# NUMERICAL IMPLEMENTATION OF THE ARBITRARY CRACK FRONT FOR THREE DIMENSIONAL PROBLEMS

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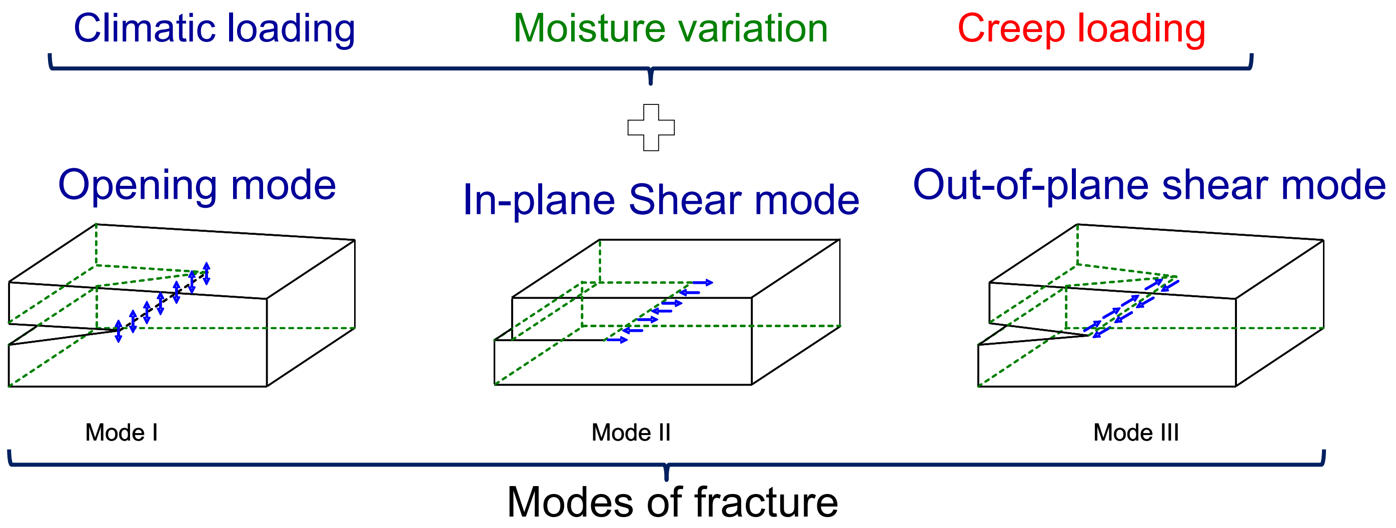
# Scientific context

- ❑ Predicting the behavior of structures under mixed mode loading
- ❑ Developing specific tools for three-dimensional configurations
- ❑ Consider thickness effect under variable environments
- ❑ Need for applications related to the inspection and diagnosis of structures



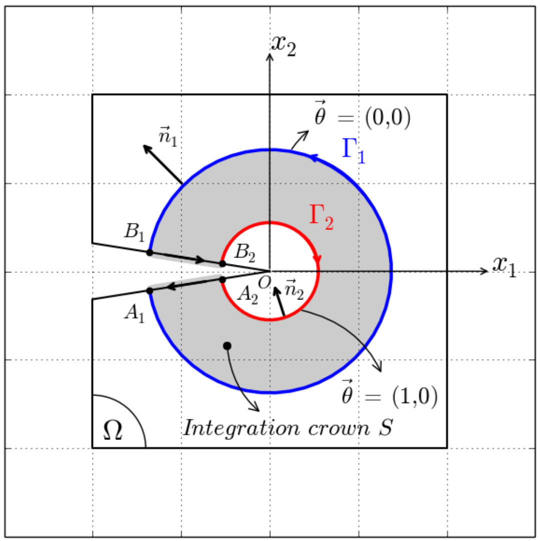
# Plan

- Scientific context
- Part I : three dimensional contour integral generalizations
  - Analytical formulation ($J^{3D}$ and $G_\theta^{3D}$ -integral)
  - Numerical implementation
- Part II : mixed mode three dimensional contour integral
  - Analytical formulation ($M_\theta^{3D}$-integral)
  - Mixed mode Numerical implementation
- Conclusions and outlooks

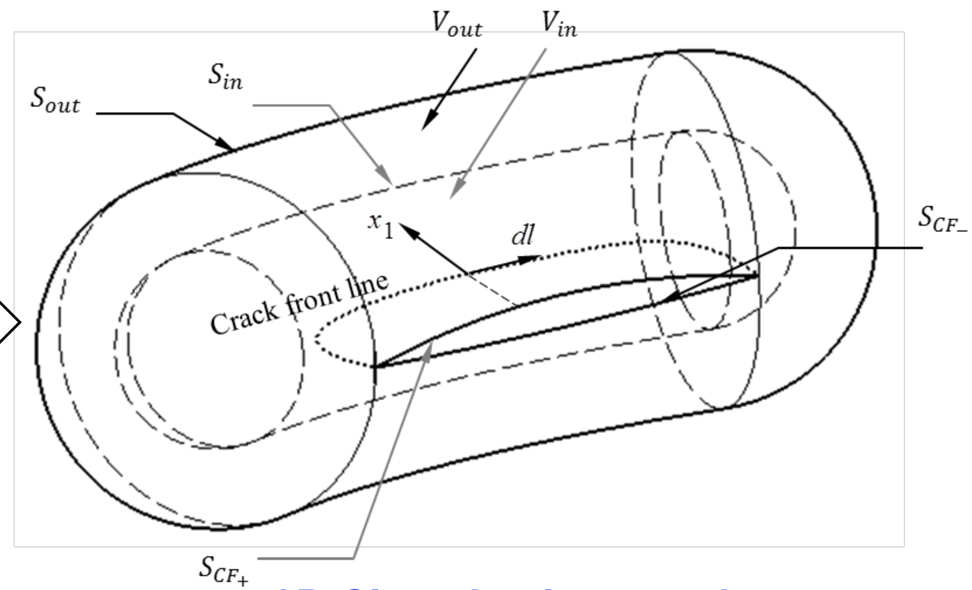
# $J$-integral in 3D configurations

Rice's integral

$$J^{2D} = \int_\Gamma \left(W.n_1 - (\sigma_{ij}.n_j.u_{i,1})\right).d\Gamma$$




**Integration domain size for 2D**




**3D Closed volume and surfaces integration domains**

The main advance of this form is the presence of an arbitrary crack front line enclosed by a 3D surface

# $J^{3D}$ and $G_\theta^{3D}$-integral description

The J-integral formulation is based on the Noether's theorem application (Noether 1918) :

$$\delta L = \int_V \int_t \delta W . dV . dt = 0$$

A Gauss-Ostrogradski transformation allows us writing the Lagrangian's invariance in the form:

$$\int_S \left(W . n_k - \left(\sigma_{ij} . n_j . u_{i,k}\right)\right) . dS + \int_V \left(\sigma_{ij} . \left(\varepsilon_{ij}\right)_{,k} - W_{,k}\right) . dV = 0$$

the $J^{3D}$-integral is defined as below:

$$J^{3D} = \int_{S_{\Gamma_{out}}} \left(W . n_1 - \left(\sigma_{ij} . n_j . u_{i,1}\right)\right) . dS - \int_{S_{cr}} \sigma_{ij} . u_{i,1} . n_j . dS - \int_{V_{\Gamma_{out}}} \left(W_{,1} - \sigma_{ij} . \left(\varepsilon_{ij}\right)_{,1}\right) . dV$$

1 Stationary crack

2 Crack lips pressure

3 Crack propagation

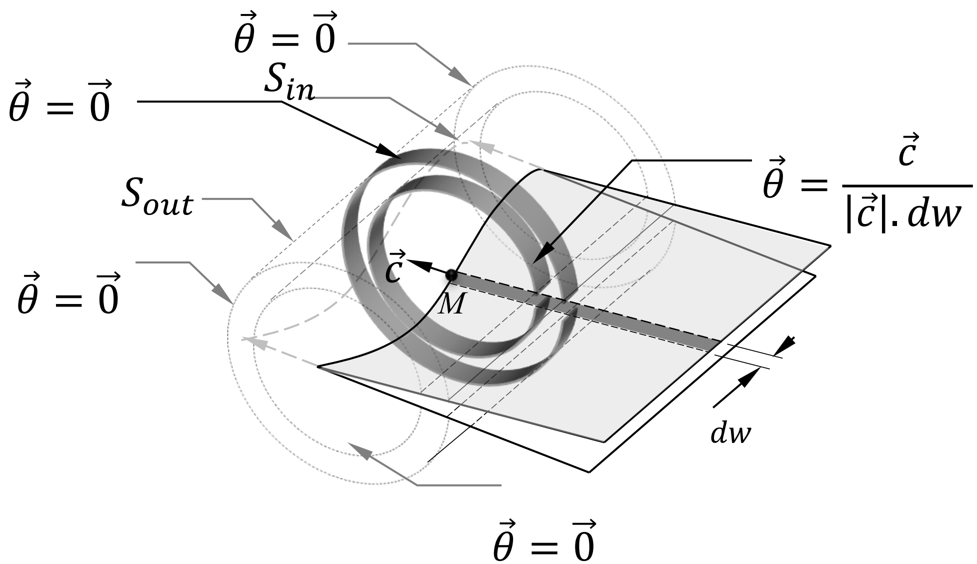
$$G_\theta = \int_V \left(P_{kj} . \theta_{k,j}\right) . n_j . dV + \int_{S_{CF}} \sigma_{ij} . u_{i,k} . n_j . \theta_k . dS - \int_{V_{\Gamma_2}} \left(W_{,k} - \sigma_{ij} . \left(\varepsilon_{ij}\right)_{,k}\right) . \theta_k . dV$$

1 2 3

# $G(M)$ integral

$$G_\theta = \int_V (P_{kj}.\theta_{k,j}).n_j.dV + \int_{S_{CF}} \sigma_{ij}.u_{i,k}.n_j.\theta_k.dS - \int_{V_{\Gamma_2}} \left(W_{,k} - \sigma_{ij}.\left(\varepsilon_{ij}\right)_{,k}\right).\theta_k.dV$$

