Club Castem 2011, 24 Novembre 2011, Vanves

GLOBAL-LOCAL X-FEM FOR 3D NON-PLANAR FRICTIONAL CRACK APPLICATION TO ROLLING FATIGUE

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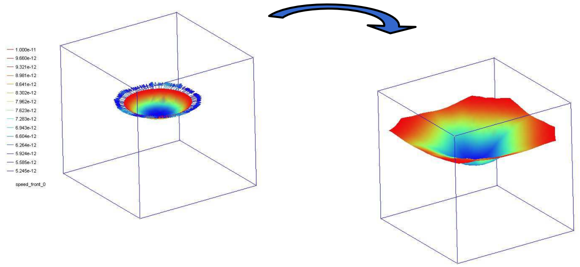
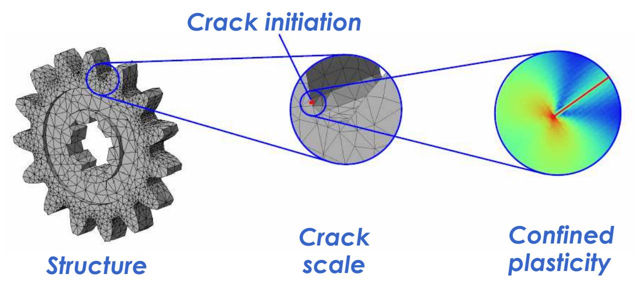
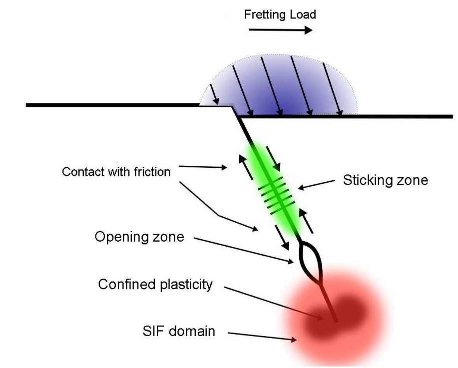
Introduction

- X-FEM coupled with a crack level set modelling greatly facilitates the simulation of 3D growing cracks
- Neither fine mesh (close to the front) nor remeshing of the domain (during the crack propagation) are required



- However, even with X-FEM, minimal requirements on the mesh design have to be taken into account. For instance:
  - Scale of the crack
  - Confined plasticity, closure effect
  - Localized non-linearities due to contact and friction along the crack faces

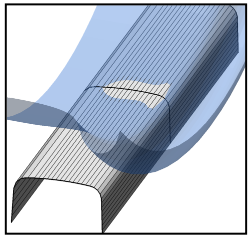




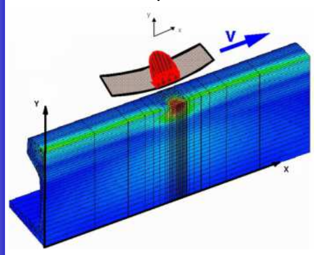
➡ When contact and friction occur along the crack faces, a discretization of the interface is required

➡ This involves a mesh dependency between the interface and the structure

Introduction



Contact et frottement entre les lèvres de la fissure



Situation de roulement

VOCOLIN
Calcul de dynamique ferroviaire

Contrainte sur le champignon

STARAIL
Calcul éléments finis

État mécanique dans tout le rail

MXFAT
Application du critère de DANG VAN

Cœfficient de danger dans tout le rail
Point le plus critique

Propagation de la fissure

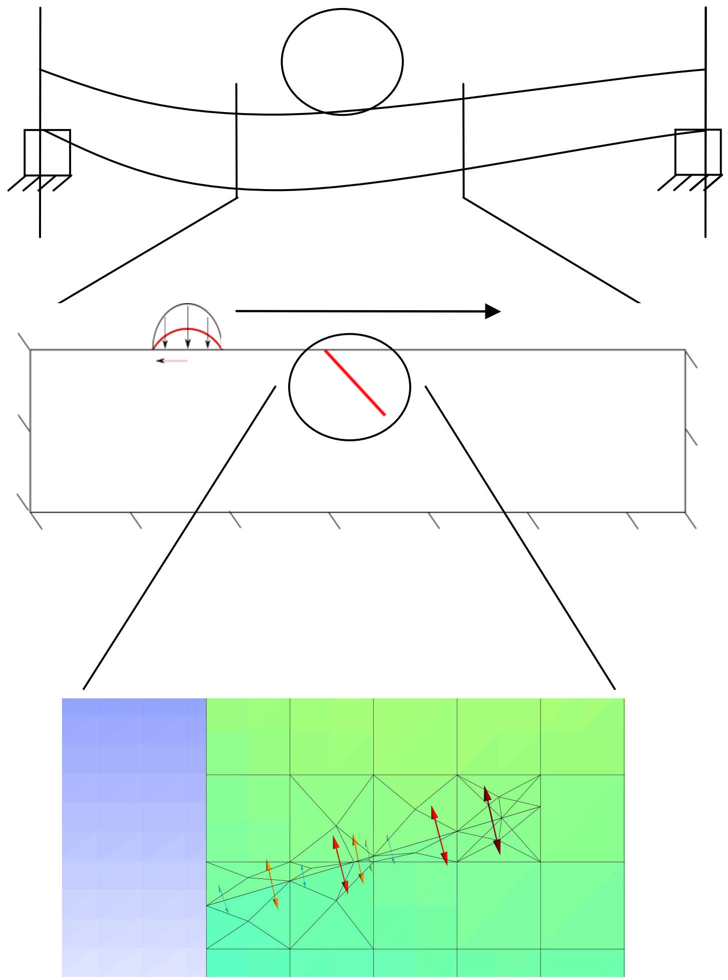
(X-FEM)

Contraintes résiduelles

Chargement
multi-axial
Non proportionnel

Loi de propagation dédiée

Stratégie multi-échelle pour la simulation de la propagation des fissures



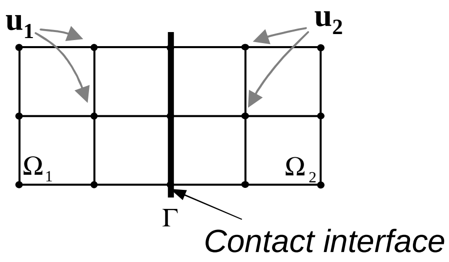
Échelle de la structure

Échelle de la discontinuité géométrique

Échelle du contact et du frottement entre les lèvres de la fissure

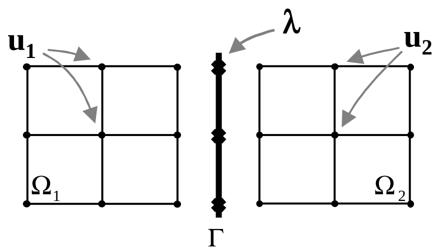
Modelling of the interfacial Fictrional contact with X-FEM

- Three mainly formulations of the contact problem with X-FEM:
  - Primal formulation:



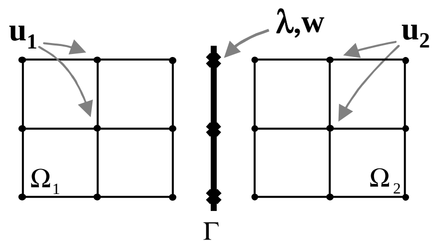
[F. Liu, R.I. Borja, IJNME 2008]

  - Dual formulation:



[F. Liu, R.I. Borja, CMAME 2010]
[E. Giner, M. Tur, J. E. Tarancón, F.J. Fuenmayor, IJNME 2009]
[N. Moës, B. Béchet, M. Tourbier, IJNME 2006]
[E. Béchet, N. Moës, B. Wohlmuth, IJNME 2009]
[I. Nistor, M.L.E. Guiton, P. Massin, N. Moës et al., IJNME 2009]
[S. Géniaut, P. Massin, N. Moës, EJCM 2007]

  - Mixed formulation:



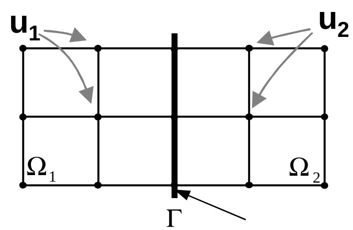
[J. Dolbow, N. Moës, T. Belytschko, CMAME 2001]
[T. Elguedj, A. Gravouil, A. Combescure, IJNME 2007]
[R. Ribeaucourt, M.C. Baietto Dubourg, A. Gravouil, CMAME 2007]

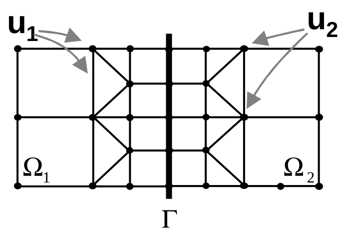
Modelling of the interfacial Frictional contact with X-FEM

- Three mainly formulations of the contact problem with X-FEM:

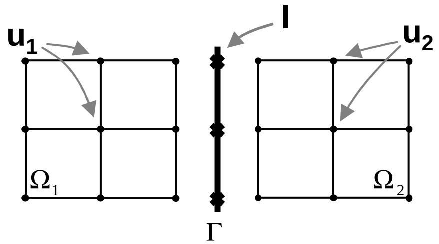
	Interface refinement		Stability
Primal formulation:		Coupled scales	Unconditionally stable
Dual formulation:		Coupled scales	Conditionally stable
Mixed formulation:		Uncoupled scales	Conditionally stable

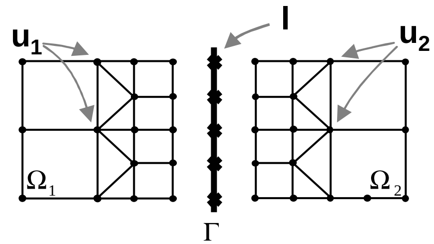
Primal formulation:



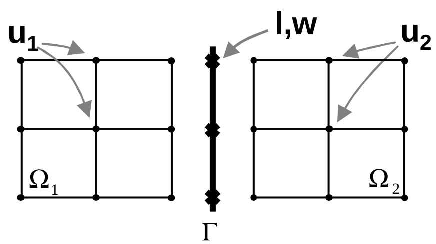


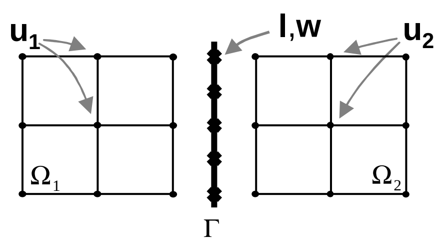
Dual formulation:





Mixed formulation:





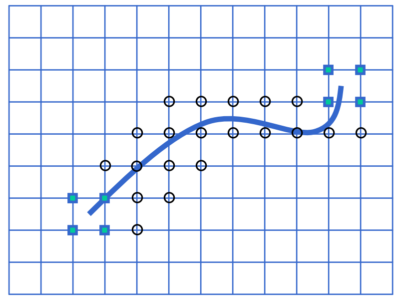
OUTLINE

1. X-FEM with level sets for 3D crack growth simulation
2. A 3 field weak formulation + 2 scales strategy
3. Examples (ELFE_3D: LaMCoS in-house Software + GMSH)
4. Applications – Cast3m implementation
5. Conclusions & perspectives

