W = C_1 (\bar{I}_1 - 3) + C_2 (\bar{I}_2 - 3) + \frac{1}{D} (J - 1)^2$$

$$C_1 = 0.183 \text{ Pa}, C_2 = 0.0034 \text{ Pa}, D = 10^{-4}$$

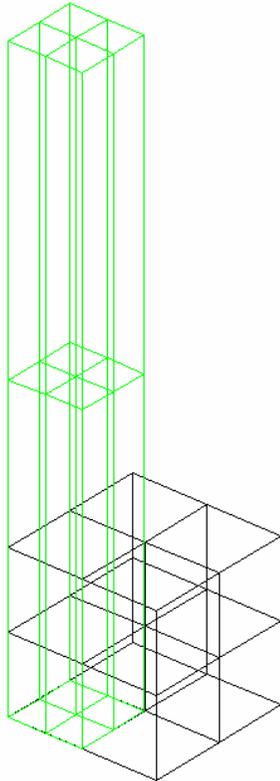
- Fonction de pénalisation (Simo 1988)

- Cas général
$$W = \bar{W}(\bar{I}_1, \bar{I}_2) + U(J)$$

Essai de Traction

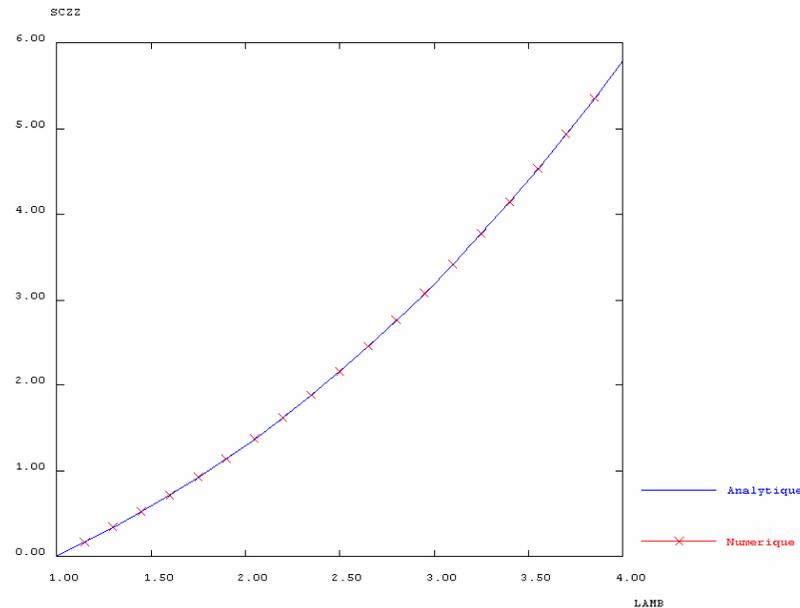
Quasi incompressible Mooney Rivlin 3D

UZ

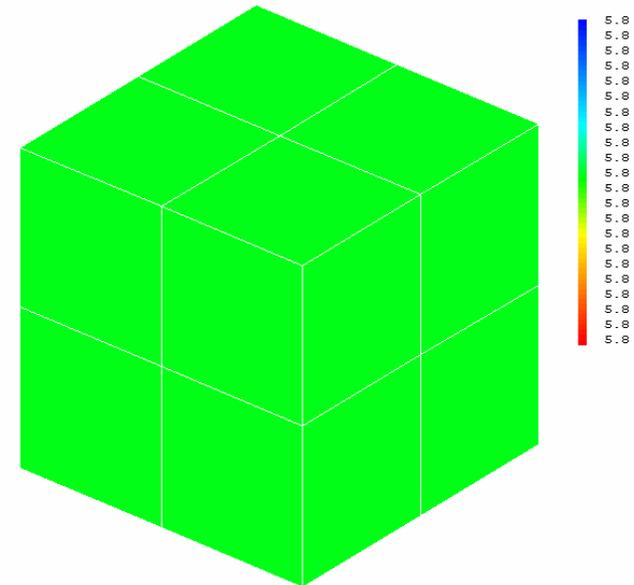


CUB8

$$W = C_1 (\bar{I}_1 - 3) + C_2 (\bar{I}_2 - 3) + \frac{1}{D} (J - 1)^2$$

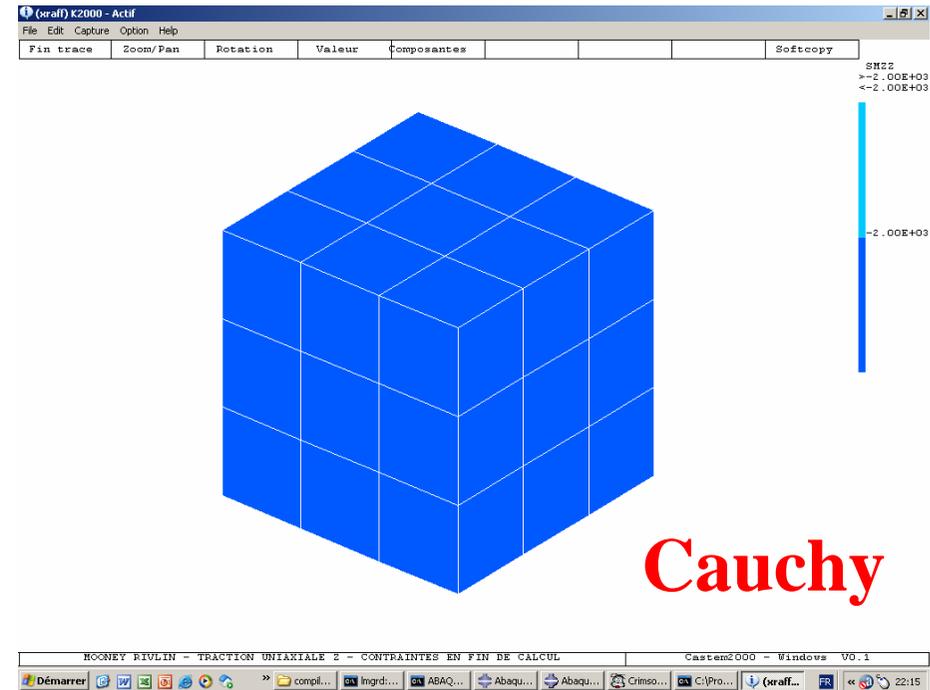
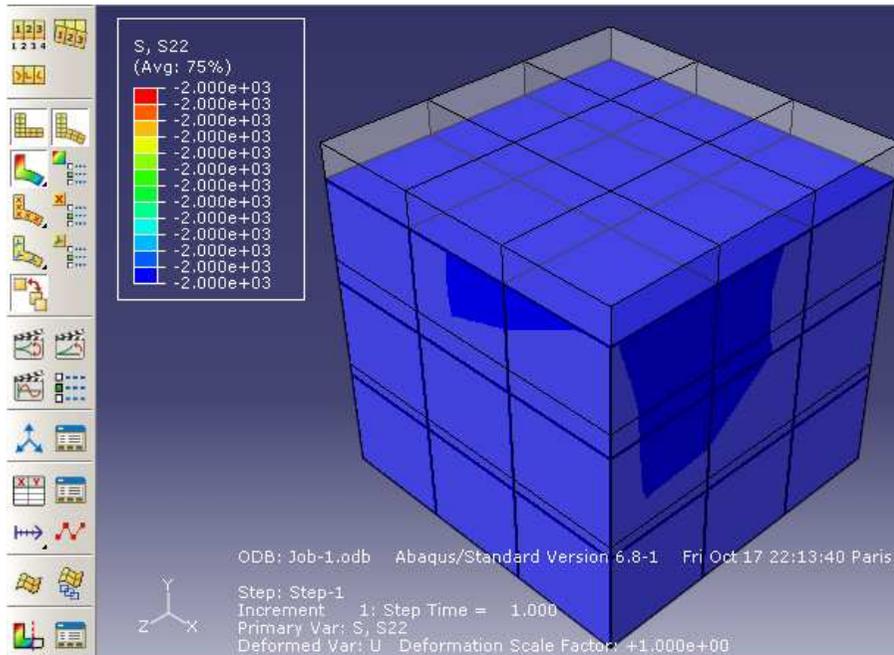