1 2 3



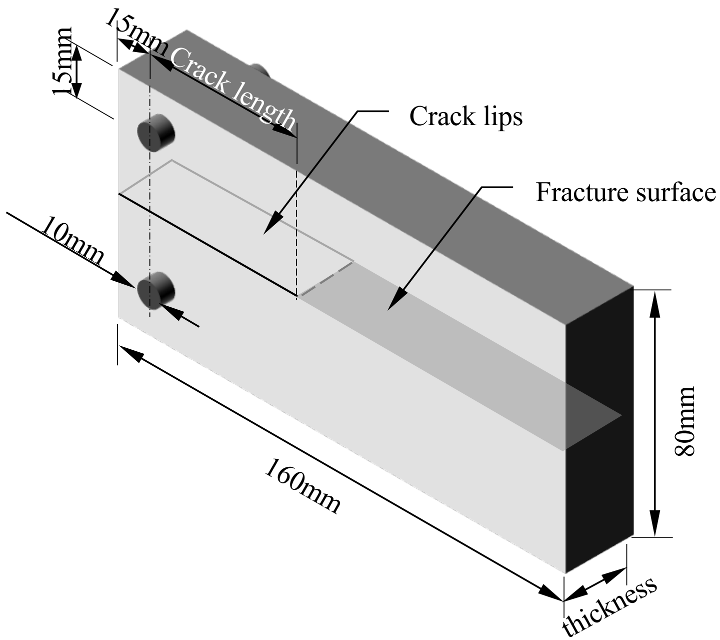

Definition of $\vec{\theta}$ around a close crown

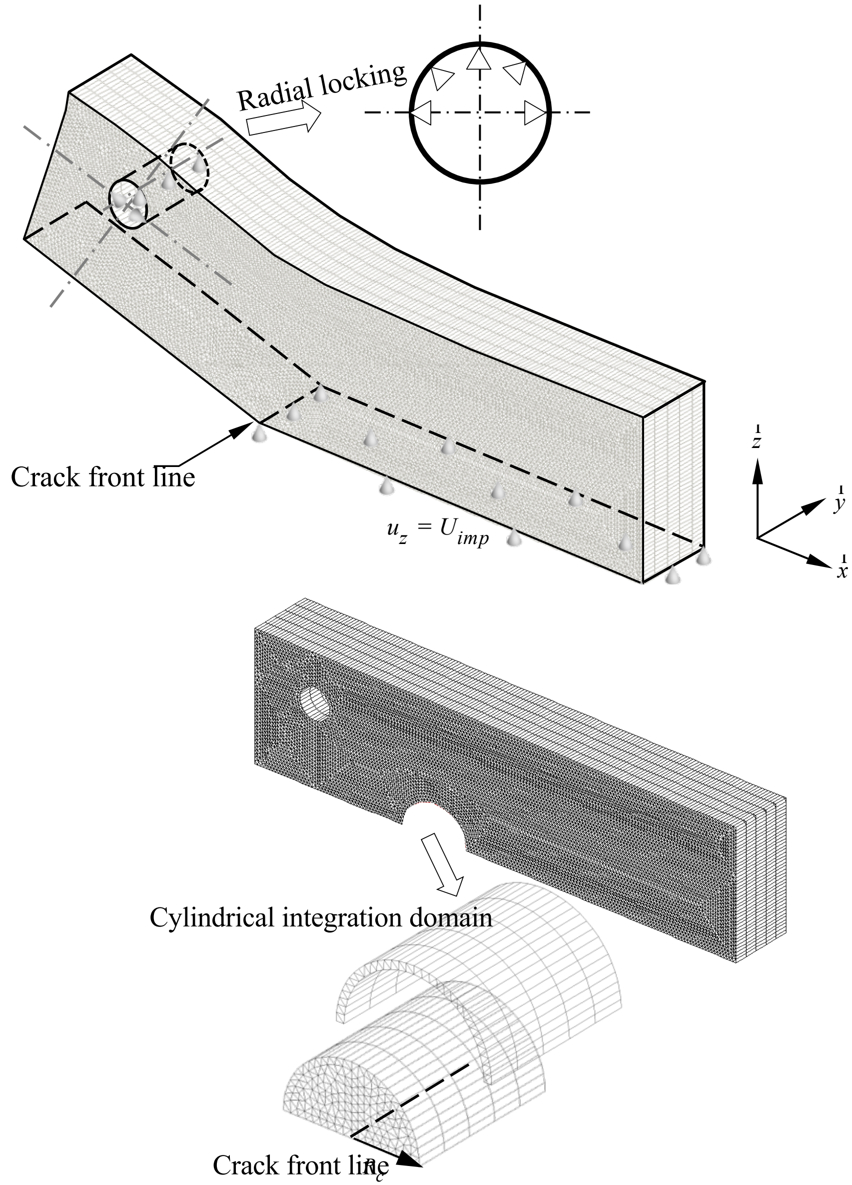
$$\vec{\theta} = 0 \ on \ S_{\Gamma_{out}}, \vec{\theta} = \frac{\vec{c}}{|\vec{c}|.dw} \ on \ C \cap S_{\Gamma_{out}} \ and \ \vec{\theta} = 0 \ on \ S_{\Gamma_{in}}$$

The average energy release rate can be calculated with an integration of along the crack front line divided by the crack width

# Numerical validation

The finite element implementation is based on a Double Cantilever Beam loaded in an open mode.



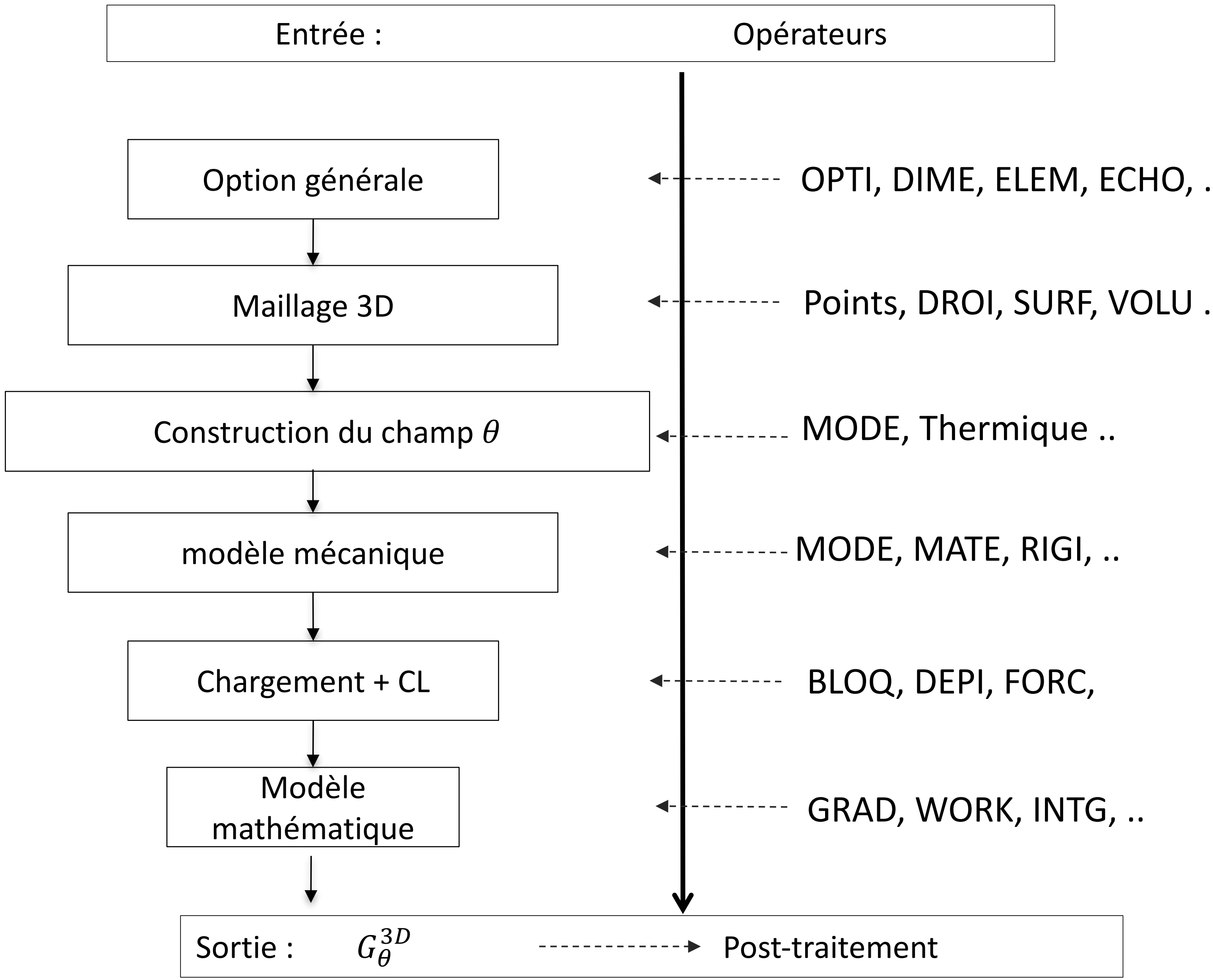




**Specimen DCB**

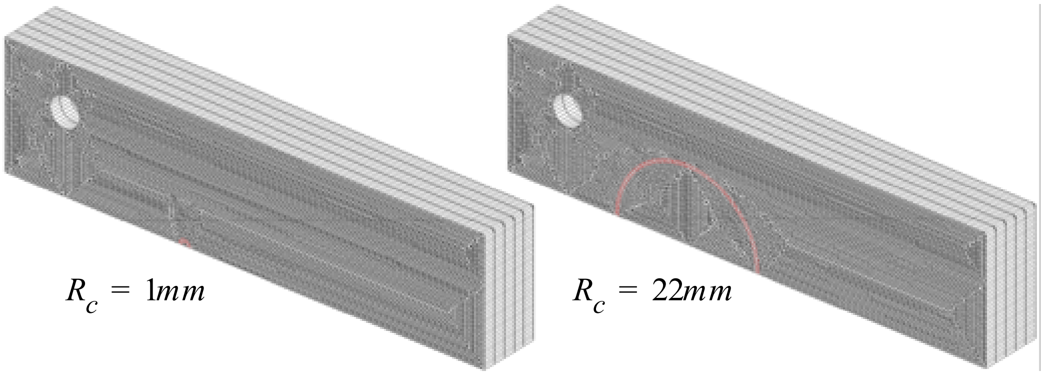
Material is characterized by :