1 Xtended Finite Element Method + level sets

- **Local partition of unity (two scale strategy):**

- *Discontinuous and asymptotic enrichment of the displacement field*



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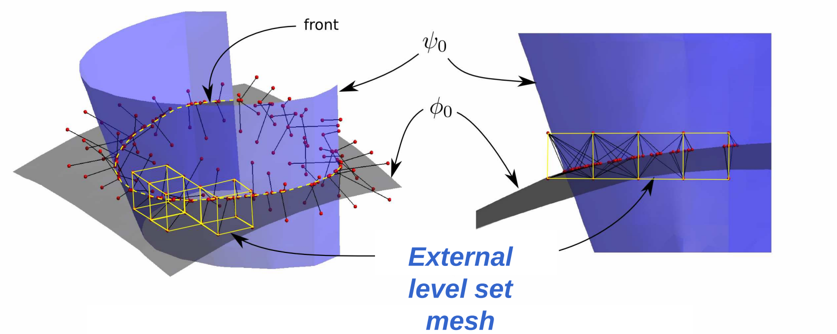
- *Crack shape modeling by two level sets:*

  ( ) ( ) *crack*

  ( ) ( ) *Crack front*

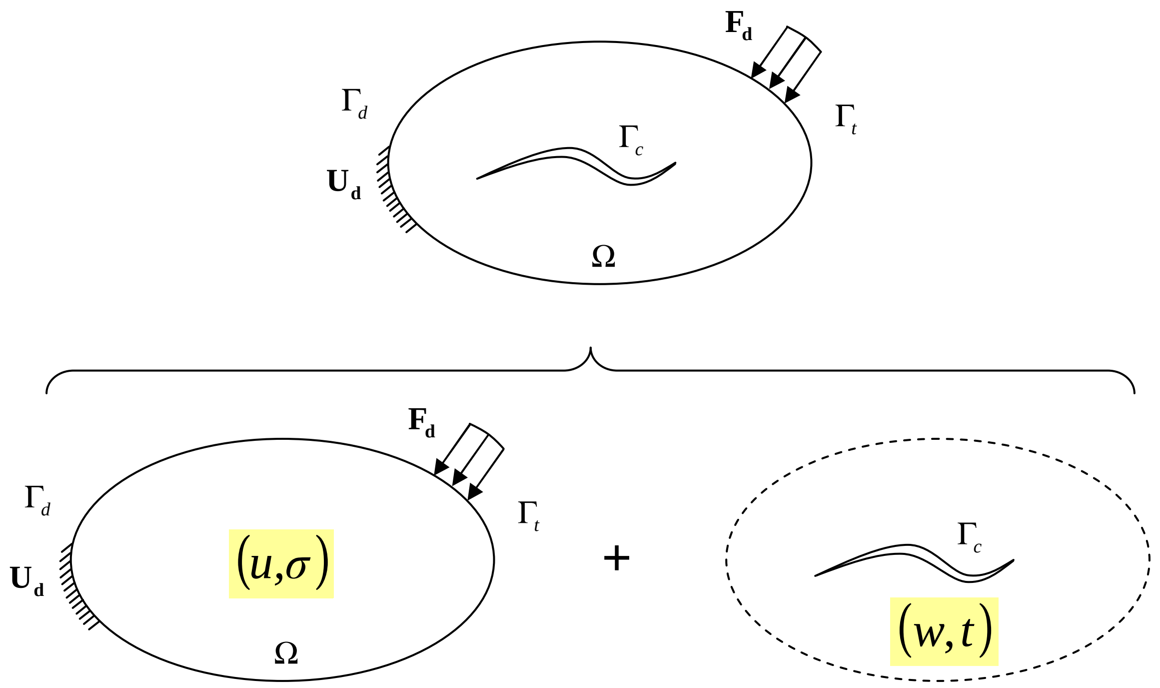
- *Level set update* — | |

- *Local orthogonality*



[Moës 1999, Stolarska 2001, Duflot 2005, Béchet 2005, Sukumar 2007] [Rannou 2009]
[Gravouil A., Moës N., Belytschko T., IJNME, 2002] [Sethian 1997]

2 Two scales strategy (Structure / crack)



- **Global problem (u,σ)**

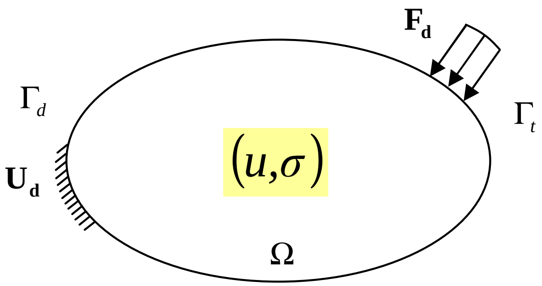
  Scale of the structure
  Equilibrium and constitutive law in the bulk (possibly nonlinear)

- **Local problem (w,t)**

  Scale of the crack
  Constitutive law at the interface (unilateral contact, contact with friction)

2 Two scale strategy: three field weak formulation

- **Global problem**

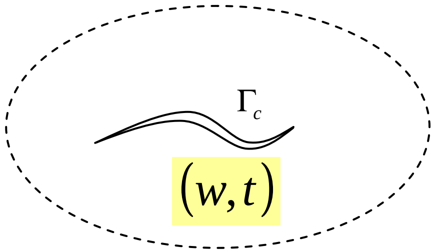


$$P^*_{int} = -\int_{\Omega} \boldsymbol{\sigma}(t) : \boldsymbol{\epsilon}(\mathbf{u}^*) d\Omega$$

$$P^*_{ext} = \int_{\Gamma^t} \mathbf{f}_t(t) \cdot \mathbf{u}^* dS$$

+ constitutive law in the bulk on (u,σ) (possibly nonlinear)

- **Local problem**



$$P^*_{crack} = \int_{\Gamma_C} \mathbf{t}(t) \cdot \mathbf{w}^* dS$$

+ interface constitutive law on (w,t) (unilateral, frictional contact)

- **Coupling between local and global problems**

Strong formulation:

$$\boldsymbol{\sigma}(t) \cdot \mathbf{n} = \mathbf{t}^+(t) \quad on \ \Gamma_C^+ \quad and \quad \boldsymbol{\sigma}(t) \cdot -\mathbf{n} = \mathbf{t}^-(t) \quad on \ \Gamma_C^-$$

$$\mathbf{u}(t) = \mathbf{w}^+(t) \quad on \ \Gamma_C^+ \quad and \quad \mathbf{u}(t) = \mathbf{w}^-(t) \quad on \ \Gamma_C^-$$

Weak coupling:

$$P^*_{coupling} = \int_{\Gamma_C} \boldsymbol{\lambda}^* \cdot (\mathbf{u}(t) - \mathbf{w}(t)) dS + \int_{\Gamma_C} \boldsymbol{\lambda}(t) \cdot (\mathbf{u}^* - \mathbf{w}^*) dS$$

2 Two scale strategy: three field weak formulation

- **Principle of virtual works:**

$$P^*_{int} + P^*_{ext} + P^*_{crack} + P^*_{coupling} = 0$$

$$\forall \mathbf{u}^* \in U_0^*,\ \forall \mathbf{w}^* \in W^*,\ \forall \boldsymbol{\lambda}^* \in \Lambda^*,\ \forall t \in [0;T]$$

- **Three field weak formulation of the fracture problem with frictional contact between the crack faces:**

$$\begin{cases} 0 = -\int_\Omega \boldsymbol{\sigma}(t) : \boldsymbol{\epsilon}(\mathbf{u}^*) d\Omega + \int_{\Gamma_t} \mathbf{f}_t(t) \cdot \mathbf{u}^* dS + \int_{\Gamma_C} \boldsymbol{\lambda}(t) \cdot \mathbf{u}^* dS \\ \quad + \int_{\Gamma_C} (\mathbf{t}(t) - \boldsymbol{\lambda}(t)) \cdot \mathbf{w}^* dS \\ \quad + \int_{\Gamma_C} (\mathbf{u}(t) - \mathbf{w}(t)) \cdot \boldsymbol{\lambda}^* dS \\ \quad \forall \mathbf{u}^* \in U_0^*,\ \forall \mathbf{w}^* \in W^*,\ \forall \boldsymbol{\lambda}^* \in \Lambda^*,\ \forall t \in [0;T] \end{cases}$$

Weak coupling between u and w (refers to the term $\int_{\Gamma_C} (\mathbf{u}(t) - \mathbf{w}(t)) \cdot \boldsymbol{\lambda}^* dS$)

+ Constitutive law in volume (u,σ) (possibly non linear)

+ Frictional contact law at the interface (w,t)

→ Allows an intrinsic description of the crack interface:
- with its own primal and dual variables (w,t)
- with its own (possibly refined) discretization

2 Discretized three field weak formulation

- *X-FEM discretization of the displacement field in the bulk*

$$\mathbf{u}(\mathbf{x},t) \simeq \sum_{i\in N_{nodes}} \mathbf{u}_i(t)\Phi_i(\mathbf{x}) + H(\mathbf{x})\cdot \sum_{j\in N_{crack}} \mathbf{a}_j(t)\Phi_j(\mathbf{x}) + \sum_{l=1} B_l \cdot \sum_{k\in N_{front}} \mathbf{b}_{lk}(t)\Phi_k(\mathbf{x})$$

- *Discretization of the displacement and load field on the interface*

$$\mathbf{w}(\mathbf{x},t) \simeq \sum_{i=1}^{3} \mathbf{w}_i(t)\Psi_i(\mathbf{x})$$

$$\mathbf{t}(\mathbf{x},t) \simeq \sum_{i=1}^{3} \mathbf{t}_i(t)\Psi'_i(\mathbf{x})$$

$$\boldsymbol{\lambda}(\mathbf{x},t) \simeq \sum_{i=1}^{3} \boldsymbol{\lambda}_i(t)\Psi'_i(\mathbf{x})$$

- *Discretized 3 field weak formulation*

(mortar method)
(unambiguously definition of the coefficients)

$$\begin{aligned} 0 \;=\; & +\mathbf{U}^{*T}(-\mathbf{F}_{int}(\mathbf{U}(t)) + \mathbf{F}_{ext}(t) + \mathbf{L}^T\boldsymbol{\Lambda}(t)) \\ & +\mathbf{W}^{*T}(\mathbf{T}(t) - \boldsymbol{\Lambda}(t)) \\ & \boxed{+\boldsymbol{\Lambda}^{*T}(\mathbf{L}\mathbf{U}(t) - \mathbf{W}(t))} \end{aligned}$$

