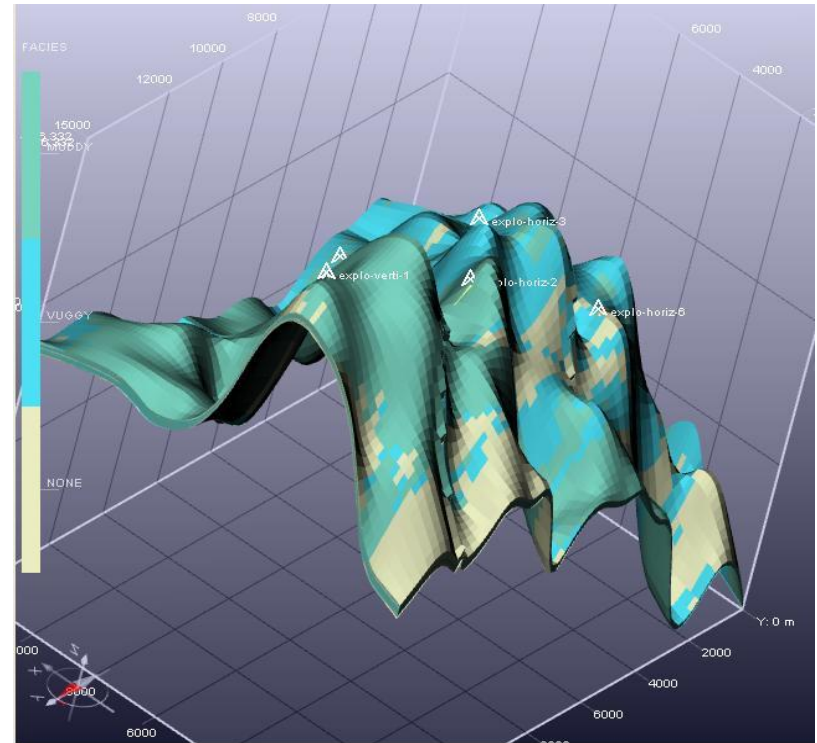
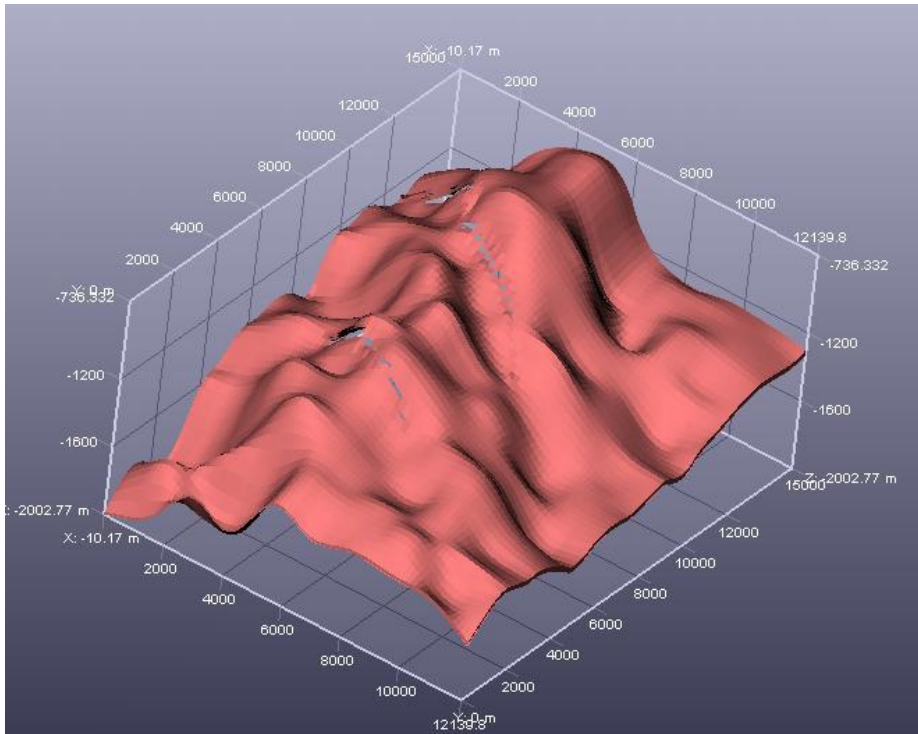


# Modélisation des écoulements dans un réseau discret de fractures par une approche continue

A. Fournou, C. Grenier, F. Delay,  
H. Benabderrahmane, B. Noetinger

# Geological context

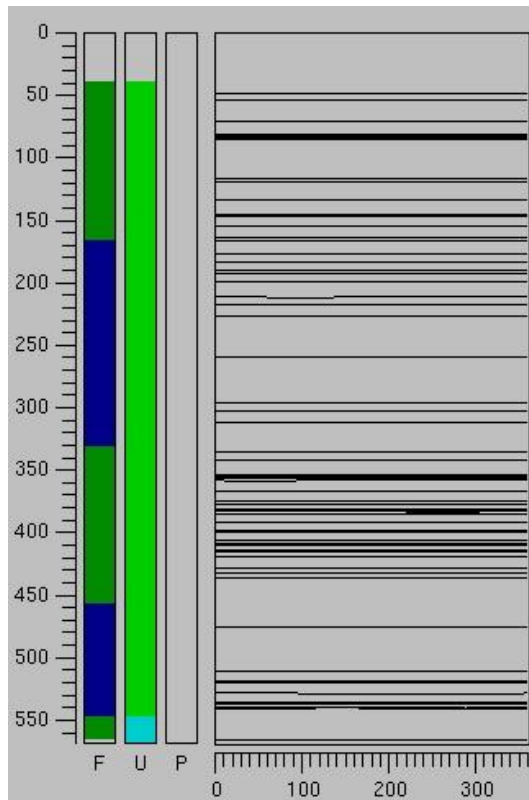


- Complex geological structures
- Different Rock types
- High heterogeneities (porosity, permeability)