- Elastic isotropic
- Thickness =20 mm
- Linear crack front line
- $E = 210MPa,\ \nu = 0,3$

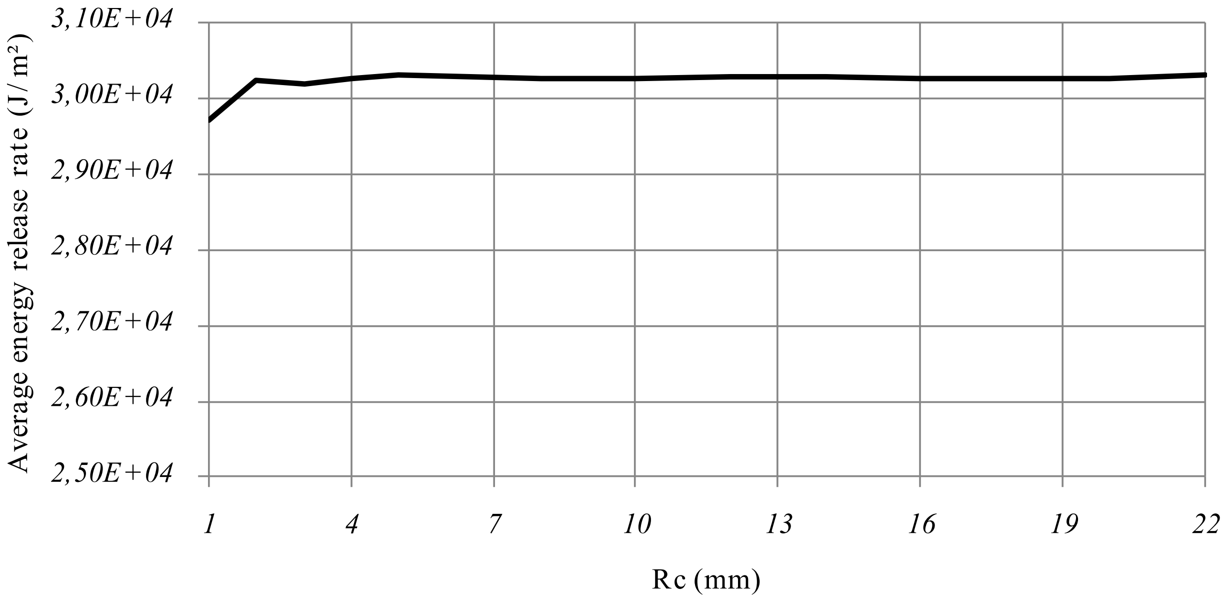
# Algorithm for crack growth process



# Numerical validation




We can shows the variations of the energy release rate versus $R_c$:

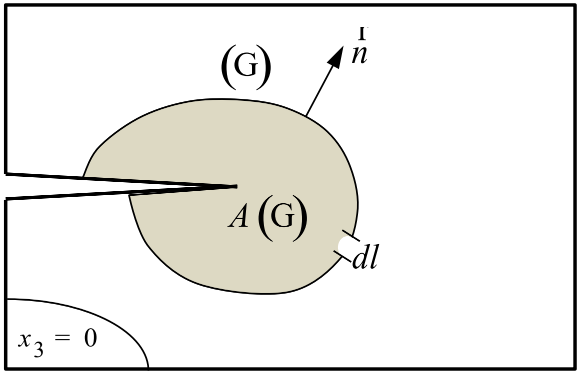



Numerical results validate the non-dependence of the integration domain with an average value of 30.3kJ/m²

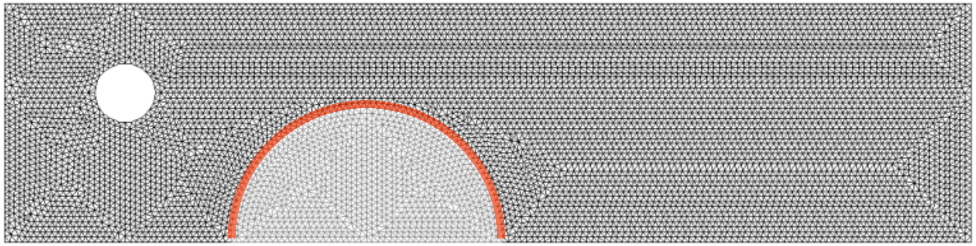
# Physical interpretation

Surface integration domains for the Bui's integral :

$$J_{Am} = \underbrace{\int_\Gamma \left(W.n_1 - (\sigma_{ij}.n_j.u_{i,1})\right).d\Gamma}_{J^{2D}} - \int_{A(\Gamma)} \frac{d}{dx_3}(\sigma_{i3}.u_{i,1}).dA(\Gamma)$$




Integration domains

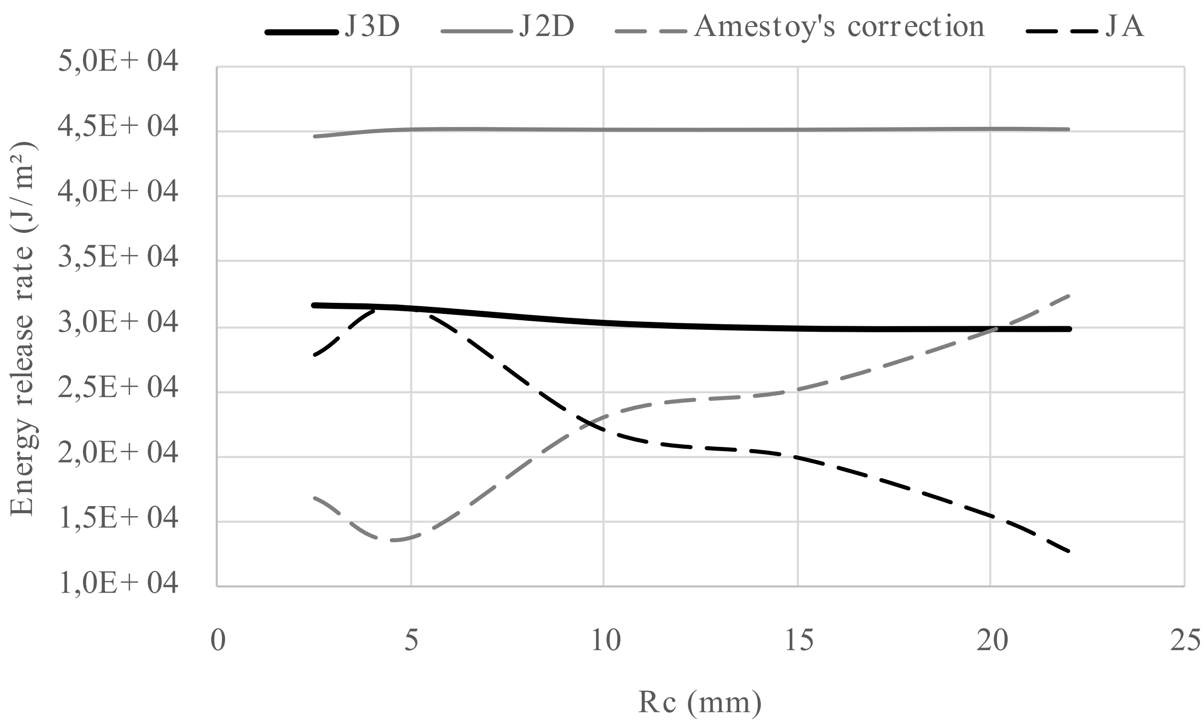



Integration domain size for 2D model

# Physical interpretation

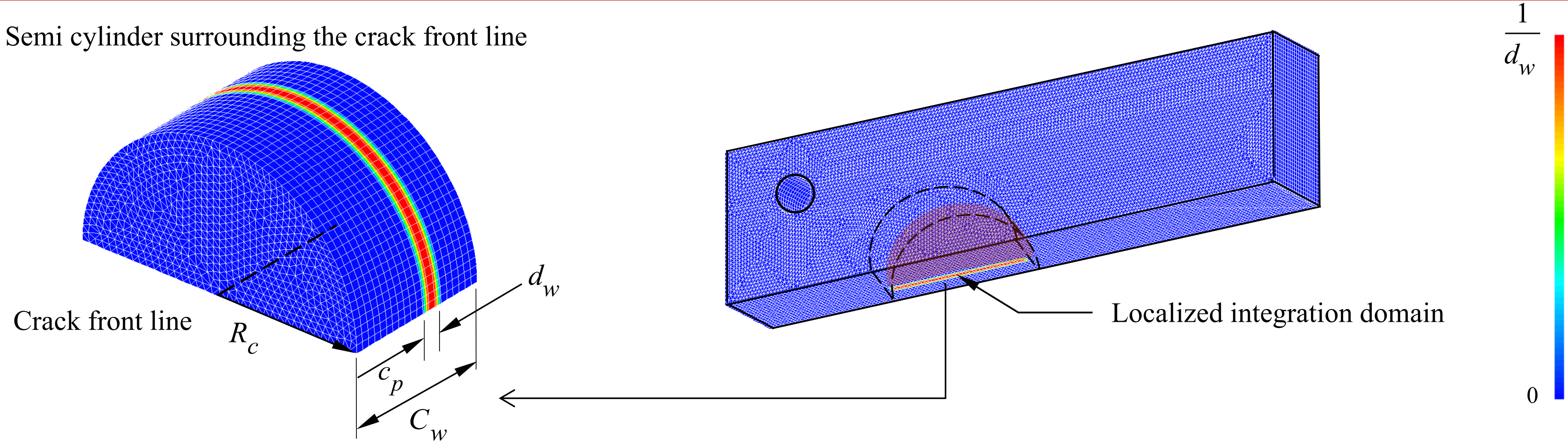
Surface integration domains for the Bui's integral :