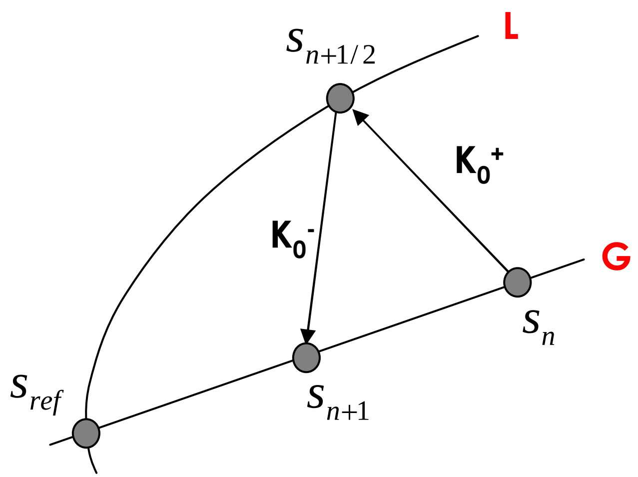
2 Non linear iterative solver (LATIN method)

- *Iterative solver for the solution of the frictional contact pro* $s = (u, \omega, t)$
(incremental LATIN Method [Ladevèze 1985])

  - *divide the equations into two subsets :*
    - global linear equations ***(G)*** *(3 field weak formulation)*
    - local possibly non linear equations ***(L)*** *(frictional contact equations)*

  - *find an approximate solution according to an iterative process in 2 stages*

- *Iterative strategy:*



- *Corresponding search directions:*

$$t_{i+\frac{1}{2}} - t_i = \mathbf{K}_0(\omega_{i+\frac{1}{2}} - \omega_i)$$

$$t_{i+1} - t_{i+\frac{1}{2}} = -\mathbf{K}_0(\omega_{i+1} - \omega_{i+\frac{1}{2}})$$

$$\mathbf{K}_0 = k_0 Id$$

2 Non linear iterative solver (LATIN method) Global stage (3 field weak formulation)

- Combination of the 3 field weak formulation and the search direction:

$$
\begin{aligned}
0 \;=\; & -\int_{\Omega} \boldsymbol{\sigma}_{i+1} : \boldsymbol{\epsilon}(\mathbf{u}^*) d\Omega + \int_{\Gamma^t} \mathbf{f}_t \cdot \mathbf{u}^* dS + \int_{\Gamma_C} \boldsymbol{\lambda}_{i+1} \cdot \mathbf{u}^* dS \\
& + \int_{\Gamma_C} (\mathbf{t}_{i+\frac{1}{2}} + k_0 \mathbf{w}_{i+\frac{1}{2}}) \cdot \mathbf{w}^* dS - \int_{\Gamma_C} (\boldsymbol{\lambda}_{i+1} + k_0 \mathbf{w}_{i+1}) \cdot \mathbf{w}^* dS \\
& + \int_{\Gamma_C} (\mathbf{u}_{i+1} - \mathbf{w}_{i+1}) \cdot \boldsymbol{\lambda}^* dS \quad \forall \mathbf{u}^* \in U_0^*, \; \forall \mathbf{w}^* \in W^* \; and \; \forall \boldsymbol{\lambda}^* \in \Lambda^*
\end{aligned}
$$

- Corresponding linear system:

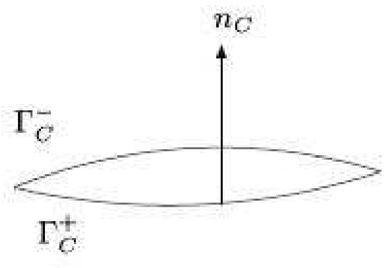
$$
\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}_{u\lambda} \\ 0 & \mathbf{K}_{ww} & \mathbf{K}_{w\lambda} \\ -\mathbf{K}_{u\lambda}^T & \mathbf{K}_{w\lambda}^T & 0 \end{bmatrix}
\begin{pmatrix} \mathbf{U}_{i+1} \\ \mathbf{W}_{i+1} \\ \boldsymbol{\Lambda}_{i+1} \end{pmatrix}
=
\begin{pmatrix} \mathbf{F}_t \\ \mathbf{K}_{w\lambda} \cdot \mathbf{T}_{i+\frac{1}{2}} + \mathbf{K}_{ww} \cdot \mathbf{W}_{i+\frac{1}{2}} \\ 0 \end{pmatrix}
$$

Mortar operators: coupling in a weak sense of interface-structure non-matching discretization

- Very close to the augmented Lagrangian formulation [Elguedj 2007]:

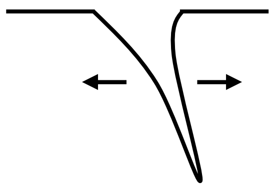
$$
\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}_{u\lambda} \\ 0 & \mathbf{K}_{ww} & \mathbf{K}_{w\lambda} \\ -\mathbf{K}_{u\lambda}^T & \mathbf{K}_{w\lambda}^T & 0 \end{bmatrix}
\begin{pmatrix} \Delta\mathbf{U}_{i+1} \\ \Delta\mathbf{W}_{i+1} \\ \Delta\boldsymbol{\Lambda}_{i+1} \end{pmatrix}
=
\begin{pmatrix} \mathbf{F}_t + \mathbf{K}_{u\lambda} \cdot \boldsymbol{\Lambda}_i \\ \mathbf{K}_{w\lambda} \cdot (\mathbf{T}_i - \boldsymbol{\Lambda}_i) \\ \mathbf{K}_{u\lambda}^T \cdot \mathbf{U}_i - \mathbf{K}_{w\lambda}^T \cdot \mathbf{W}_i \end{pmatrix}
$$

2 Non linear iterative solver (LATIN method)
Local stage (frictional contact equations)



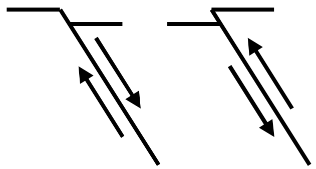
- **Notations for the local interface fields:**

$$[w] = \omega^- - \omega^+$$
$$\Delta\, w = \omega^n - \omega^{\,n-1}$$
$$\Delta\,[w_T] = \Delta\omega_T^{-n} - \Delta\omega_T^{+n} = (\omega_T^{-n} - \omega_T^{+n}) - (\omega_T^{-(n-1)} - \omega_T^{+(n-1)})$$



- **Unilateral contact at crack interface (w, t)**

$$\begin{array}{lcl} [w_N] & := & \omega_N^- - \omega_N^+ \geq 0 \\ F_N & := & t_N^+ = -t_N^- \leq 0 \\ F_T & := & t_T^+ = -t_T^- \\ [w_N]\,.F_N & = & 0 \end{array}$$



- **Frictional conditions at crack interface (w, t)**

$$\begin{array}{lcrl} \| F_T \| < \mu_C \,|\, F_N \,| & \Rightarrow & \Delta\,[w_T] = & 0 \\ \| F_T \| = \mu_C \,|\, F_N \,| & \Rightarrow & \exists \lambda \geq 0,\ \Delta\,[w_T] = & \lambda\, F_T \end{array}$$

2 Specific convergence indicator

- *Convergence indicator:*
  - *distance between the global and local approximations*
  - *allows to stop the iterative process when the error is lower than a prescribed tolerance*
  - *ensures the convergence both on the normal and tangential problems*

$$\eta_N = \frac{\| s_{N,i+1} - s_{N,i+\frac{1}{2}} \|_\infty^2}{\| s_{N,i+1} \|_\infty^2 + \| s_{N,i+\frac{1}{2}} \|_\infty^2} \quad , \quad \eta_T = \frac{\| s_{T,i+1} - s_{T,i+\frac{1}{2}} \|_\infty^2}{\| s_{T,i+1} \|_\infty^2 + \| s_{T,i+\frac{1}{2}} \|_\infty^2}$$

$$\| s \|_\infty^2 = max(k_0 \mathbf{t}^2 + \frac{1}{k_0}\mathbf{w}^2)$$

$$max(\eta_N ; \eta_T) < \varepsilon$$

[Ribeaucourt R., Baietto M.C., Gavouil A., CMAME 2007]

2 Stabilization of the three field weak formulation

- *The X-FEM three field weak formulation can be unstable wathever the non-linear*

$$\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}_{u\lambda} \\ 0 & \mathbf{K}_{ww} & \mathbf{K}_{w\lambda} \\ -\mathbf{K}_{u\lambda}^T & \mathbf{K}_{w\lambda}^T & 0 \end{bmatrix} \begin{pmatrix} \mathbf{U}_{i+1} \\ \mathbf{W}_{i+1} \\ \Lambda_{i+1} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{K}_{w\lambda} \cdot \mathbf{T}_{i+\frac{1}{2}} + \mathbf{K}_{ww} \cdot \mathbf{W}_{i+\frac{1}{2}} \\ 0 \end{pmatrix}$$

- *Introduction of a stabilization term on the local-global coupling condition in order to satisfy the LBB condition:*

$$\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}_{u\lambda} \\ 0 & \mathbf{K}_{ww} & \mathbf{K}_{w\lambda} \\ -\mathbf{K}_{u\lambda}^T & \mathbf{K}_{w\lambda}^T & \boxed{\mathbf{K}_{\lambda\lambda}} \end{bmatrix} \begin{pmatrix} \mathbf{U}_{i+1} \\ \mathbf{W}_{i+1} \\ \Lambda_{i+1} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{K}_{w\lambda} \cdot \mathbf{T}_{i+\frac{1}{2}} + \mathbf{K}_{ww} \cdot \mathbf{W}_{i+\frac{1}{2}} \\ \boxed{\mathbf{K}_{\lambda\lambda} \cdot \Lambda_i} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B}^T \\ \mathbf{B} & \boxed{-\varepsilon \mathbf{D}} \end{pmatrix} \begin{pmatrix} \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \boxed{-\varepsilon \mathbf{d}} \end{pmatrix}$$

The exact solution is obtained at convergence

- *Stability condition of Ladyzhenskaya-Babuška-Brezzi (LBB):*

$$\inf_{\mathbf{Z}\in\mathcal{Z}\backslash 0} \sup_{\mathbf{Y}\in\mathcal{Y}\backslash 0} \frac{\mathbf{Y}^T\,\mathbf{B}^T\,\mathbf{Z}}{\|\mathbf{Y}\|_{\mathcal{Y}} \cdot \|\mathbf{Z}\|_{\mathcal{Z}}} \geq \beta > 0 \quad \textit{with} \quad \begin{cases} \|\mathbf{Y}\|_{\mathcal{Y}} \leq \dfrac{1}{\alpha} \dfrac{4\,M_a\,M_b}{M_a\,\varepsilon + \beta^2} \cdot \|\varepsilon \mathbf{d}\| \\ \|\mathbf{Z}\|_{\mathcal{Z}} \leq \dfrac{4\,M_a^{1/2}\,M_b}{2\,M_a^{1/2}\,\alpha\,\varepsilon + \alpha^{1/2}\,\beta\,M_b} \cdot \|\mathbf{F}\| + \dfrac{4\,M_a}{M_a \varepsilon + \beta^2} \cdot \|\varepsilon \mathbf{d}\| \end{cases}$$