# Fractured reservoir

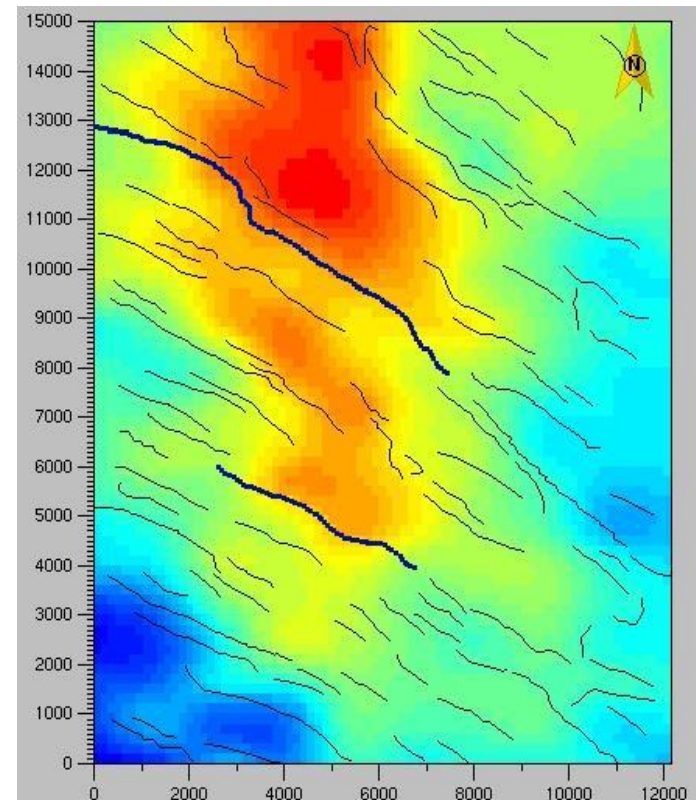
Some reservoirs present complex fracture network

Local scale (well)



Fracture log

Field scale

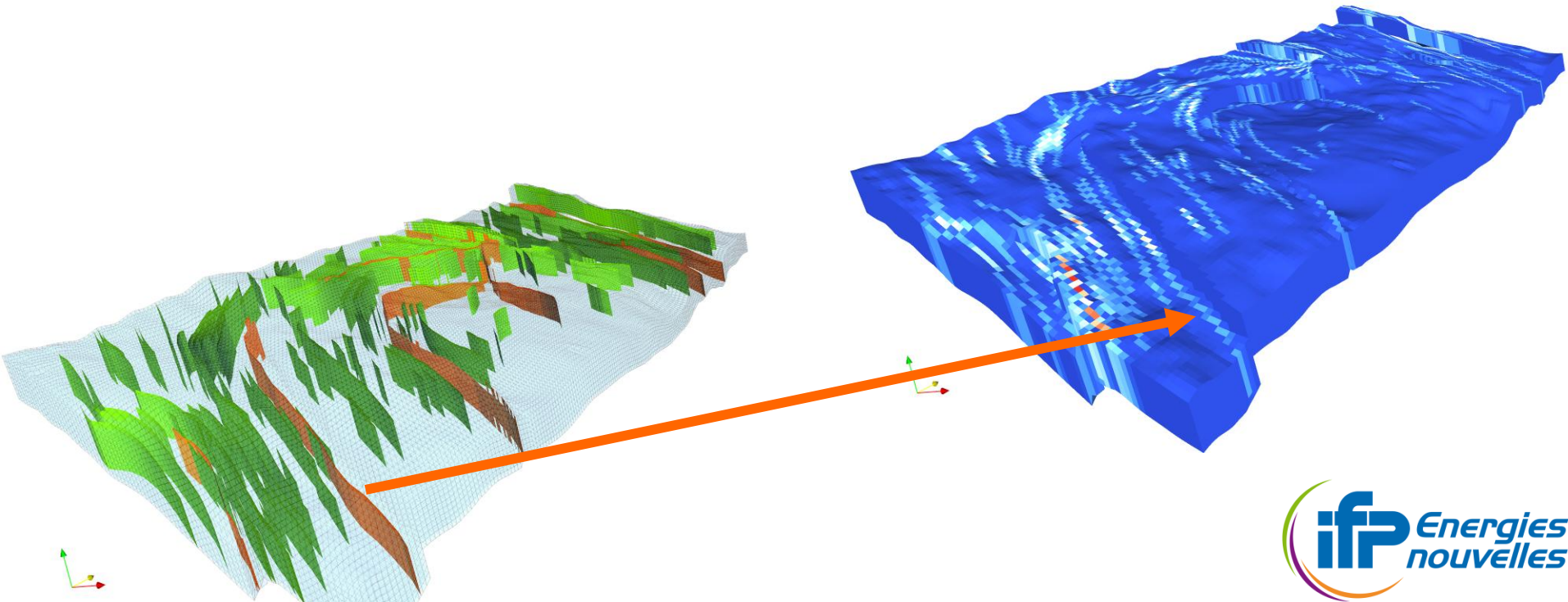


Faults and structural maps

# Flow simulation model : equivalent properties

Reservoir modeling softwares don't model flow on discrete fracture network.

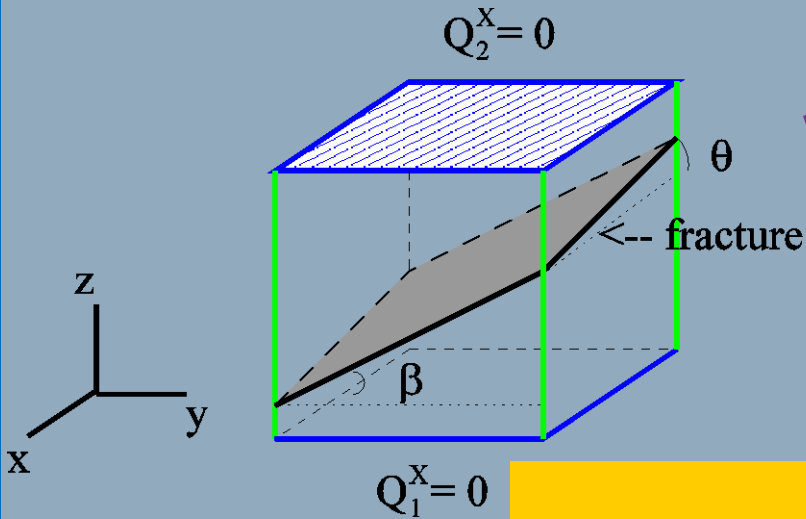
→ Equivalent flow properties have to be computed for each cell