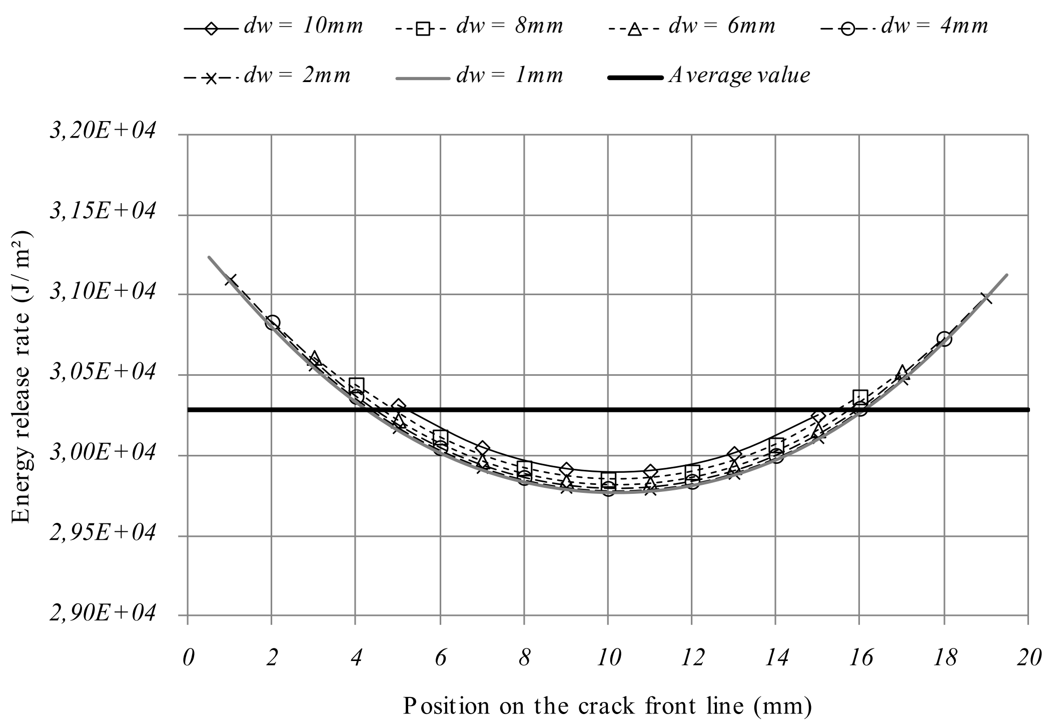
$$J_{Am} = \underbrace{\int_{\Gamma} \left(W.n_1 - \left(\sigma_{ij}.n_j.u_{i,1}\right)\right).d\Gamma}_{J^{2D}} - \int_{A(\Gamma)} \frac{d}{dx_3}\left(\sigma_{i3}.u_{i,1}\right).dA(\Gamma)$$



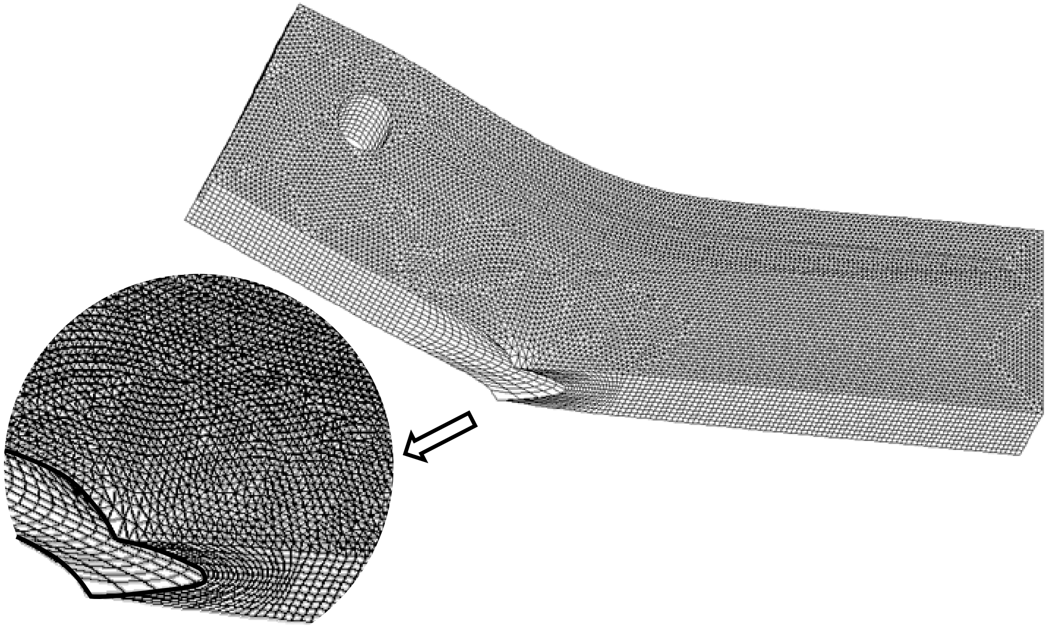

Comparison between $J^{2D}$ and $J^{3D}$ approaches

# Numerical validation



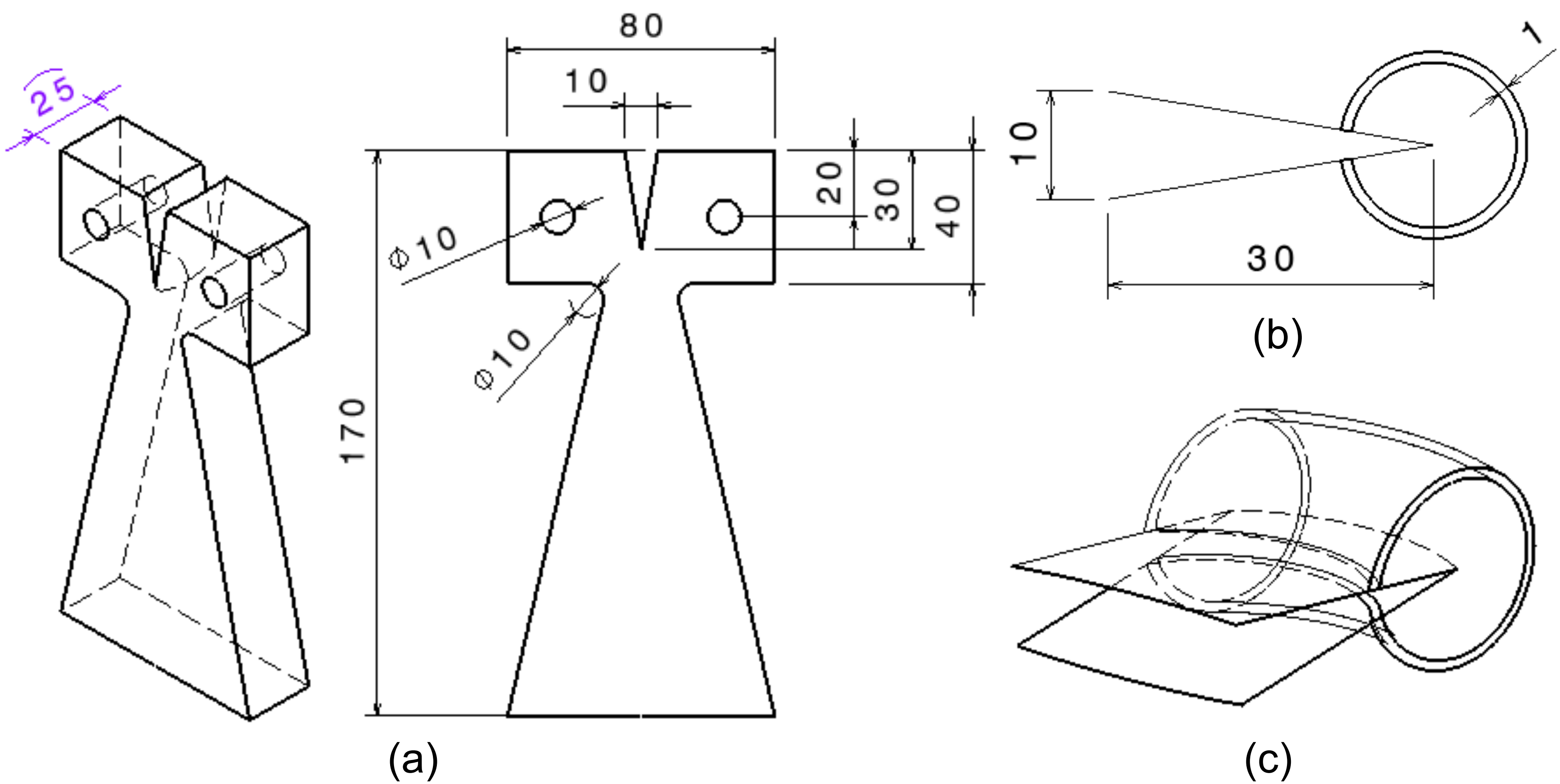
# Numerical validation




| Layer | Average crack length (mm) | $J^{3D}$ | $J^{2D}$ | $JA$ |
|---|---|---|---|---|
| 1 | 61.8 | 42394 | 37646 | 30065 |
| 2 | 65 | 21687 | 31710 | 22148 |
| 3 | 67.4 | 14523 | 28034 | 19118 |
| 4 | 69 | 12025 | 25835 | 17417 |
| 5 | 69.8 | 11041 | 24835 | 16673 |