Cauchy



Essai Œdométrique

Mooney Rivlin quasi-incompressible



Cauchy

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

Validations
Abaqus et Cast3M

$$\sigma = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

Plan

- **Contexte**
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Modèle avec effet Mullins

Mécanique de l'endommagement

Thèse G. Chagnon 2003, JMPS 2004

Densité d'énergie hyperélastique : W_0

avec accommodation : $(1-D)W_0$

$$\boldsymbol{\sigma} = -p\mathbf{I} + (1-D)2\mathbf{B} \frac{\partial W_0}{\partial \mathbf{B}}$$

Critère prenant en compte toutes les directions de l'espace

Mesures $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ $\alpha = \sqrt{I_1/3} - 1$ $I_1 = \text{tr}(\mathbf{B})$

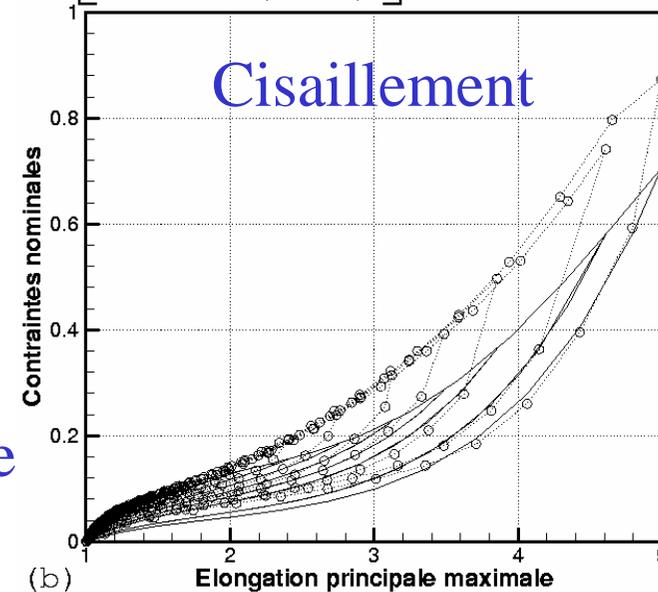
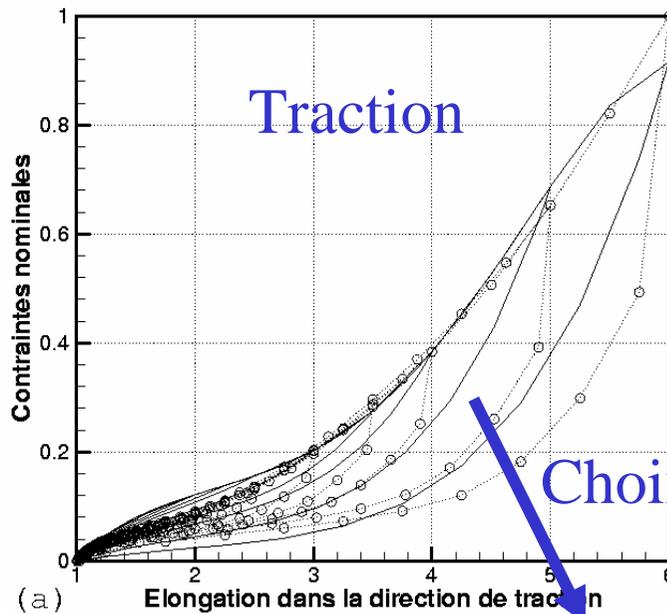
Forme de l'endommagement $D = D(\bar{\alpha}) = D(I_1^{\max})$