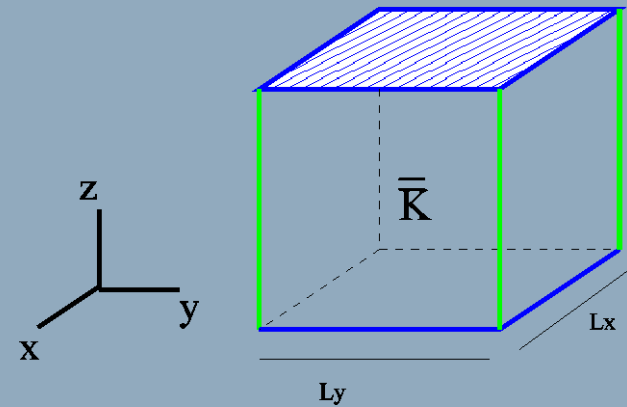
# Goal : to propose a new equivalent permeability

Flow :  $\vec{q} = -k \cdot \vec{\nabla} h$

$$Q_i^{frac} = - \sum_{fractures} k \vec{\nabla} h \cdot \vec{n}_i^f \cdot S_i^f$$

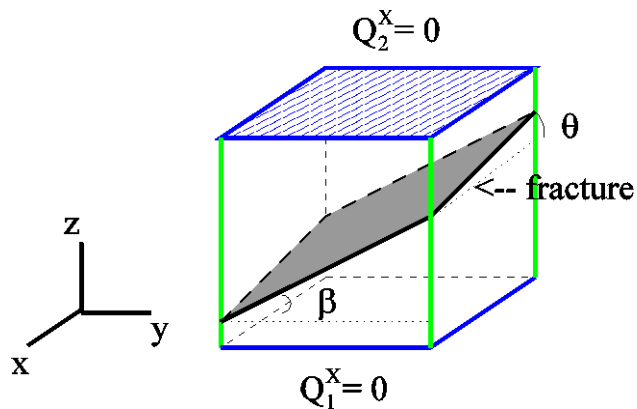


$$Q_i^{eq} = -\bar{K} \vec{\nabla} h \cdot \vec{n}_i \cdot S_i$$



$$\bar{K} = \frac{k \cdot a}{(\cos^2 \beta + \sin^2 \beta \cos^2 \phi)^{\frac{1}{2}}} \begin{bmatrix} \frac{\cos \theta}{L_z \cos \beta} & \frac{\sin \beta \sin \theta}{L_z} & 0 \\ \frac{\sin \beta \sin \theta}{L_z} & \frac{\cos \beta}{L_z \cos \theta} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# Some comments



$$\bar{K} = - \frac{k.a}{(\cos^2 \beta + \sin^2 \beta \cos^2 \phi)^{\frac{1}{2}}} \begin{bmatrix} \frac{\cos \theta}{L_z \cos \beta} & \frac{\sin \beta \sin \theta}{L_z} & 0 \\ \frac{\sin \beta \sin \theta}{L_z} & \frac{\cos \beta}{L_z \cos \theta} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- To correctly take into account a fracture, a full tensor have to be used by cells

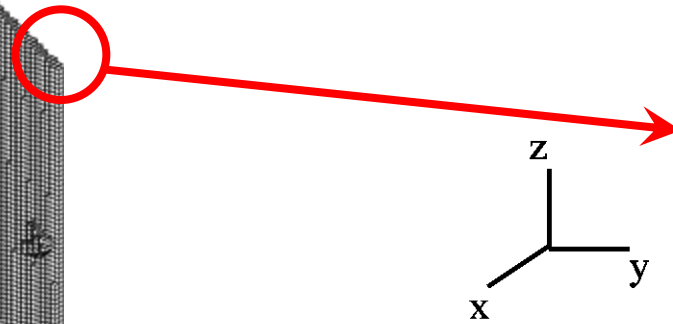
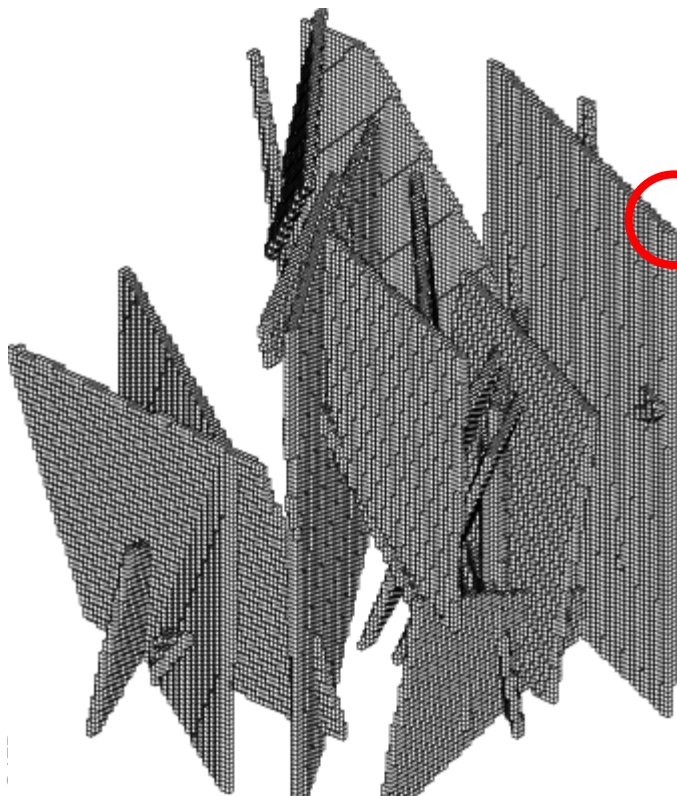
## Nevertheless

- If the dip or azimuth is closed to 0, K can be approximated by a diagonal tensor.

# Smearred Fractures

This work was initiated by CEA  
(DEN/DM2S/MTMS)

The idea behind this approach is to represent a fracture network by heterogeneous properties on a regular mesh