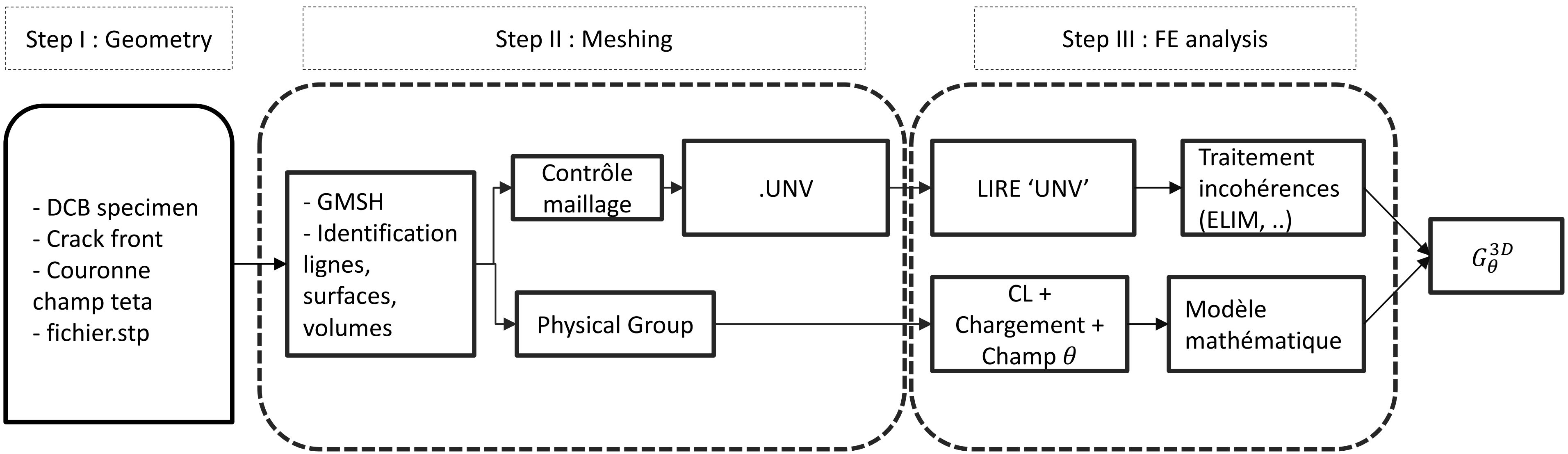
# Numerical validation

Using CAD software we can define our specimen as :




DCBVI specimen (a), two dimensional crack tip (b)
and three-dimensional elliptical crack front (c)

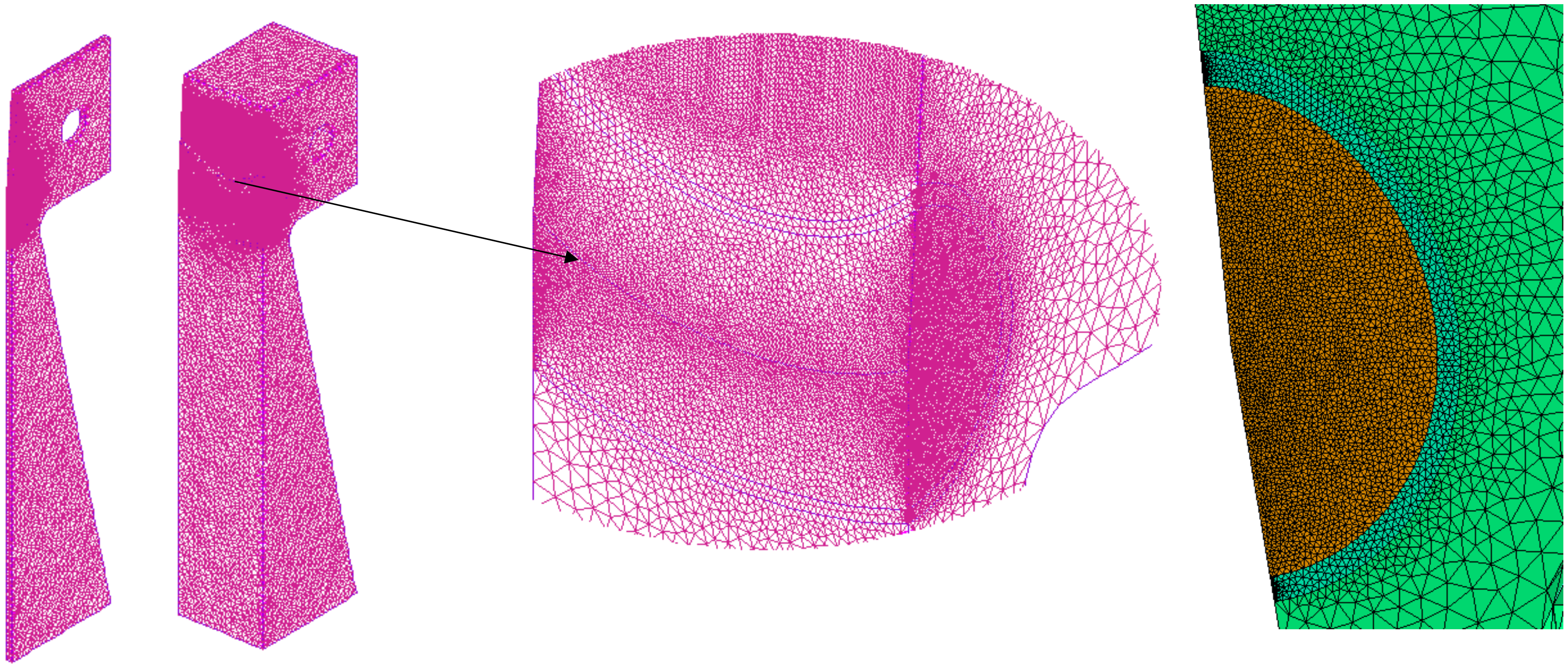
# Algorithm for crack growth process




Post traitement :

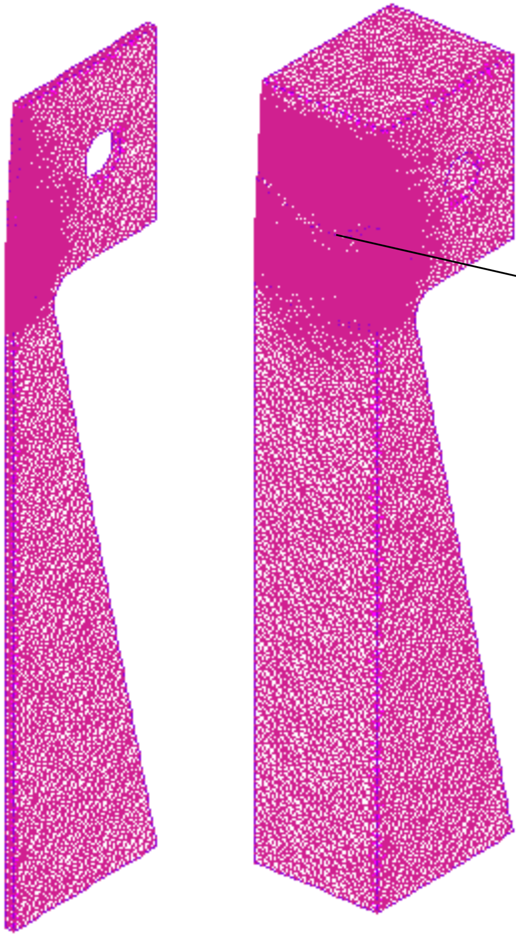
- ✓ Calcul de G à partir du l'intégrale G_teta_3D
- ✓ Vérification de l' indépendance du chemin d'intégration
- ✓ Tracer l'évolution de K1 le long du cfl
- ✓ …