2 Stabilization of the three field weak formulation – elements of

- *Consider the following linear system:* $\begin{pmatrix} \mathbf{A} & \mathbf{B}^T \\ \mathbf{B} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ 0 \end{pmatrix}$
- *Block condensation:* $\begin{pmatrix} \mathbf{A} & \mathbf{B}^T \\ 0 & \mathbf{CS} \end{pmatrix} \begin{pmatrix} \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{FS} \end{pmatrix}$
- *Schur complement:* $\mathbf{CS} = \mathbf{B}\ \mathbf{A}^{-1}\ \mathbf{B}^T$ $\quad \mathbf{FS} = \mathbf{B}\ \mathbf{A}^{-1}\ \mathbf{F}$
- *CS invertible i* $\mathrm{kernel}\left(\mathbf{B}^T\right) = 0$ *That is to say:* $\max_{\mathbf{Y}} (\mathbf{B}\ \mathbf{Y}, \mathbf{Z}) = \max_{\mathbf{Y}} \left(\mathbf{Y}, \mathbf{B}^T\ \mathbf{Z}\right) > 0 \quad \forall \mathbf{Z}$
- *Case of finite element:* $\begin{pmatrix} \mathbf{A}_h & \mathbf{B}_h^T \\ \mathbf{B}_h & 0 \end{pmatrix} \begin{pmatrix} \mathbf{Y}_h \\ \mathbf{Z}_h \end{pmatrix} = \begin{pmatrix} \mathbf{F}_h \\ 0 \end{pmatrix}$ $\quad \max_{\mathbf{Y}_h \in \mathcal{Y}_h} \frac{\mathbf{Y}_h^T\ \mathbf{B}_h^T\ \mathbf{Z}_h}{\|\mathbf{Y}_h\|_{\mathcal{Y}_h} \cdot \|\mathbf{Z}_h\|_{\mathcal{Z}_h}} > 0$
- *When h tends to zero, one obtains the LBB condition:* $\inf_{\mathbf{Z} \in \mathcal{Z} \setminus 0} \sup_{\mathbf{Y} \in \mathcal{Y} \setminus 0} \frac{\mathbf{Y}^T\ \mathbf{B}^T\ \mathbf{Z}}{\|\mathbf{Y}\|_{\mathcal{Y}} \cdot \|\mathbf{Z}\|_{\mathcal{Z}}} \geq \beta > 0$
- *Error Estimator:*

$$\|\mathbf{Y}_{exact} - \mathbf{Y}_h\|_1 + \|\mathbf{Z}_{exact} - \mathbf{Z}_h\|_0 \leq C_Y h^k \cdot \|\mathbf{Y}_{exact}\|_{k+1} + C_Z h^{l+1} \cdot \|\mathbf{Z}_{exact}\|_{l+1}$$

2 Stabilization of the three field weak formulation – elements of

- Consider the following linear system $\begin{pmatrix} \mathbf{A} & \mathbf{B}^T \\ \mathbf{B} & -\varepsilon\mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ -\varepsilon\mathbf{d} \end{pmatrix}$

- Ellipticity condition on A: $\alpha \|\mathbf{Y}\|^2_{\mathcal{Y}} \leq \mathbf{Y}^T \; \mathbf{A} \; \mathbf{Y} \quad \forall \mathbf{Y} \in \mathcal{Y}$

- inf-sup condition: there exists a positive constant β independent on the mesh (h) such that:

  (Ladyzhenskaya-Babuška-Brezzi stability conditi $\inf_{\mathbf{Z}\in\mathcal{Z}\setminus 0} \; \sup_{\mathbf{Y}\in\mathcal{Y}\setminus 0} \dfrac{\mathbf{Y}^T \; \mathbf{B}^T \; \mathbf{Z}}{\|\mathbf{Y}\|_{\mathcal{Y}} \cdot \|\mathbf{Z}\|_{\mathcal{Z}}} \geq \beta > 0$

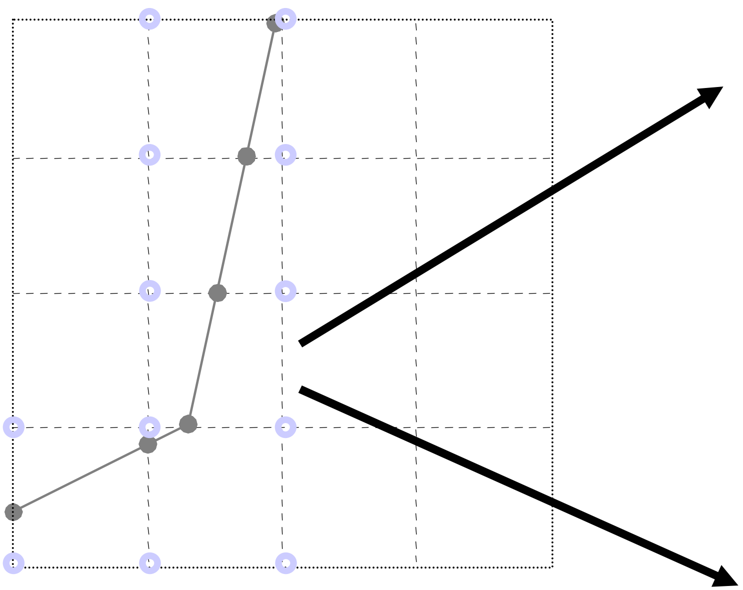
- Continuity condition on A and B: there exists two constants Ma and Mb independent on h such that:

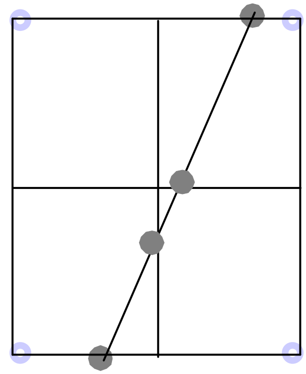
$$\forall(\mathbf{Y},\mathbf{Z}) \in \mathcal{Y}\times\mathcal{Z} \quad \mathbf{Y}^T \; \mathbf{A} \; \mathbf{Z} \leq M_a \|\mathbf{Y}\|_{\mathcal{Y}} \cdot \|\mathbf{Z}\|_{\mathcal{Z}}$$
$$\forall(\mathbf{Y},\mathbf{Z}) \in \mathcal{Y}\times\mathcal{Z} \quad \mathbf{Y}^T \; \mathbf{B}^T \; \mathbf{Z} \leq M_b \|\mathbf{Y}\|_{\mathcal{Y}} \cdot \|\mathbf{Z}\|_{\mathcal{Z}}$$

- Property:

$$\begin{cases} \|\mathbf{Y}\|_{\mathcal{Y}} \leq \dfrac{1}{\alpha} \dfrac{4\,M_a\,M_b}{M_a\,\varepsilon + \beta^2} \cdot \|\varepsilon\mathbf{d}\| \\ \|\mathbf{Z}\|_{\mathcal{Z}} \leq \dfrac{4\,M_a^{1/2}\,M_b}{2\,M_a^{1/2}\,\alpha\,\varepsilon + \alpha^{1/2}\,\beta\,M_b} \cdot \|\mathbf{F}\| + \dfrac{4\,M_a}{M_a\varepsilon + \beta^2} \cdot \|\varepsilon\mathbf{d}\| \end{cases}$$

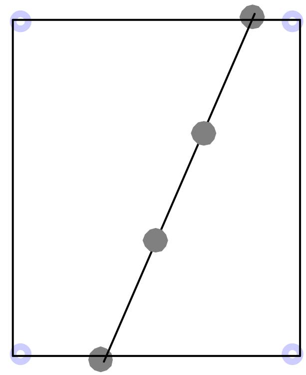
2 X-FEM discretization of the crack interface





Standard approach: Refinement of the bulk mesh

[Dolbow J, Moës N, Belytschko T. CMAME 2002]



New approach: Independent refinement

Enriched nodes

Interface element

[Ladeveze 1985]
[Pierres E., Baietto M.C., Gravouil A., CMAME 20

- *The distribution of Gauss points is the support of the primal and dual interface fields (w,t).*

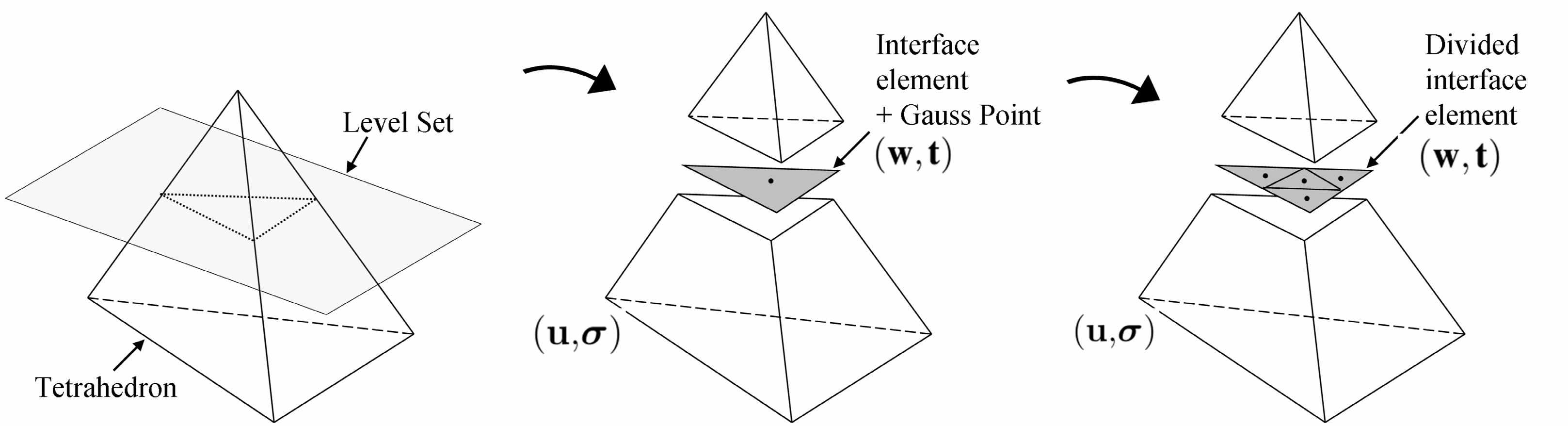
2 Two scales X-FEM discretization of the crack interface

New approach: Independent discretization of the interface

- *Discretization of the interface independent of the underlying X-FEM mesh*
- *Three field weak formulation: Non matching discretizations authorized*

Interface elements divided according to size and shape.