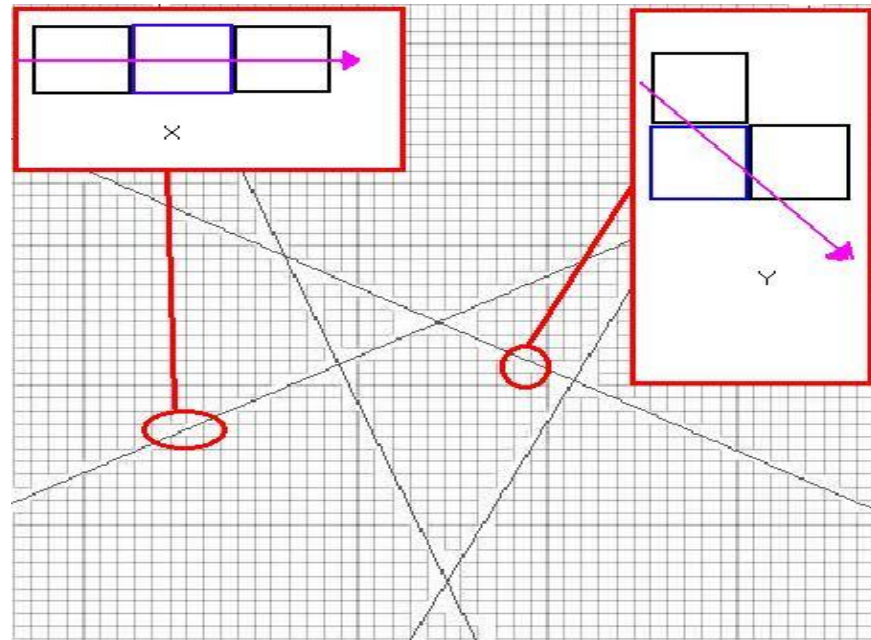


$$\bar{K}_{SF} = \begin{bmatrix} K_1^{SF} & 0 & 0 \\ 0 & K_2^{SF} & 0 \\ 0 & 0 & K_2^{SF} \end{bmatrix}$$

Energies  
nouvelles

# Exemple for the 2D

- Two sets of cell are identified

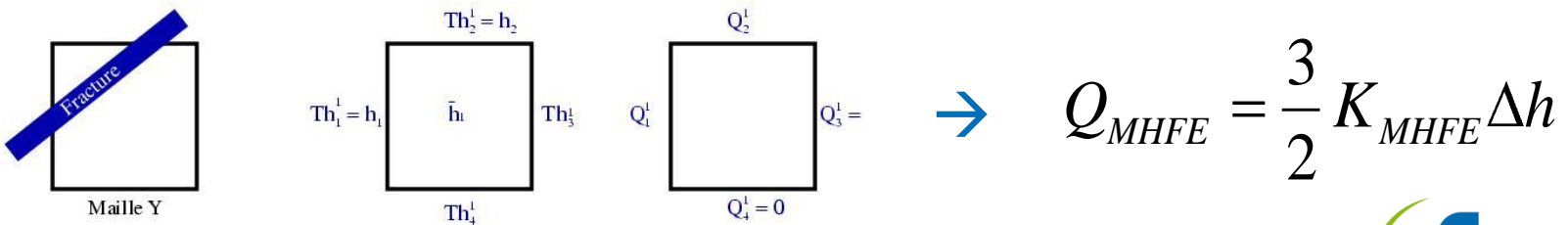
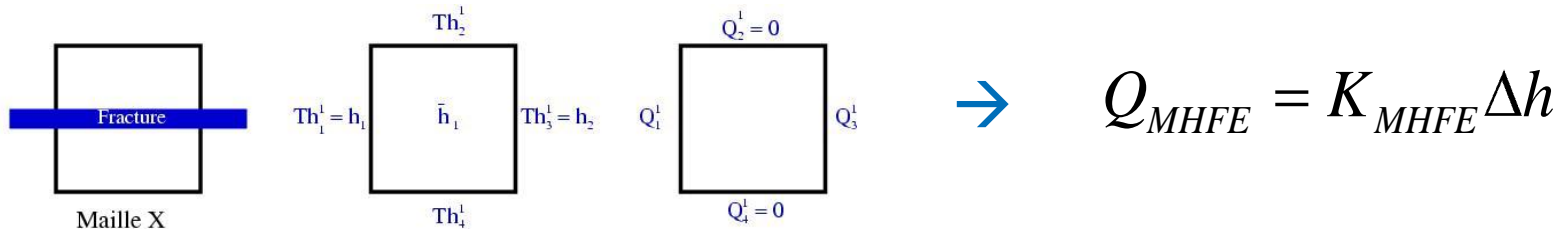




# Equivalent permeability $K_{MHFE}$

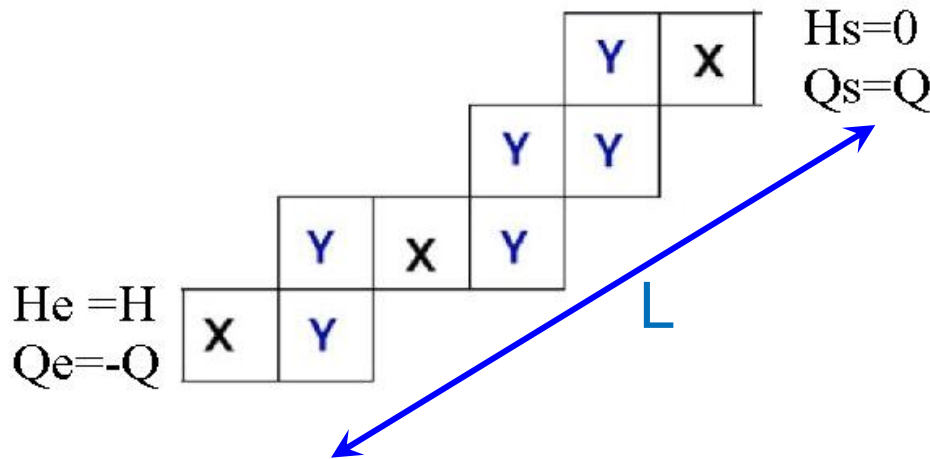
$$Q_i = \int K \cdot \vec{\nabla} h \cdot \vec{n}_i \cdot \partial s \rightarrow Q_i = \bar{h}_i \sum_j M_{ij}^{-1} - \sum_j M_{ij}^{-1} Th_j$$