# Numerical validation

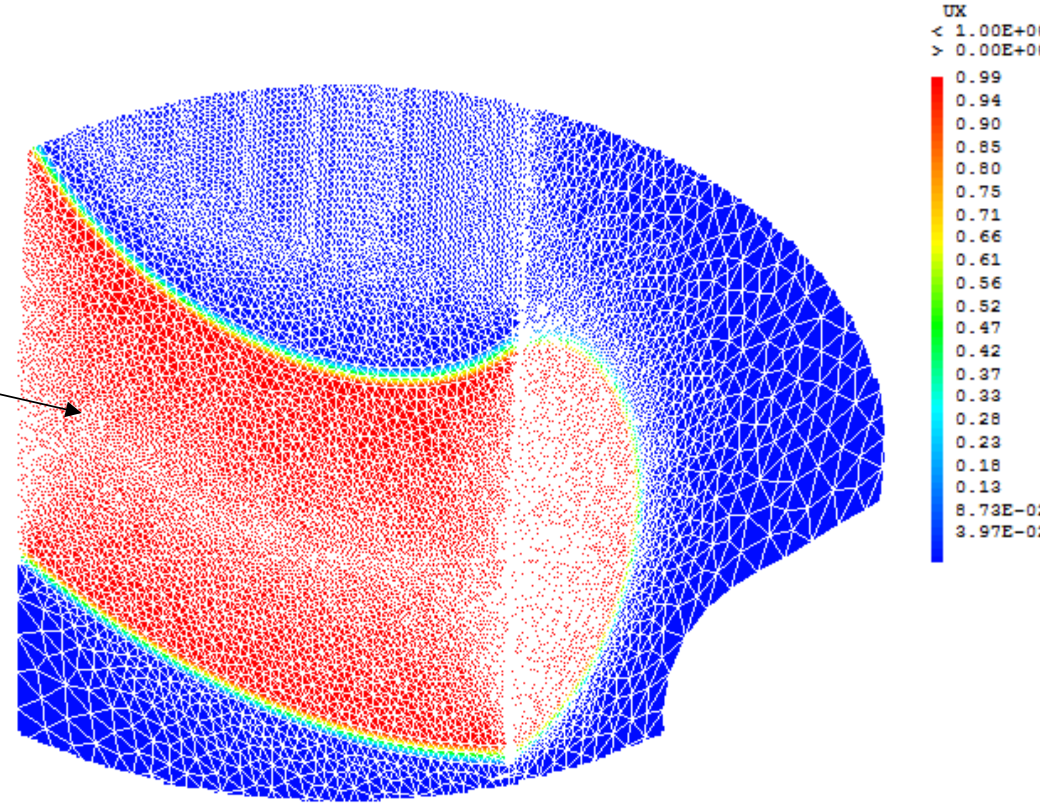


DCB Mesh with theta field : Typical FE meshes of the ½ half DCBVI specimen

# Numerical validation

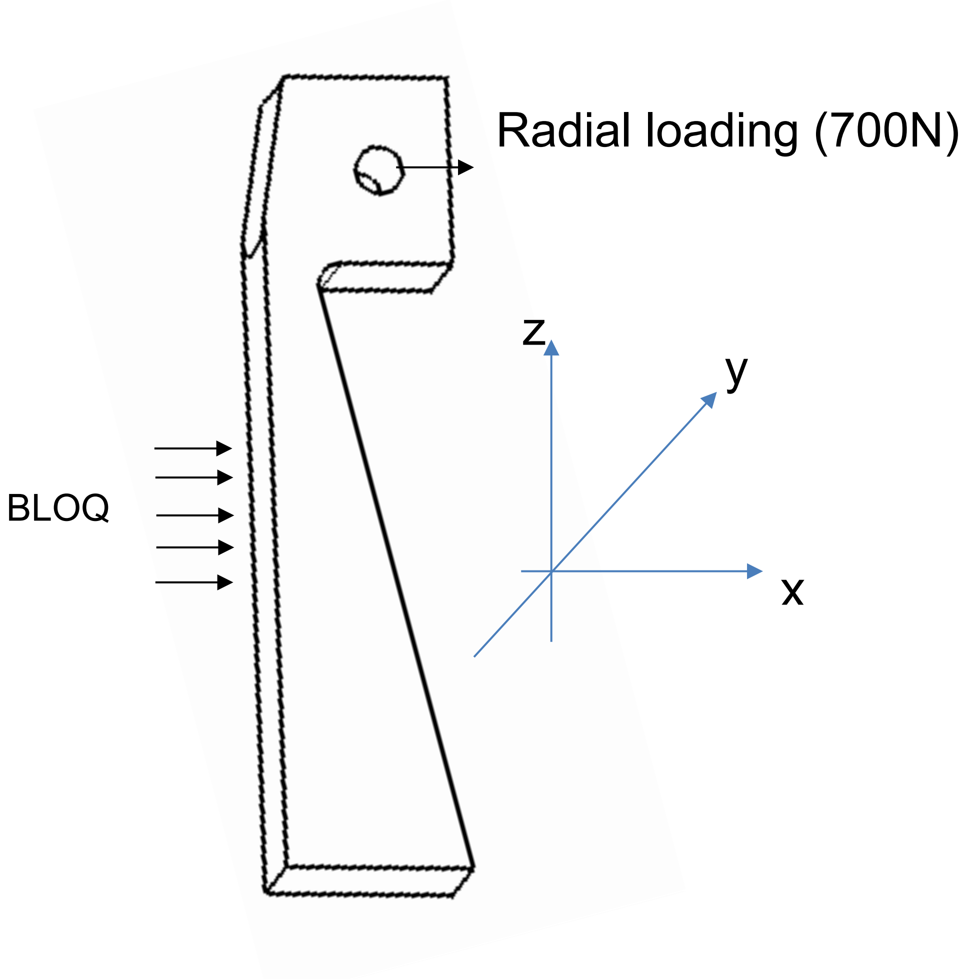


Maillage 30mm

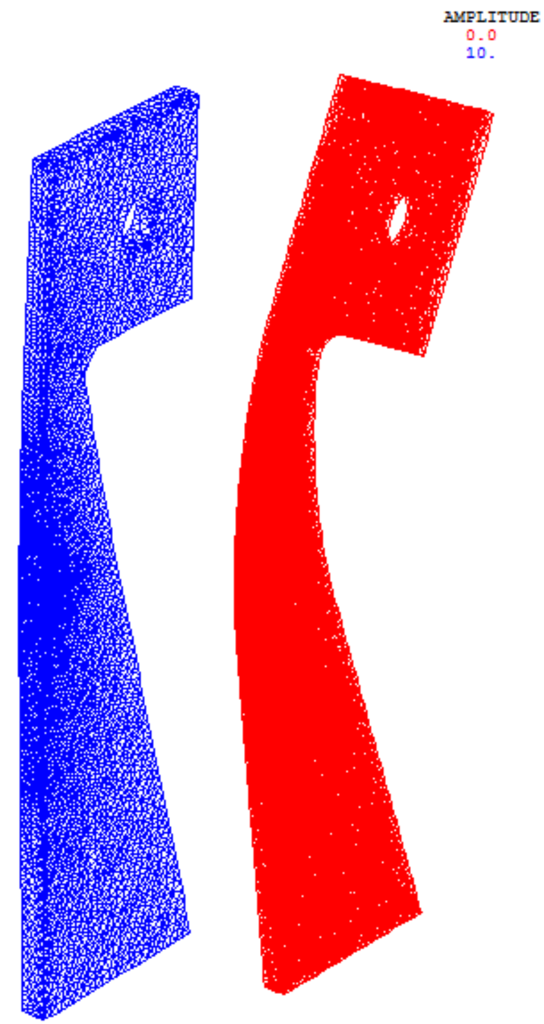



Theta fields definition (30mm)

# Numerical validation

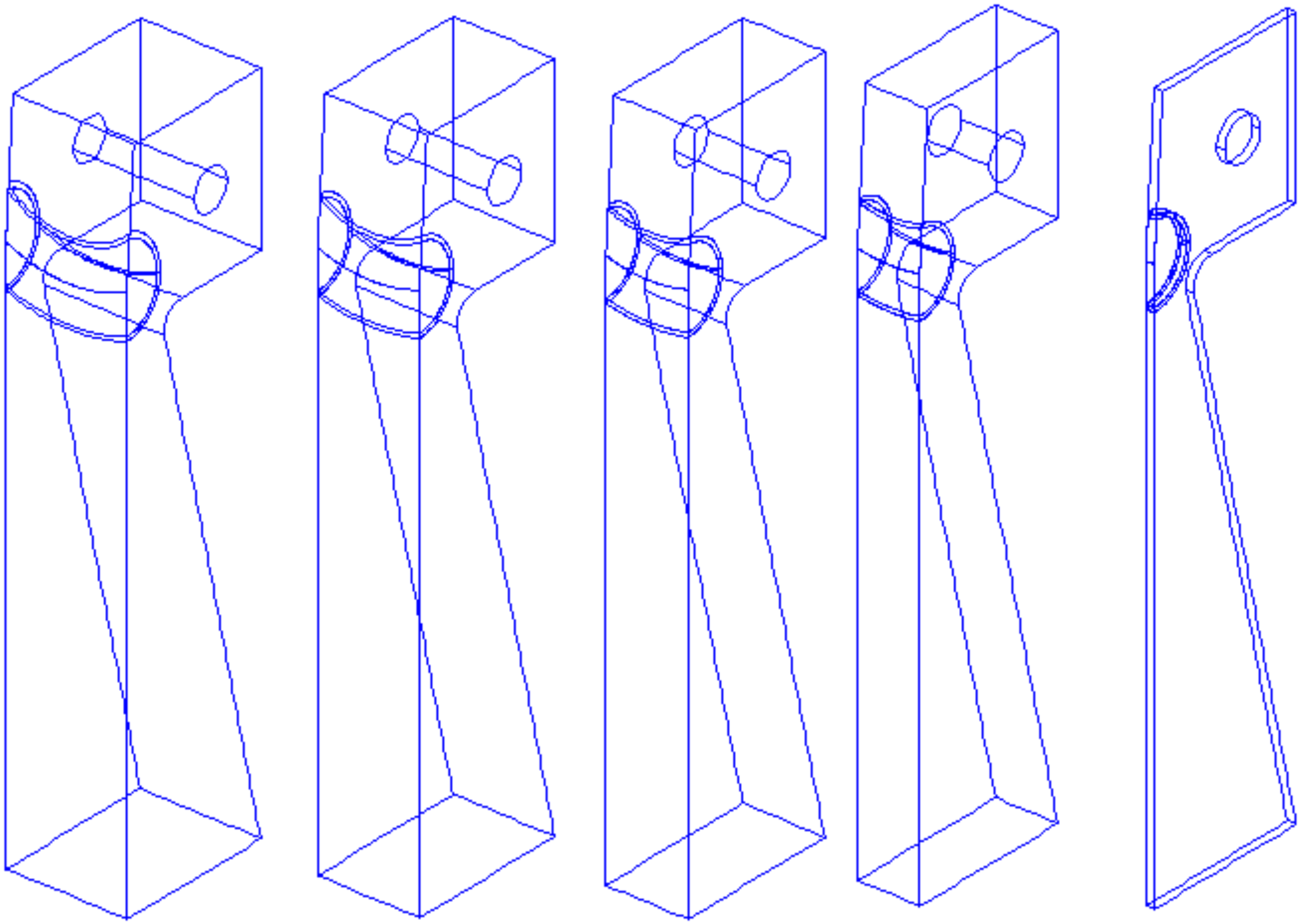



Boundary conditions




La deformée(Castem)

# Numerical validation




Vue filiaire des segments

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# $M^{3D}$ Integral

The M-integral formulation is based on the Noether's theorem application :

$$\delta L = \int_V \int_t \delta W . dV . dt = 0$$

A Gauss-Ostrogradski transformation allows us writing the Lagrangian's invariance in the form:

$$\int_S \left( W^* . n_k - \left( \frac{\partial W^*}{\partial u_{i,j}} . n_j . u_{i,k} \right) - \left( \frac{\partial W^*}{\partial v_{i,j}} . n_j . v_{i,k} \right) \right) . dS$$

$$+ \int_V \left( \left( \frac{\partial W^*}{\partial u_{i,\alpha}} . \delta u_{i,k} \right)_{,\alpha} + \left( \frac{\partial W^*}{\partial v_{i,\alpha}} . \delta v_{i,k} \right)_{,\alpha} - W^*{}_{,k}(u) - W^*{}_{,k}(v) \right) . dV = 0$$