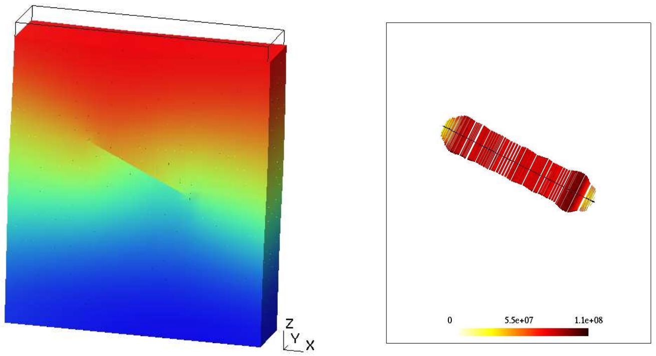
Uniform distribution of Gauss points along the interface.
Refinement adapted to the local frictional contact



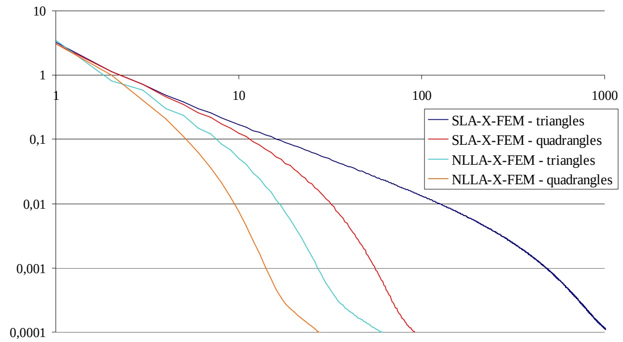
[Pierres E., Baietto M.C., Gavouil A., CMAME 2009]

3 Efficiency and robustness of the two-scales X-FEM model for frictional cracks

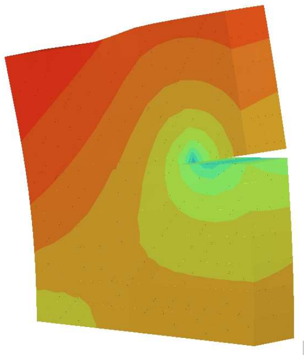
- Numerical stability of the model



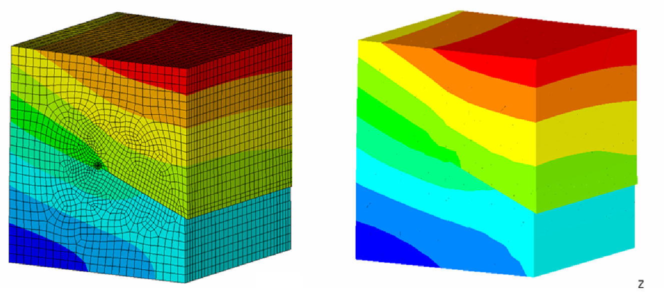
- Non-linear convergence



- Accuracy and CPU saving

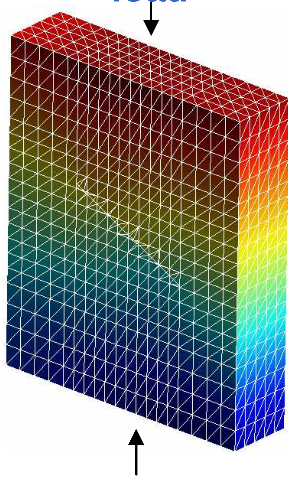


- Efficiency of the Global-local X-FEM



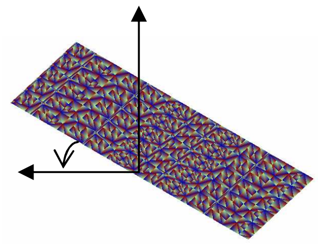
3 Example 1: Case of an inclined crack submitted to a compressive load

- Definition of geometry, material, crack and load



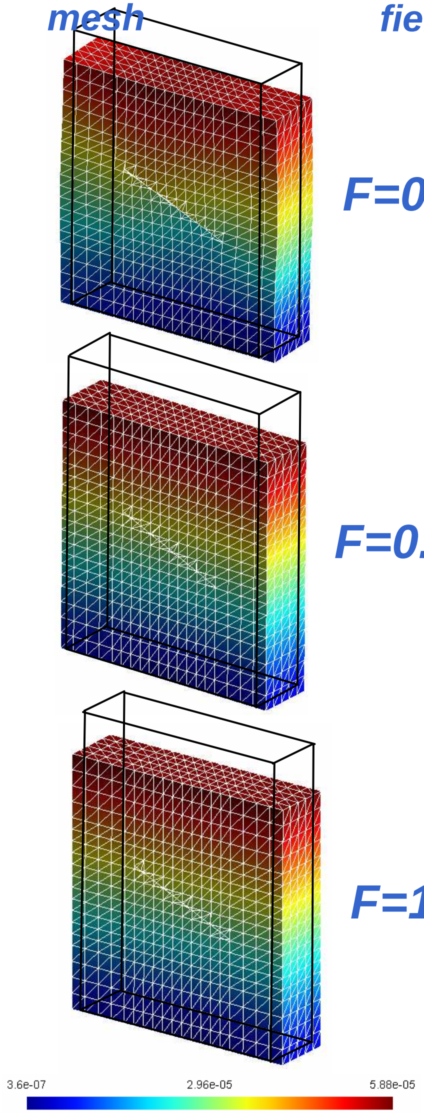
p=100.MPa
E=206.GPa ν=0.3

- Refined interface discretization



α=30.

Deformed mesh

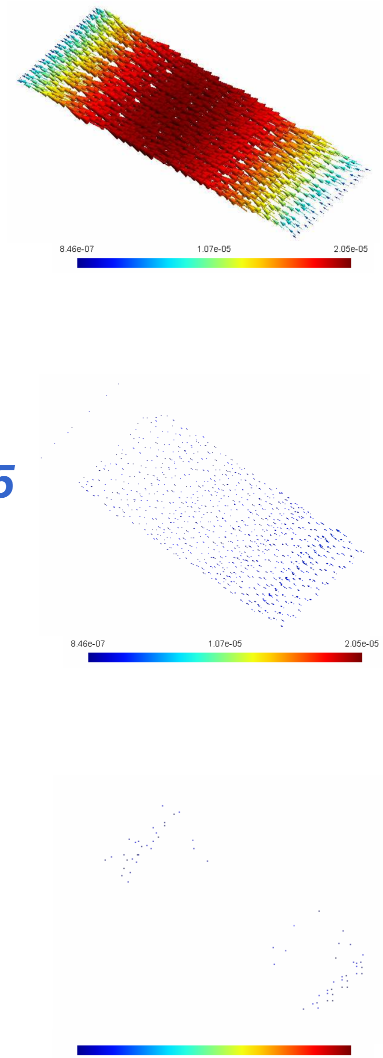


F=0.

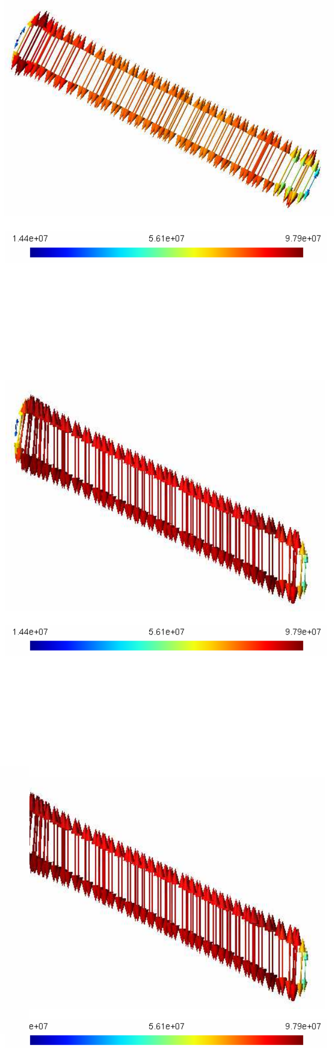
F=0.5

F=1.