- Mixed and Hybrid Finite Element scheme (MHFE) Flow



# 2D: equivalent permeability $K_{MHFE}$

The flow balance give the equivalent permeability

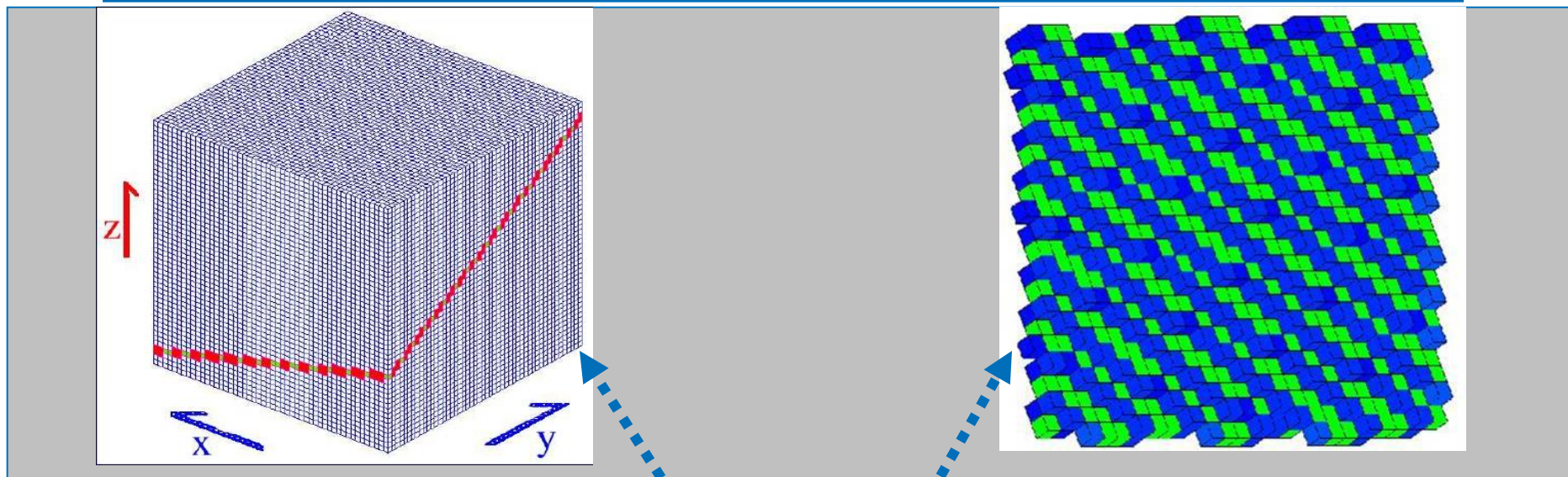


$a$  = fracture aperture  
 $k$  = fracture permeability  
 $L$  = fracture length  
 $\Delta h$  = head difference

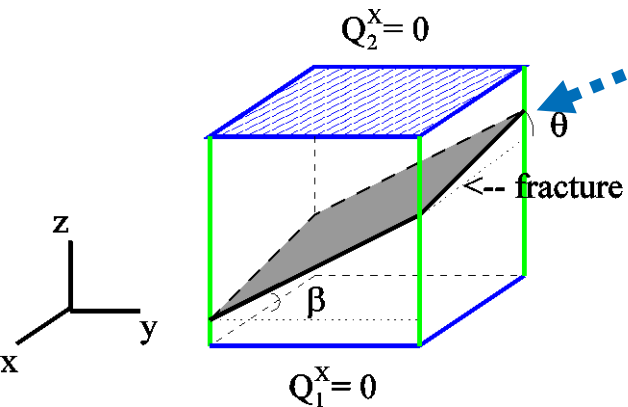
$Nb_x$  = number of X cells  
 $Nb_y$  = number of Y cells

$$Q_{MHFE} = -\frac{3}{3Nb_x + 2Nb_y} K_{MHFE} \Delta h \leftrightarrow Q_{ref} = -\frac{a}{L} k \Delta h$$

# 3D fracture mesh

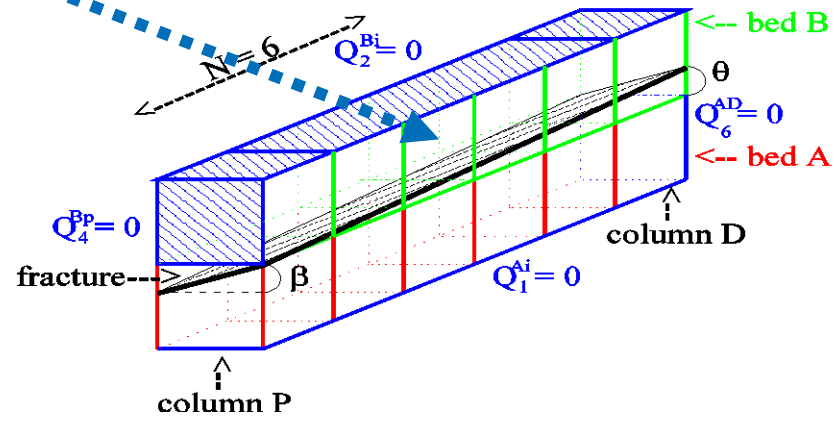


**S Cells (green)**

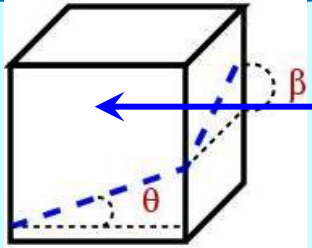


Fracture

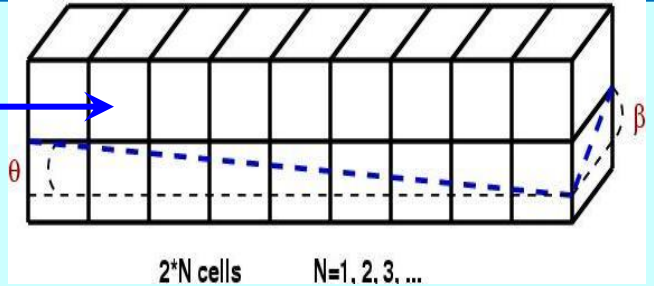
**C Cells (blue)**



# Analytical and MHFE flow



$$\overline{\overline{K}}_{SF} = \begin{bmatrix} K_1^{SF} & 0 & 0 \\ 0 & K_2^{SF} & 0 \\ 0 & 0 & K_2^{SF} \end{bmatrix}$$



**S (simple) cell**

**C (complex) cells**

2\*N cells    N=1, 2, 3, ...