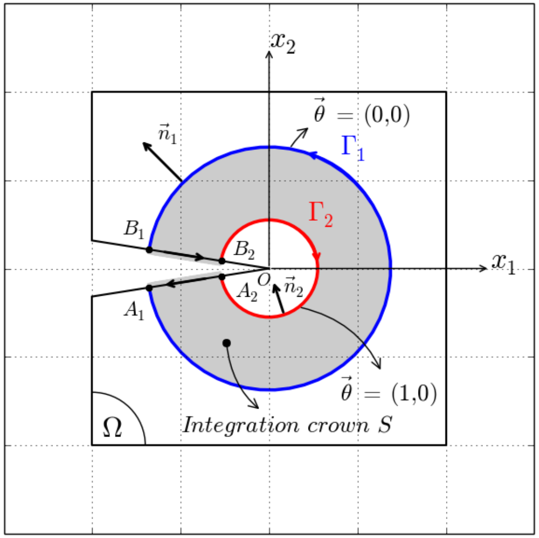
$M^{3D}$-integral

$$M^{3D} = \int_{S_{\Gamma_1}} \left( \sigma_{ij}^{v} . u_{i,j} . n_k - \frac{1}{2} \left( \sigma_{ij}^{v} . u_{i,k} + \sigma_{ij}^{u} . v_{i,k} \right) . n_j \right) . dS$$ 1

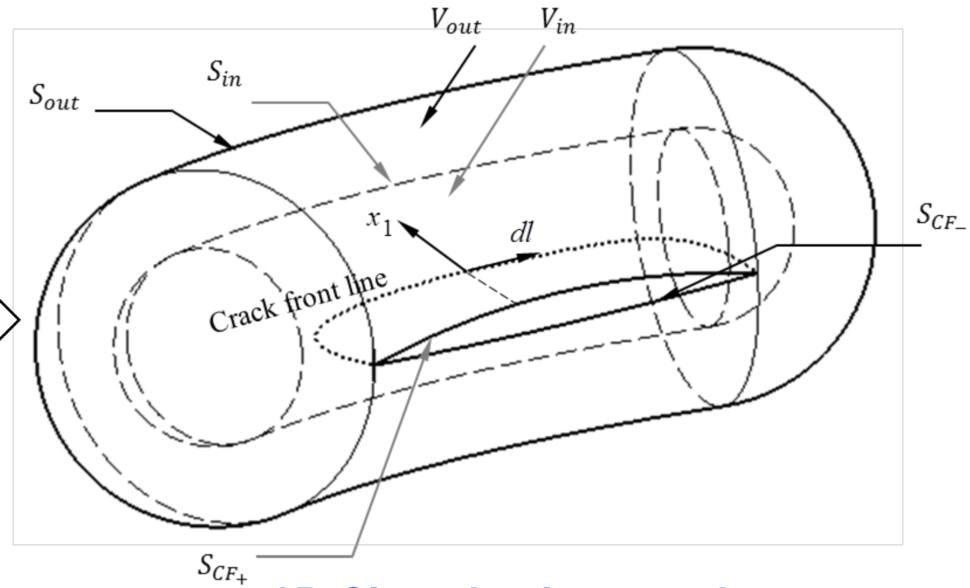
$$+ \int_{S_{CF}} \left( \sigma_{ij}^{v} . u_{i,j} . n_k - \frac{1}{2} \left( \sigma_{ij}^{v} . u_{i,k} + \sigma_{ij}^{u} . v_{i,k} \right) . n_j \right) . dS$$ 2

$$+ \frac{1}{2} \int_{V_{\Gamma_1}} \left( \left( \sigma_{ij}^{v} . \left( \varepsilon_{ij}^{u} \right)_{,k} + \sigma_{ij}^{u} . \left( \varepsilon_{ij}^{v} \right)_{,k} \right) - \left( \left( \sigma_{ij}^{v} . \varepsilon_{ij}^{u} \right)_{,k} + \left( \sigma_{ij}^{u} . \varepsilon_{ij}^{v} \right)_{,k} \right) \right) . dV$$ 3

# $M_{\theta}^{3D}$ Integral




**Integration domain size for 2D**




**3D Closed volume and surfaces integration domains**

$M_{\theta}^{3D}$-integral

$$M_{\theta}^{3D} = \frac{1}{2}\int_{V} P_{kj}.\theta_{k,j}.dV \qquad 1$$

$$-\frac{1}{2}\int_{S_{CF}} \left(\sigma_{ij}^{v}.u_{i,k} + \sigma_{ij}^{u}.v_{i,k}\right).n_j.\theta_k.dS \qquad 2$$

$$+\frac{1}{2}\int_{V_{\Gamma_2}} \left(\left(\sigma_{ij}^{v}.\left(\varepsilon_{ij}^{u}\right)_{,k} + \sigma_{ij}^{u}.\left(\varepsilon_{ij}^{v}\right)_{,k}\right) - \left(\left(\sigma_{ij}^{v}.\varepsilon_{ij}^{u}\right)_{,k} + \left(\sigma_{ij}^{u}.\varepsilon_{ij}^{v}\right)_{,k}\right)\right).\theta_k.dV \qquad 3$$

# Numerical validation

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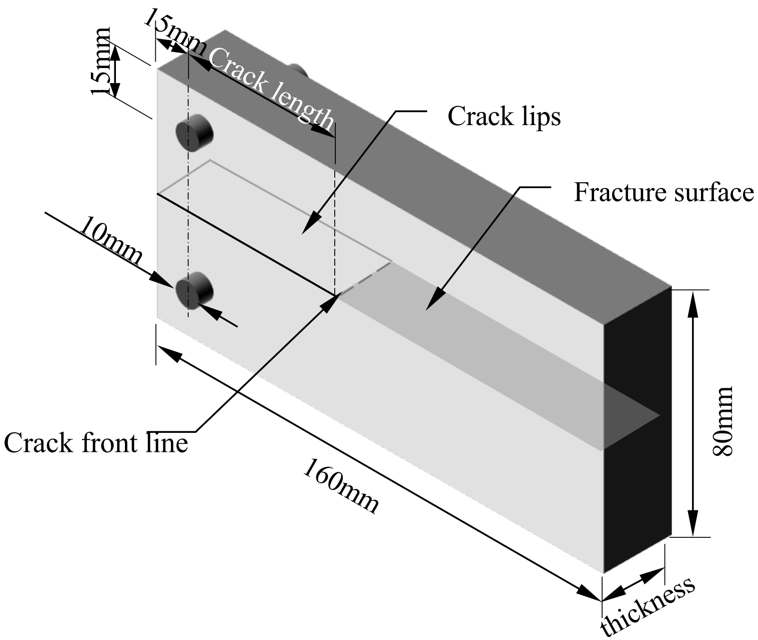
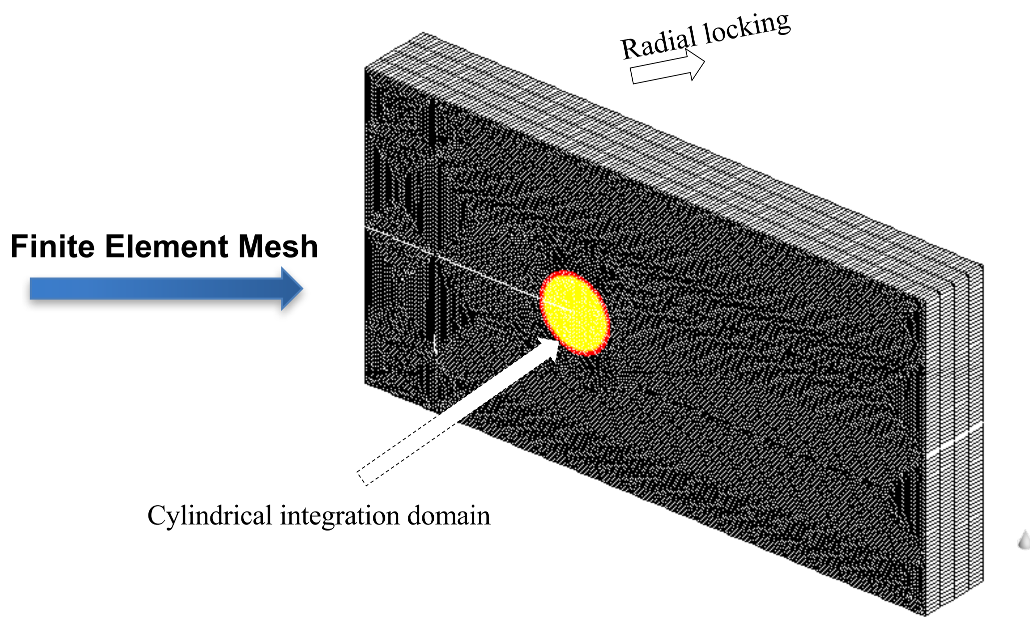
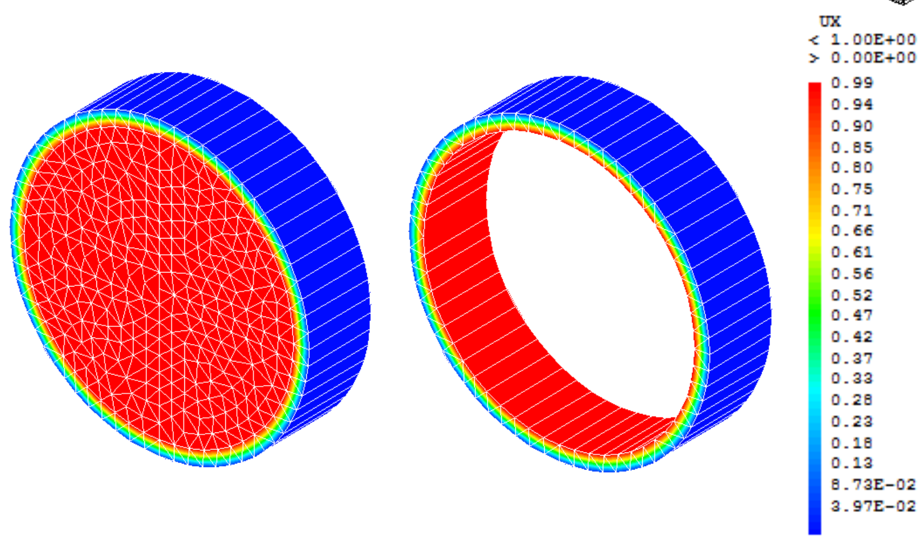