Tangential displacement field

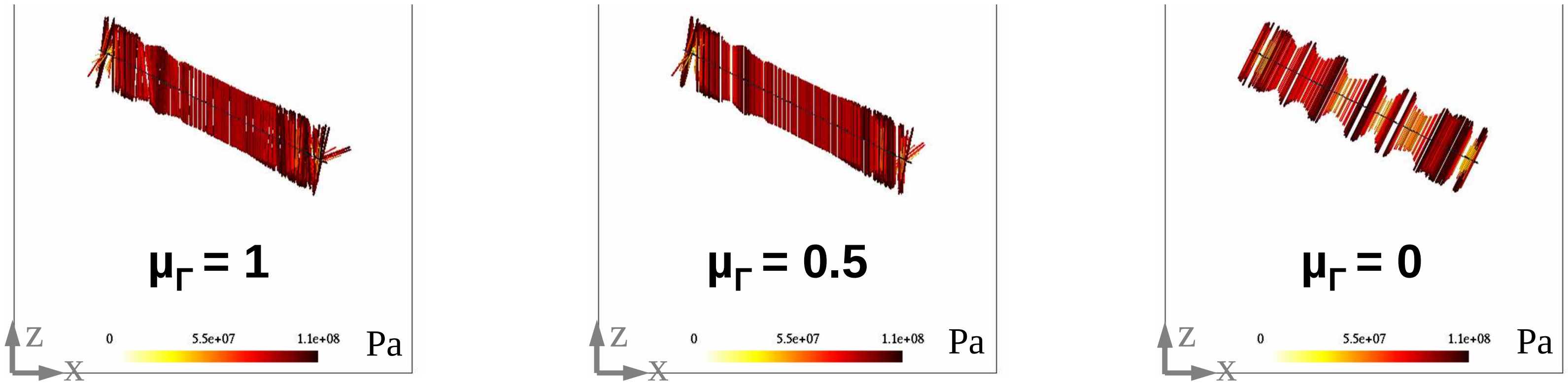


Interface load

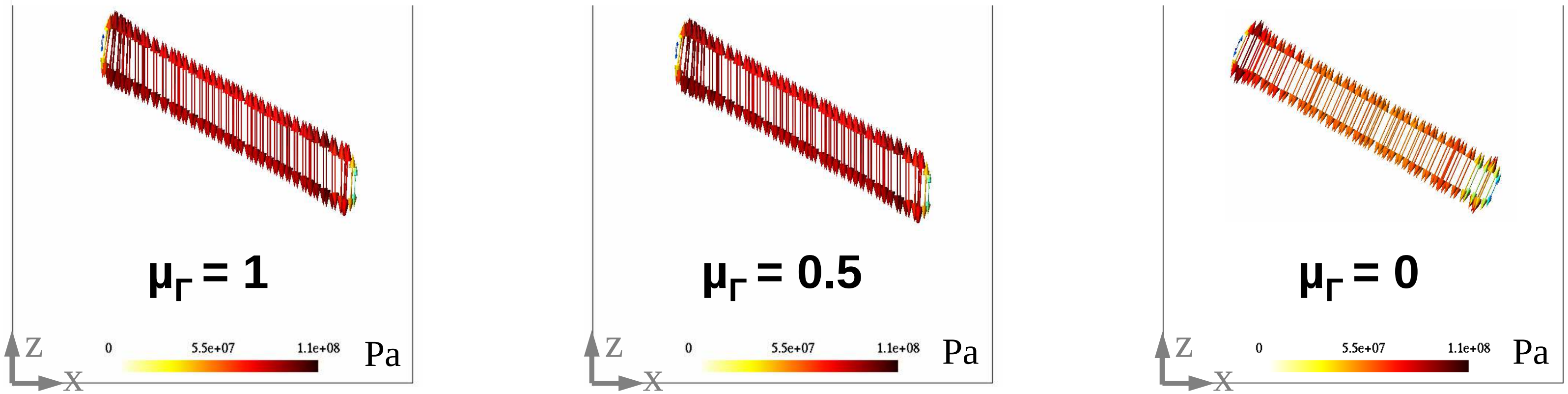


3 Stability and hability of the model to capture accurately different contact solutions

- *Non stabilized model: contact load field with numerical oscillations*

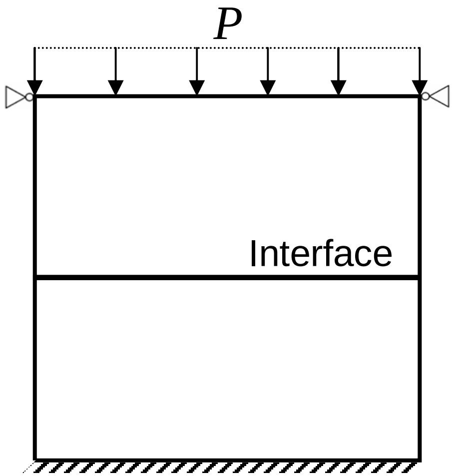


- *Stabilized model: contact load field without numerical oscillation*

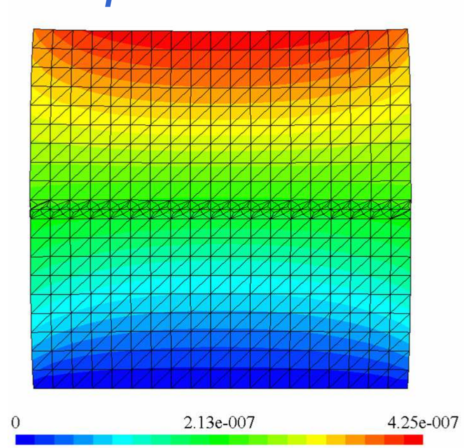


Sticking case ***Partial sliding*** ***Gross sliding***

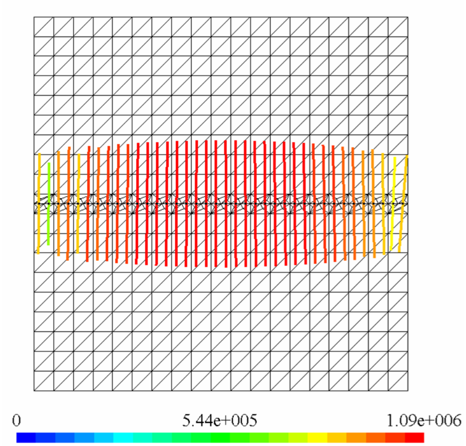
3 Example 2: Convergence property of the global-local X-FEM



Displacement field

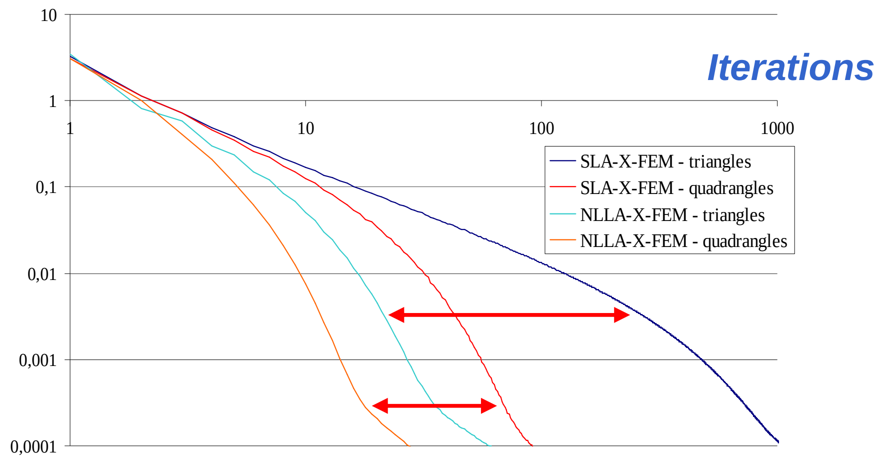


Contact loads

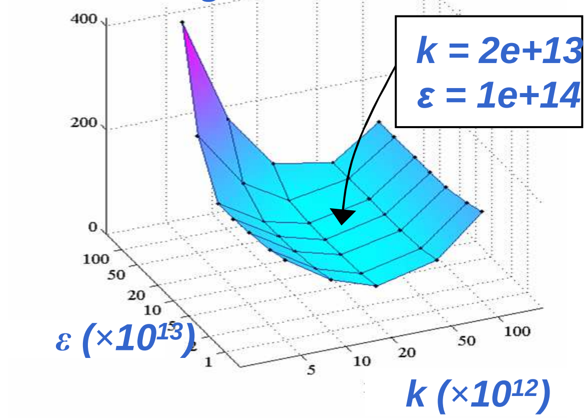


LATIN + stabilization

Local error

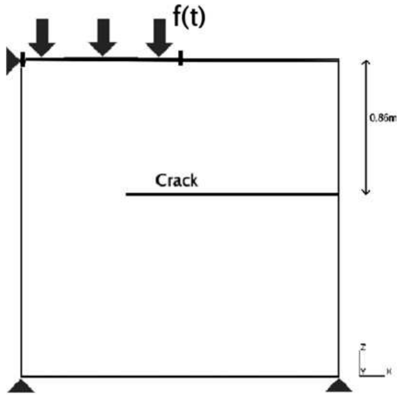


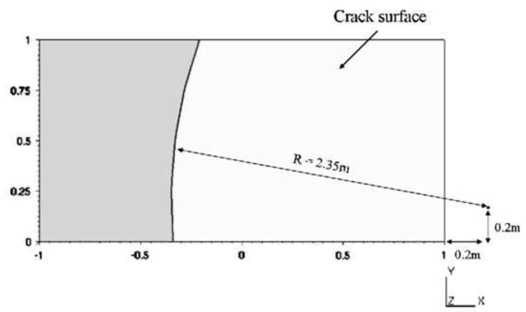
Number of iterations at convergence



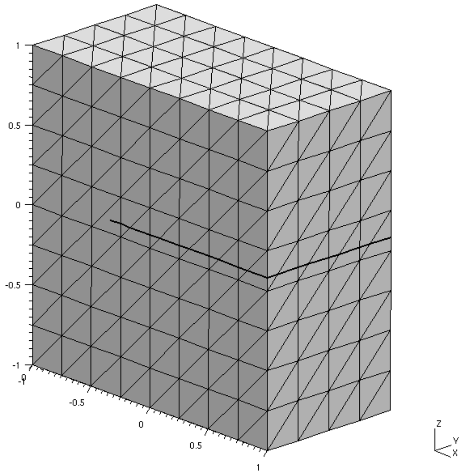
3 Example 3: A 3D crack submitted to a compressive load

- Structure and crack geometry

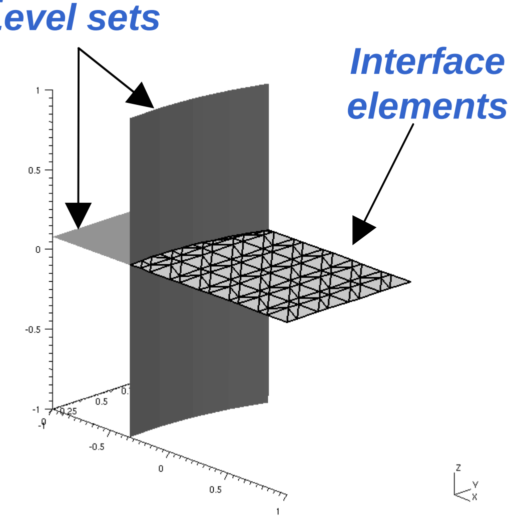




- Mesh of the structure and crack:

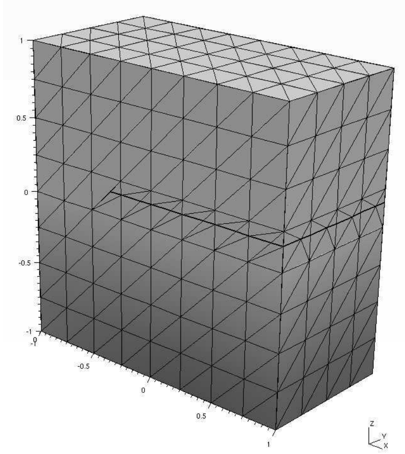


- Intersection of the crack and the X-FEM mesh



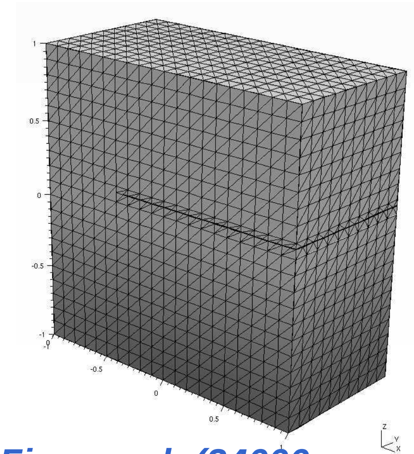
3

CASE A:

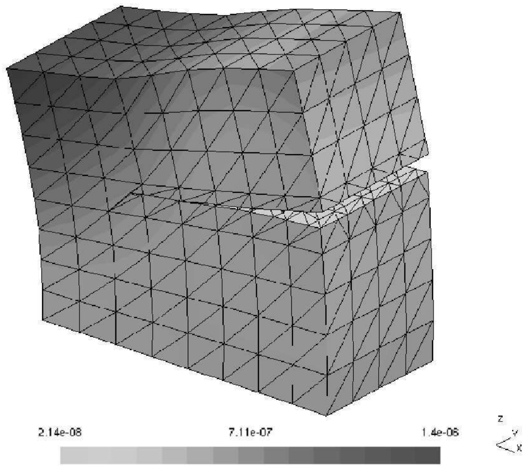


Coarse mesh (1536 elements)

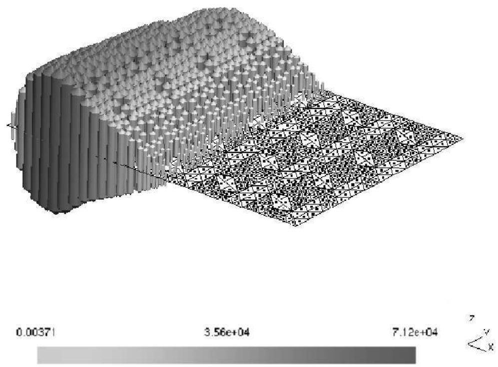
CASE B:



Fine mesh (24000 elements)

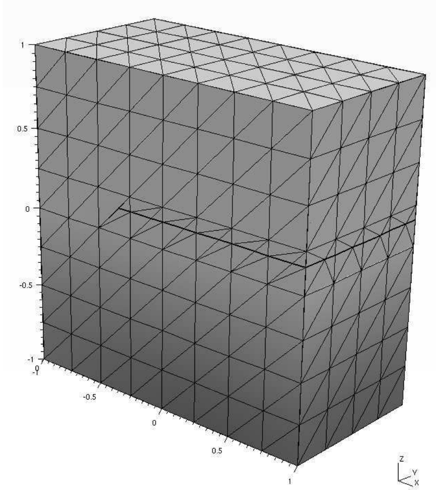


Amplified representation of the deformed mesh (case A)



Three-dimensional representation of the loads along the crack interface (case B).