## **X** S Cells (green)

$$Q_1^{MHFEX} = \Delta.K_1.\Delta h \iff Q_1^{ref,X} = \frac{a \cos \theta}{c_n \cos \beta} .k.\Delta h$$

$$Q_2^{MHFEX} = \Delta.K_2.\Delta h \iff Q_2^{ref,X} = \frac{a \cos \beta}{c_n \cos \theta} .k.\Delta h$$

$$N = \frac{\tan \beta}{\tan \theta}$$

$$c_n = (\cos^2 \beta + \sin^2 \beta \cos^2 \theta)^{\frac{1}{2}}$$

## **X** C Cells (blue)

$$Q_2^{MHFE,Y} = -\frac{3}{4} N.\Delta.K_2.\Delta h \iff Q_2^{ref,Y} = -N \frac{\cos \beta}{\cos \theta} .a.k.\Delta h$$

$$Q_1^{MHFE,Y} = -\frac{2K_2}{(N+1 - \frac{2}{3N})K_2 + \frac{4}{3N}K_1} .\Delta.K_1.\Delta h \iff Q_1^{ref,Y} = -\frac{1}{N} \frac{\cos \theta}{\cos \beta} a.k.\Delta h$$



# A 3D equivalent permeability

$$\overline{\overline{K}}_{SF} = \begin{bmatrix} K_1^{SF} & 0 & 0 \\ 0 & K_2^{SF} & 0 \\ 0 & 0 & K_2^{SF} \end{bmatrix}$$

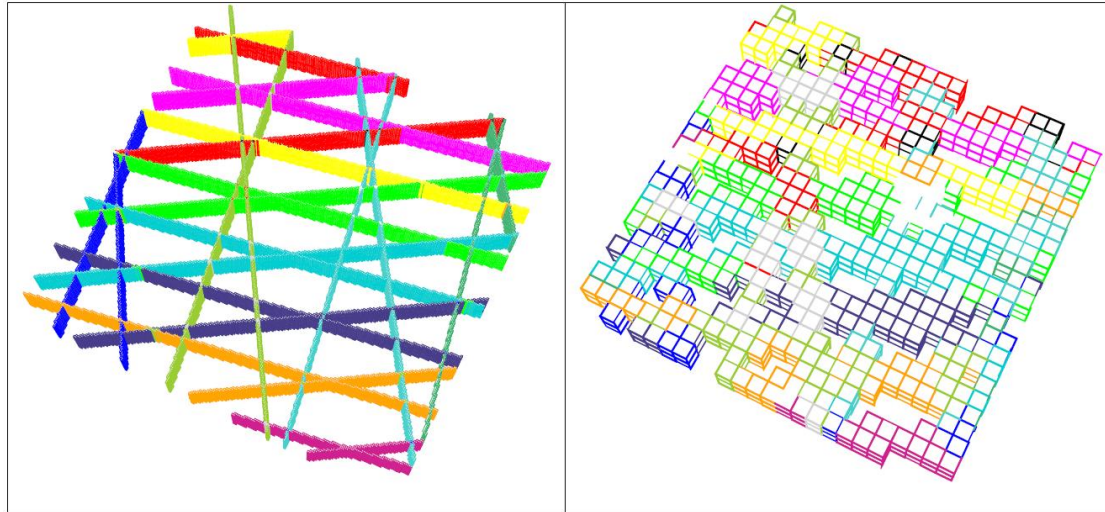
S Cells	$K_1^{SF} = \frac{\cos \theta}{c_n \cos \beta} \frac{a}{\Delta} k$	$K_2^{SF} = \frac{\cos \beta}{c_n \cos \theta} \frac{a}{\Delta} k$	$K_3^{Sf} = K_2^{Sf}$
C Cells	$K_1^{SF} = \frac{\cos \theta}{c_n \cdot \cos \beta} \frac{\left(1 + \frac{\tan \theta}{\tan \beta} - \frac{2 \tan^2 \theta}{3 \tan^2 \beta}\right)}{\left(2 - \frac{\sin^2 \theta}{\sin^2 \beta}\right)} \frac{a}{\Delta} k$	$K_2^{SF} = \frac{4}{3} \frac{\cos \beta}{c_n \cos \theta} \frac{a}{\Delta} k$	$K_3^{Sf} = K_2^{Sf}$



# Validation case

**Sensitivity study on the dip and azimuth value.  
Numerical and analytical equivalent permeabilities  
are compared**

- Single fracture : dip and strike
- Regular fracture network : cubic element size



# Precision of the results : single fracture

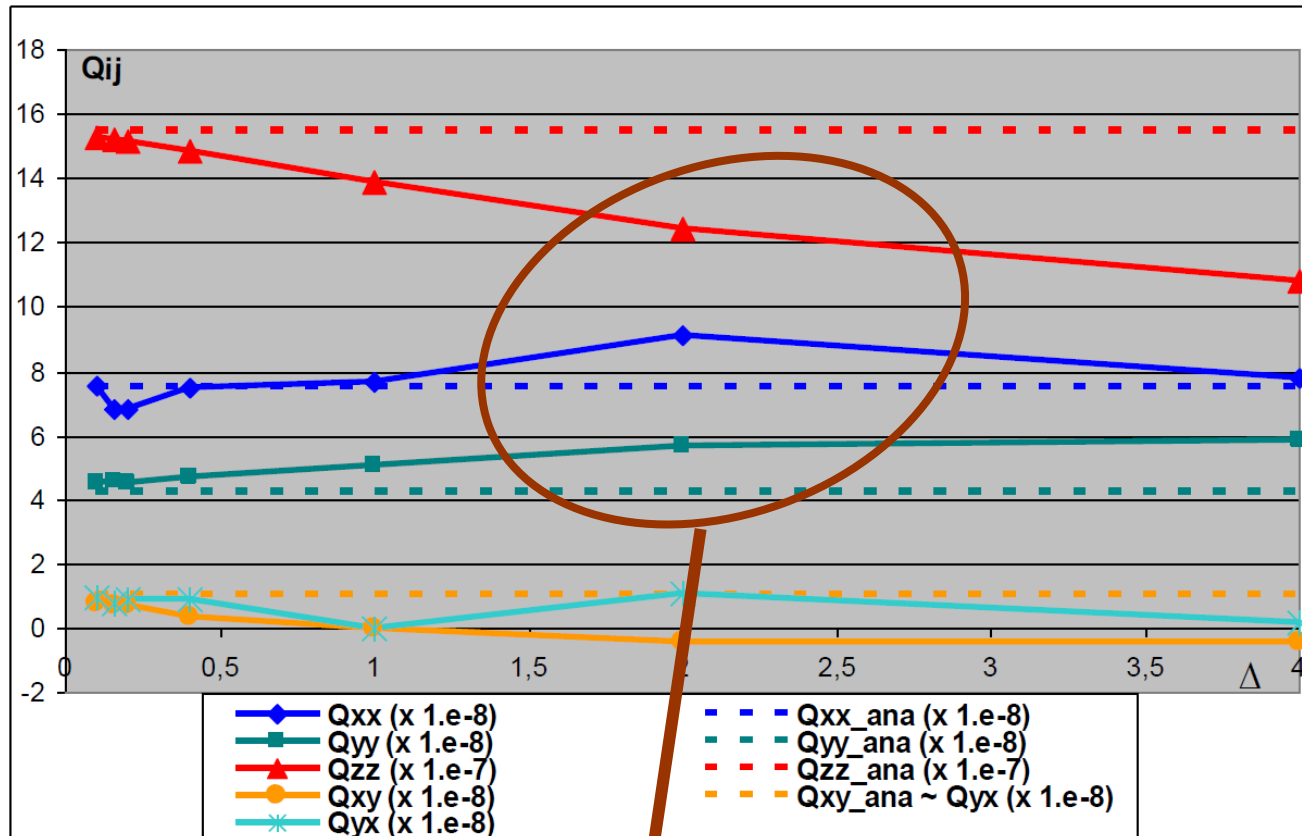
Err %	Strike (°)			
	0	15	30	45
$k_{max}$	0	15	30	45
$k_{int}$	0	0.	0.	0.
$k_{min}$	0	0.6	2.8	13.8
Dip(°)	0	0	0.	0.
		0	0.6	2.8
		0	0.	0.
	10	0	0	0
		1.1	5.1	15.2
		8.7	4.4	3.7
30	0	0	0	
	8.5	24.2	13.9	
	9.2	11.9	22.4	