Simulation avec effet Mullins

Mécanique de l'endommagement

Forme de la loi choisie

$$D = D_{\infty} \left[1 - \exp\left(-\frac{\bar{\alpha}}{\eta}\right) \right]$$



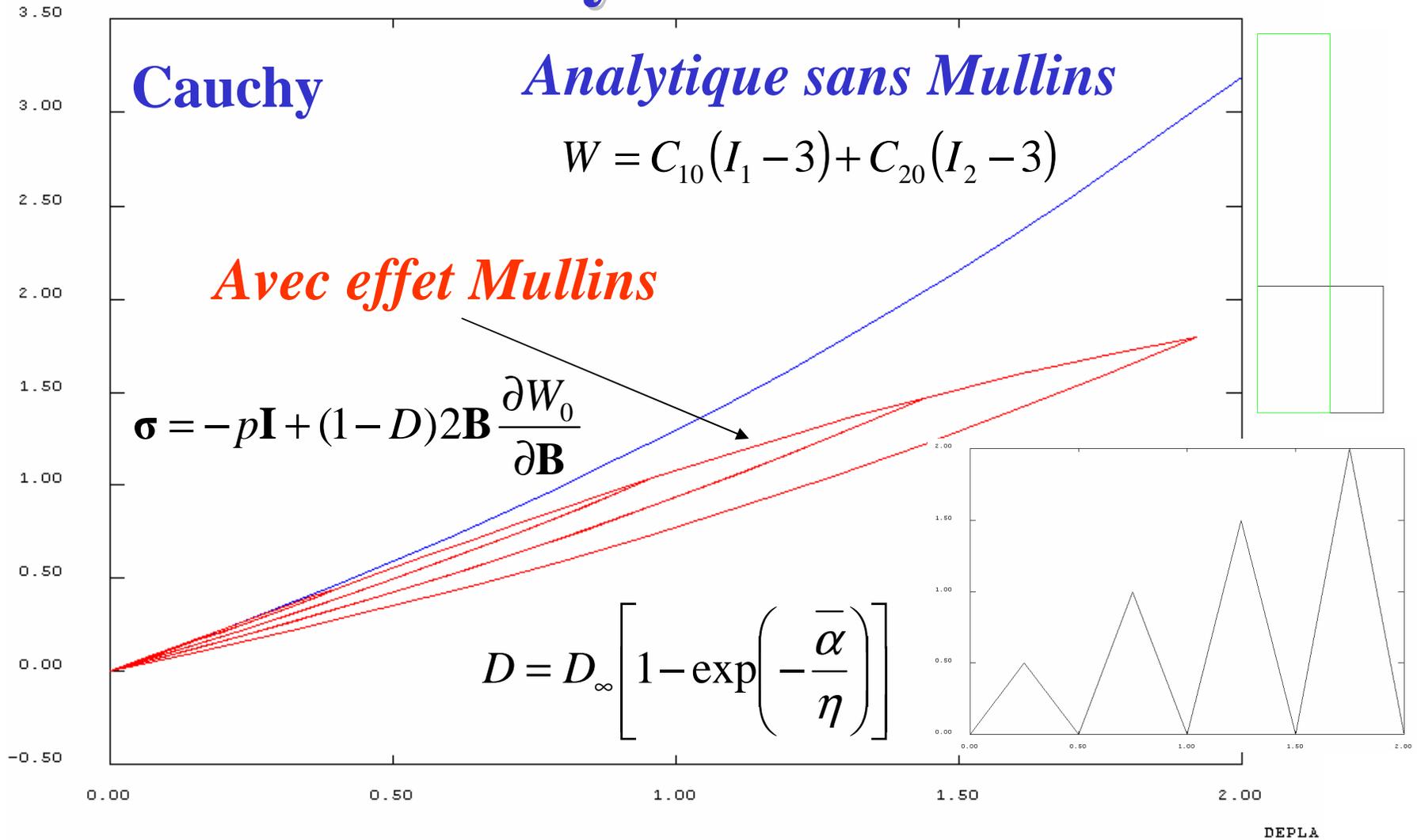
Choix d'une forme

$$W_0 = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$$

Modèle hyperélastique - Yeoh



Mooney et Mullins



Conclusion

- Performance des modèles hyperélastiques
- Simulations des essais : Treloar, Kawabata
- Matériaux Incompressibles
 - Traction, Cisaillement, Biaxiale...
- Matériaux Quasi incompressibles
 - Déformations Planes
 - Tridimensionnel