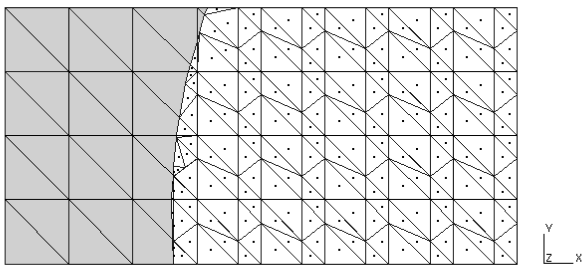
3 *CASE C: Independent discretization of the interface at a given scale (recursive refinement of the Gauss points distribution at the interface)*

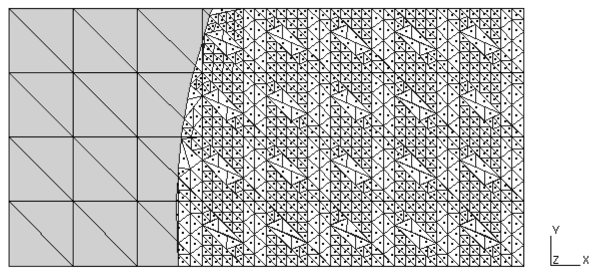


The proposed three field weak formulation authorizes non matching discretizations between the bulk and the crack

CASE A:

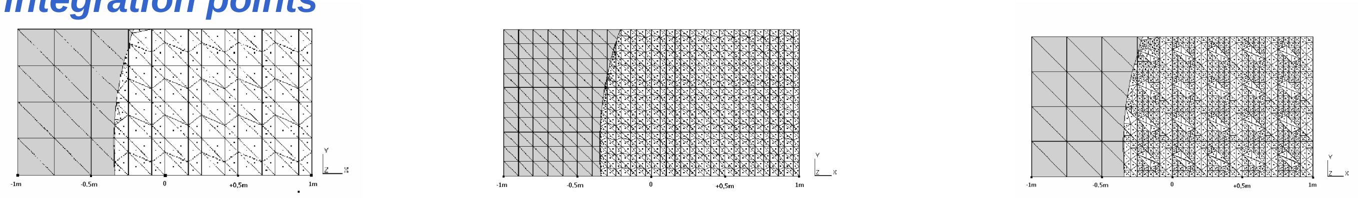
CASE C:



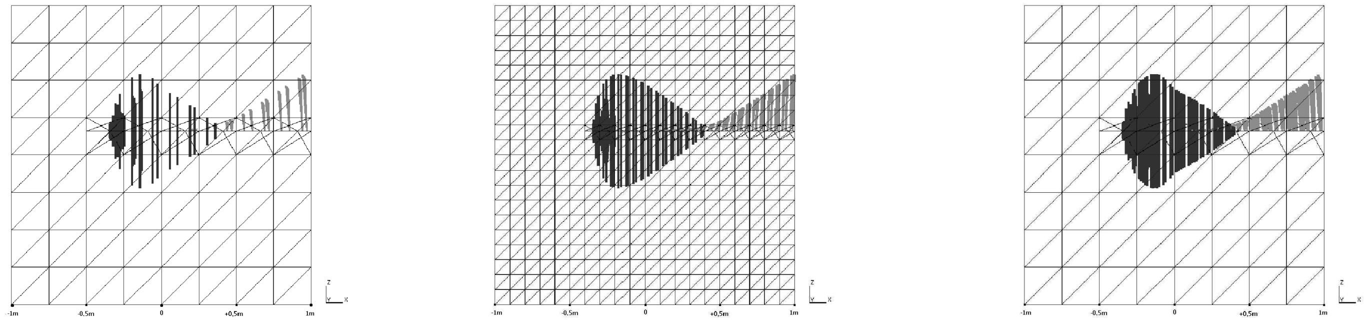


Intersection of the crack geometry and the X-FEM mesh for CASE A and CASE C

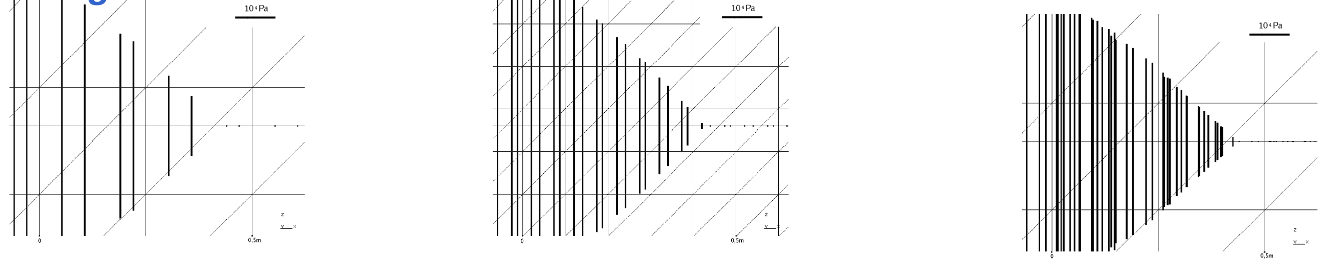
- **Case A: coarse interface discretization: 188 integration points** — **Case B: fine interface discretization: 1098 integration points** — **Case C: refined interface discretization: 1260 integration points**



- **Projection of the interface fields obtained on plane y = 0.**



- **View of the interfacial traction field zoomed on the transition between the open and contacting zone**



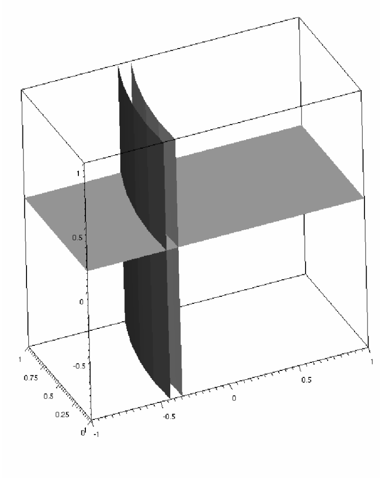
In both cases, the 3 field weak formulation authorizes contact discontinuities inside the 3D r

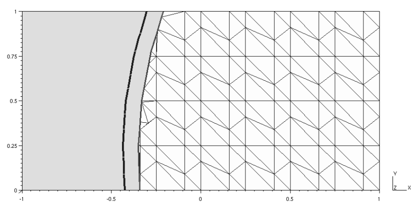
- **Numerical details relative to 3D test cases A, B and C**

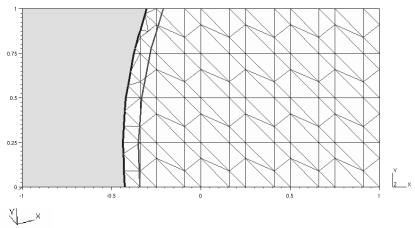
	Case A	Case B	Case C
3D element number	1536	24000	1536
Characteristic tetrahedron size (m)	0.25	0.1	0.25
Integration point number	188	1098	1260
Interface element size (m)	≃0.15	≃0.06	≃0.06
Contact/open border location (m)	0.356	0.386	0.399
and relative error	7.7%	*ref*	3.3%
along X axis, on $y = 0$ plane			
Relative CPU time	0.07	1	0.26

→ 70% CPU saving

- **Example of crack propagation: update of the level sets and definition of new interface elements + calculation of 3D SIFs based on 3D path independent integrals**

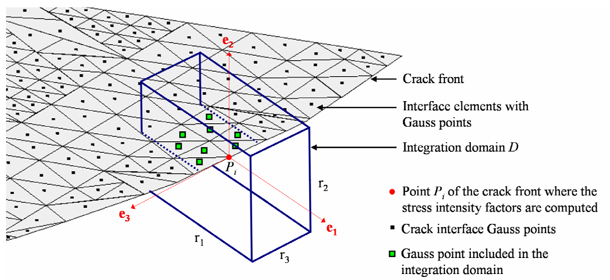






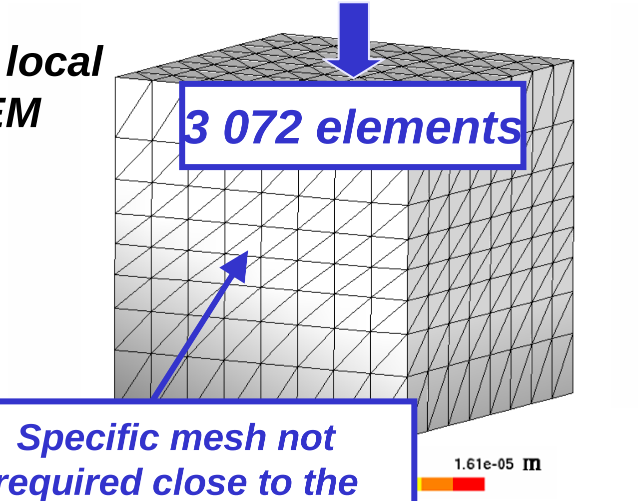
$$I_h = -\int_D (\sigma^h_{kl}\epsilon^{aux}_{kl}\delta_{ij} - \sigma^h_{kj}u^{aux}_{k,i} - \sigma^{aux}_{kj}u^h_{k,i})q_{i,j}dV$$

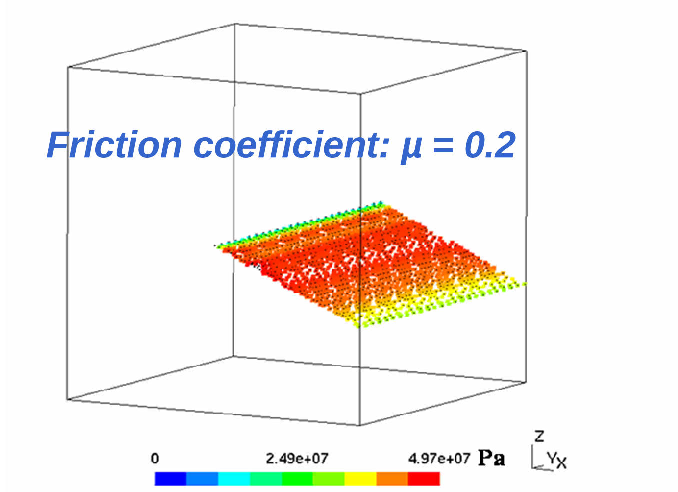
$$-\int_{\Gamma^+_C \cup \Gamma^-_C} (\sigma^h_{k2}u^{aux}_{k,1} + \sigma^{aux}_{k2}u^h_{k,1})qn_C dS$$