Increase of the error

$$\bar{K} = - \frac{k.a}{(\cos^2 \beta + \sin^2 \beta \cos^2 \phi)^{\frac{1}{2}}} \begin{bmatrix} \frac{\cos \theta}{L_z \cos \beta} & \frac{\sin \beta \sin \theta}{L_z} & 0 \\ \frac{\sin \beta \sin \theta}{L_z} & \frac{\cos \beta}{L_z \cos \theta} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

As attempted, error depends on dip and strike values due to extra diagonal terms that are neglected in our approach

# Precision of the results : regular fracture network



Huge and minor connectivity changes  
due to the spatial cell size





# Conclusions

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## ☆ Performance of the method

- Fractured media mesh easily obtained
- Quick results and low computer cost (coarse discretizations)
- Precision depends on head gradient orientation, discretization.

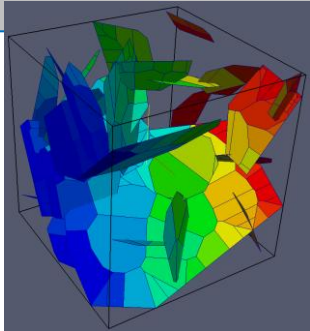
## 🕒 Modeler point of view

- For huge fracture density, weak space discretization have to be required (increase the computer cost).
- The number of cell required is frequently an handicap

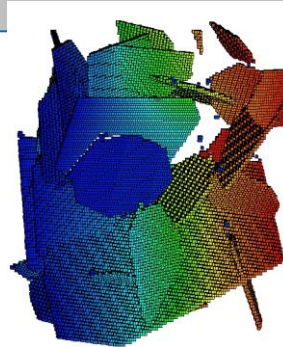
# Perspectives

## ☀ Perspectives :

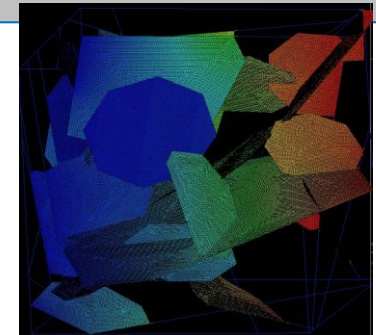
- Reduce the number of cells
- Simulations of transfers in the fractured media
- Benchmark



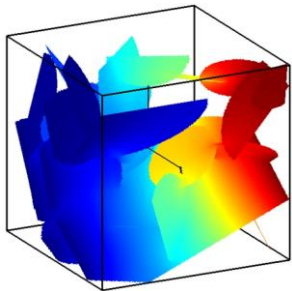
Approche « explicite optimisée »  
MD, NK,[1] (100 mailles)



Approche « Voxel »  
AF et al., (10<sup>6</sup> mailles)



Approche « explicite fin »  
AF et al. [2] (2.10<sup>5</sup> mailles)



Approche extérieure

- [1] N. Khvoenkova & M. Delorme (2011), méthode pour construire le maillage d'un réseau de fractures a partir de diagrammes de voronoï,FR11/01.686
- [2] A. Fournu, B. Noetinger, C. La Borderie. Publication prévue
- [3] G. Pichot, J. Erhel and J. R. de Dreuzy, A mixed hybrid Mortar method for solving flow in discrete fracture networks, Applicable Analysis An International Journal, 89 Issue 10, 1629, doi:10.1080/00036811.2010.495333