Specimen DCB



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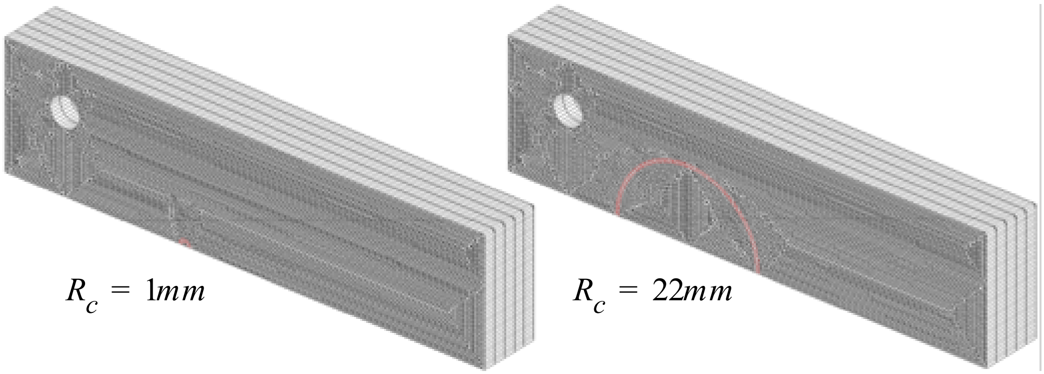


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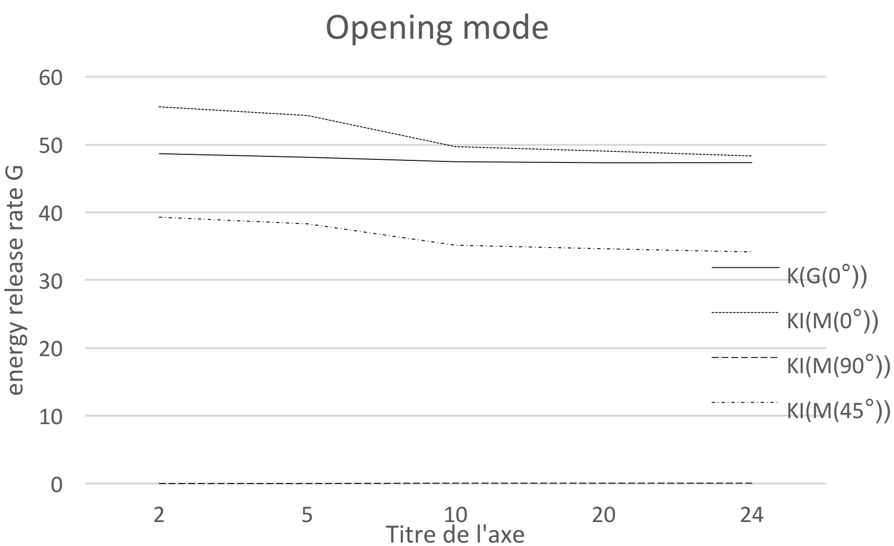
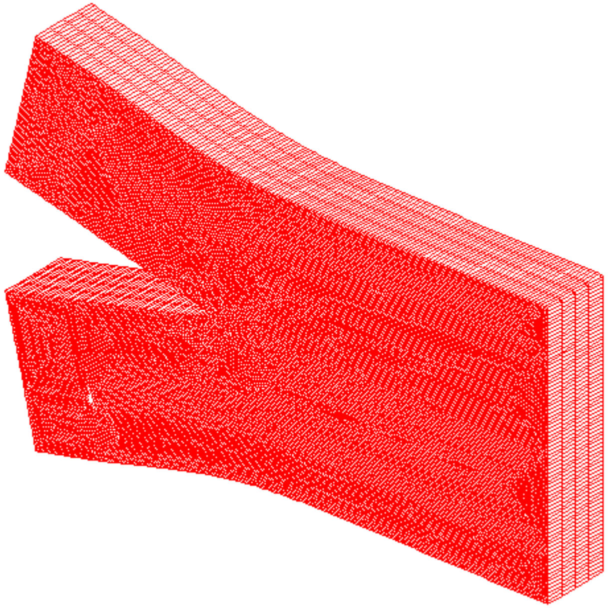
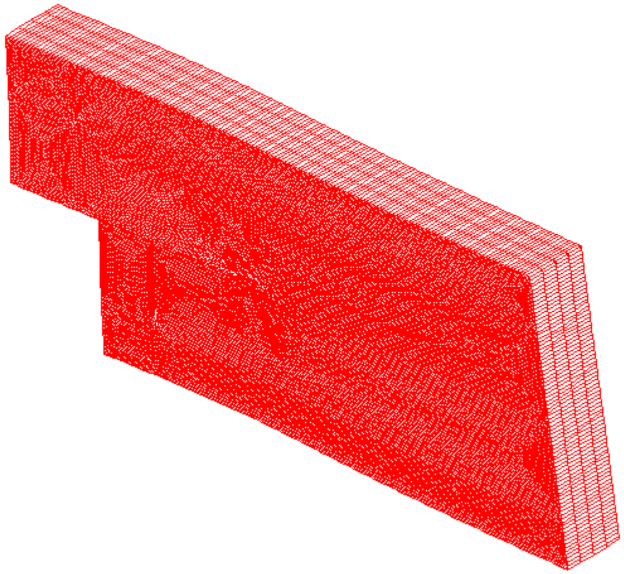


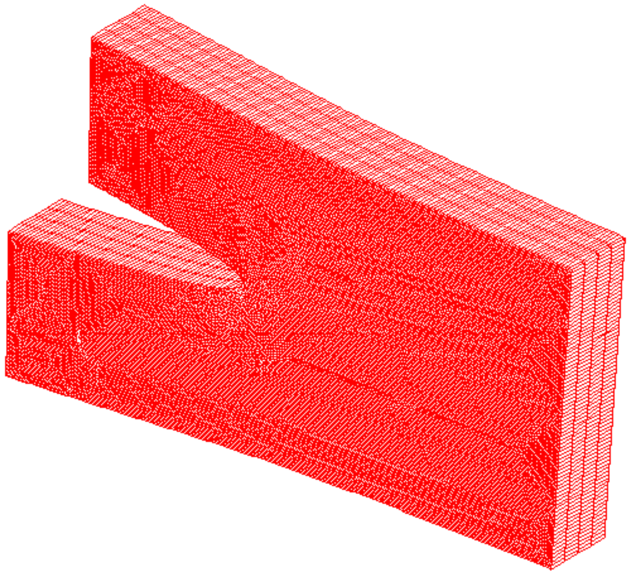



Numerical results validate the non-dependence of the integration domain with an average value.

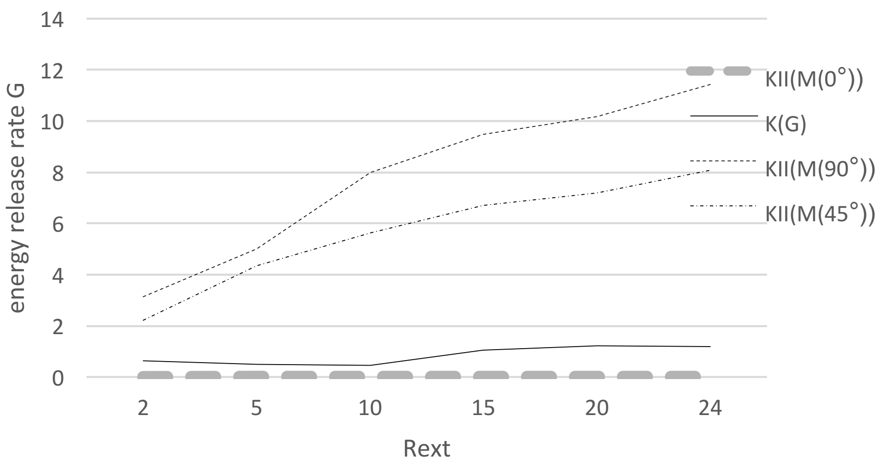
# Numerical validation

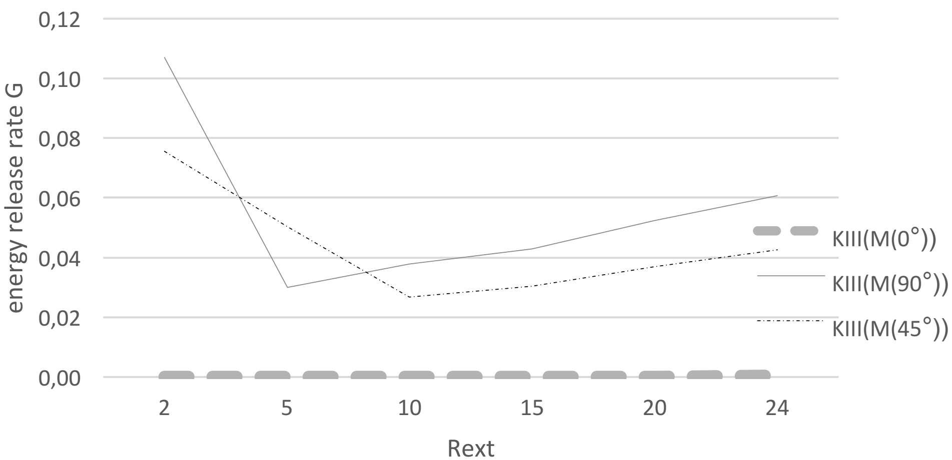


Shear mode



out-of-plan shear mode




Numerical results validate the non-dependence of the integration domain with an average value.

# Conclusions and outlook

- Numerical development of the contour integral concept for 3D problems
- The generalization toward its $G_{\theta}^{3D}$ and $M_{\theta}^{3D}$ implementation form
- Several numerical applications are proposed
- Toward implementation three dimensional mixed mode crack problem
- Elliptical crack front

- Numerical development of the contour integral concept for 3D problems.
- Generalization of the local mechanical fields
- New integral taking into account climatic effect
- 3D fractures coupling hygrothermal with mechanics effects
- Coupled 3D fracture mechanic – probabilistic methodology
- Confrontation FE / Experimental results