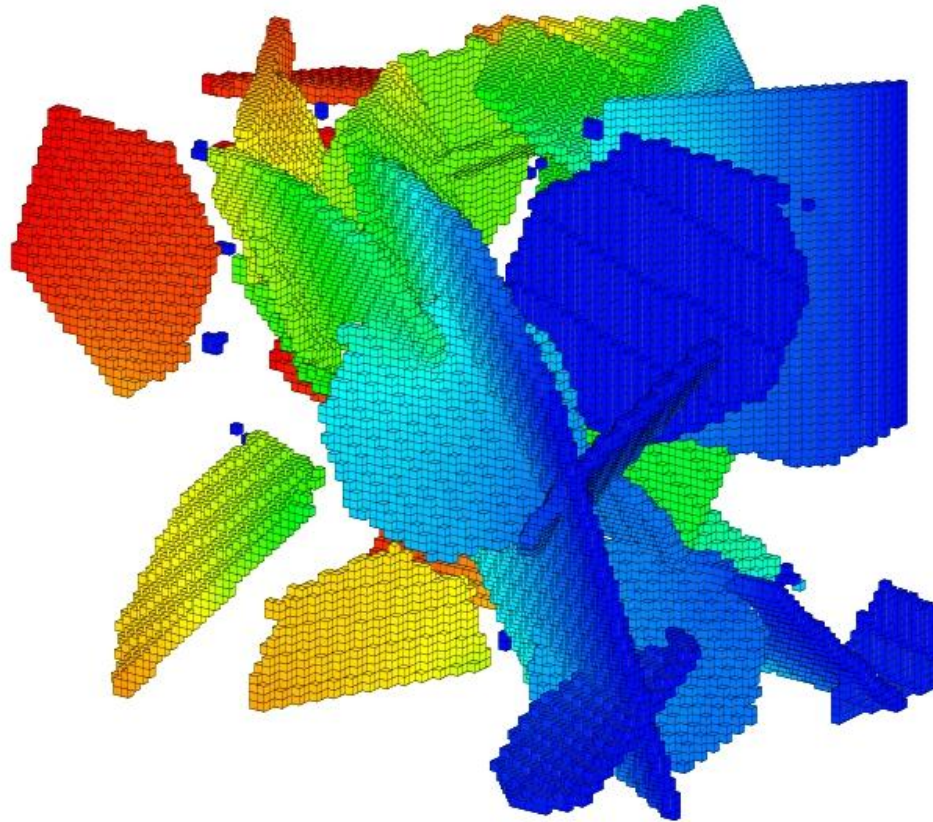
# Bibliography

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- 2011. A. Fournou, C. Grenier, F. Delay, H. Benabderrahmane. A novel and efficient 3D fracture continuum model for flow in fracture networks. **MAMERN11: 4th International Conference on Approximation Methods and Numerical Modelling in Environment and Natural Resources. Saidaia (Morocco).**
- 2007. A. Fournou, C. Grenier, F. Delay and H. Benabderrahmane. Development and qualification of a smeared fracture modelling approach for transfers in fractured media. **Gronwater in fractured rocks, IAH selected papers volume 9, section 6; Numerical modelling of fractured environment.**
- 2005. A. Fournou, directeur de thèse F. Delay, responsable C. Grenier. **Modélisation multi-échelles de l'écoulement et du transport dans un milieu granitique ; application au site d'Äspö en Suède. Thèse financée par l'ANDRA et effectuée au CEA/DEN dans le laboratoire de Modélisation des Transferts en Milieux Solides.**



# Spatial sensitivity study

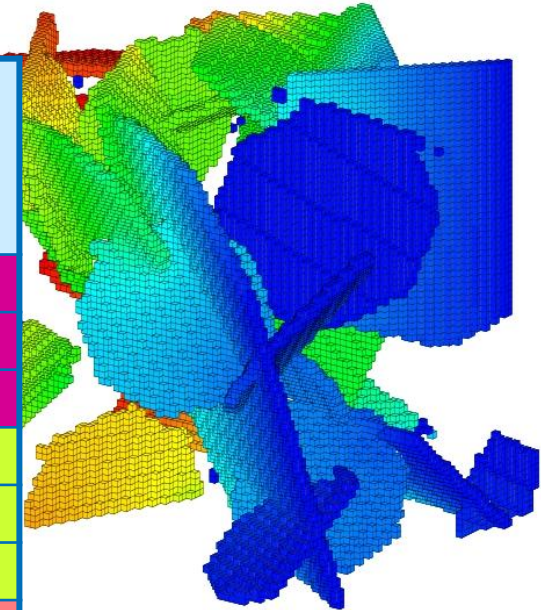


The DFN was generated using J.R. de Dreuzy tools

# Spatial sensitivity study

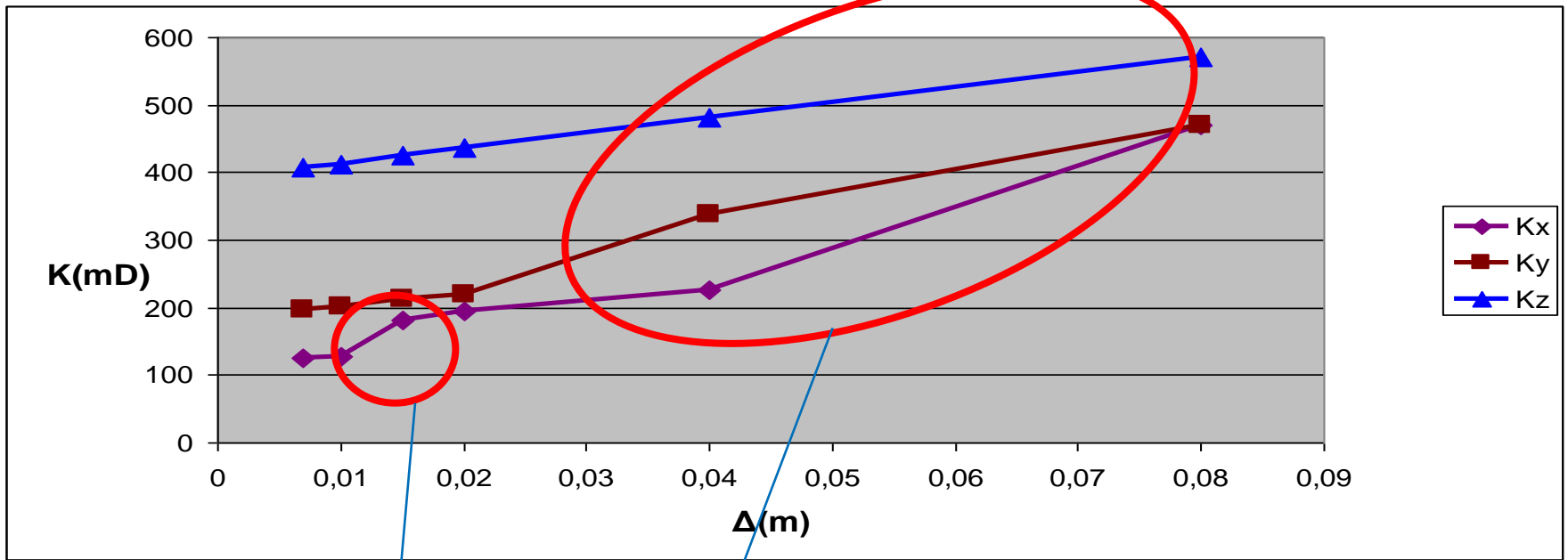
## Equivalent permeability tensor

$\Delta$ (m)	Equivalent permeability K (mD)			Diagonal tensor (mD)		
	Kmin	Kmax	Kz	Kmin	Kmax	Kz
0.007	313.37	-88.67	-36.6	211		
	-56.381	289.64	-174.20		374	
	-38.72	41.92	540.78			559
0.01	320	-90	-37.5	218		
	-56.5	298.6	-173.5		382	
	-37.7	42	548			566
0.02 cell number (1.628.973)	351	-92	-43	243		
	-57	326	182		413	
	-37	35	580			601
0.04	386	-115	74	296		
	-53	425	166		491	
	-45	33	633			657
0.08	590	-59	-14	495		
	-35	533	-175		615	
	-9	54	821			833



Fracture conductivities  
Cf = 1000 mD.m

# Spatial sensitivity study sensitivity analysis



Huge and minor connectivity changes  
due to the spatial cell size