3 Example 4: Efficiency of the Global-local X-FEM

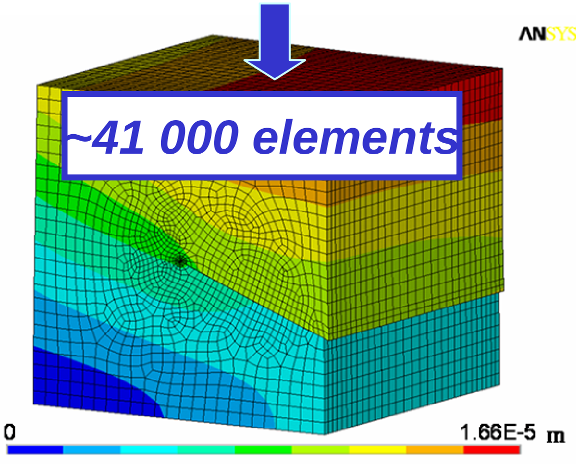
Global- local X-FEM



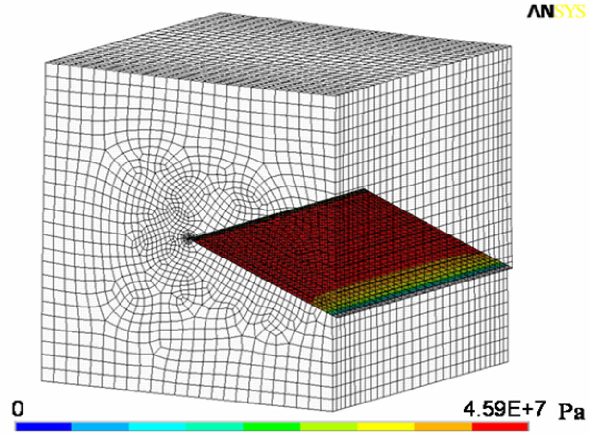


[E. Pierres et al., Tribology International 2010]

ANSYS



Displacement field



Tangential contact load

[F. Galland et al., IJNME 2010]

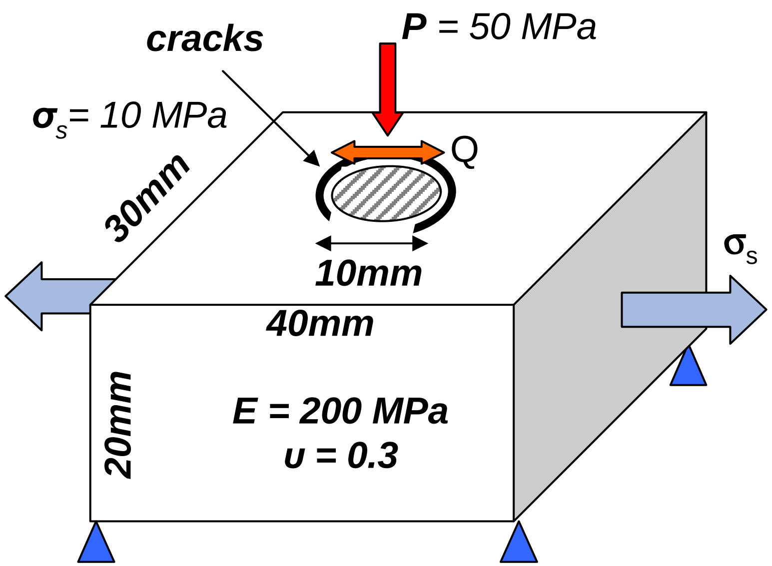
4 *Numerical modeling of experimental fretting fatigue tests.*

(.AVI)

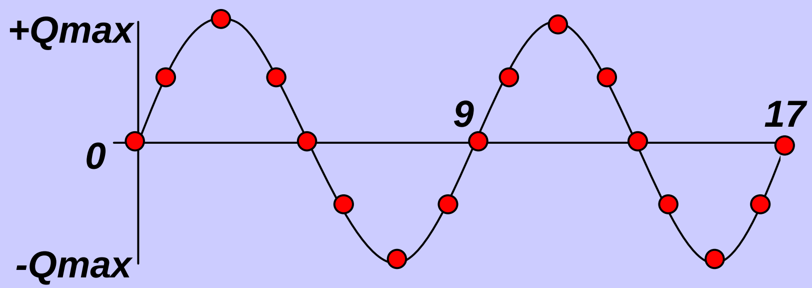
Numerical simulation with the Global – local X-FEM

[Chateauminois, Baietto, 2005]

Goal: Account for 3D complex crack geometries, local fretting loading, frictional contact conditions, multi-scale effects



Constant normal pressure P = 50 MPa and Cyclic tangential pressure Qmax = 59 MPa on a circular area on the surface:



4 Multi–model strategy for the prediction of fretting crack life time

Controled fretting experiments
cylinder/plane or sphere/plane

⬇

Data loads are saved
Fretting loop: partial sliding
alculation of the local friction coefficient μ(t)

⬇

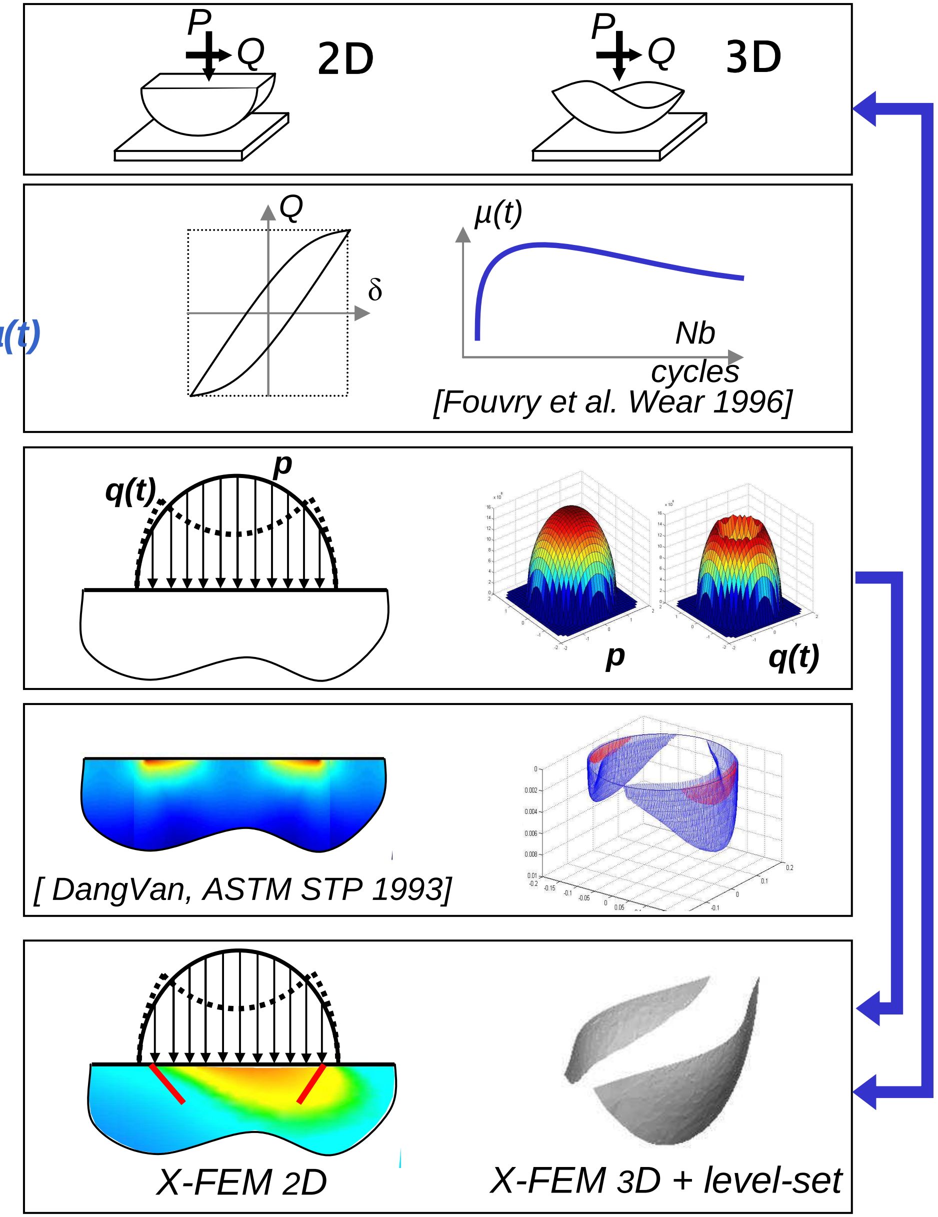
Solving of the two-body contact pb:
Calculation of normal and tangential
stress fields (p and q(t))

⬇

Crack initiation locations and angles:
Dang Van multi-axial fatigue criterion

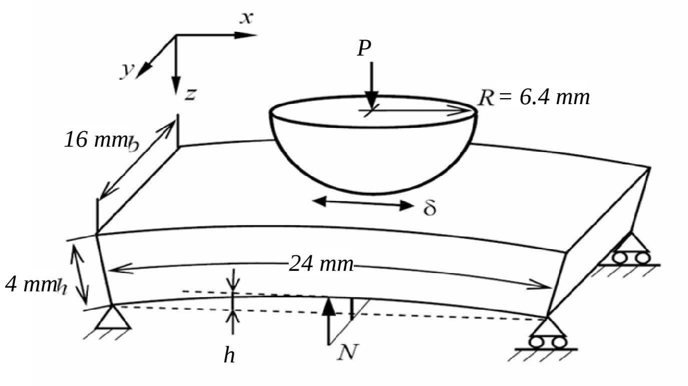
⬇

Global - Local X-FEM
Stress Intensity Factors
2D and 3D crack propagation

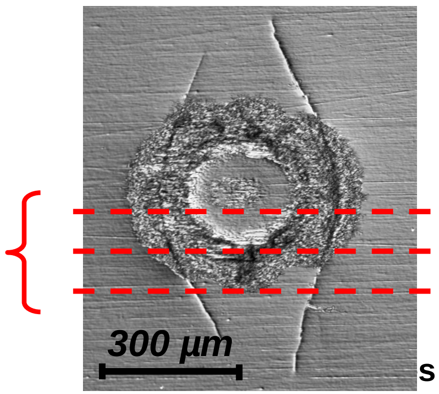


4 3D fretting fatigue experiments: sphere / plane contact (ERC SKF research center)

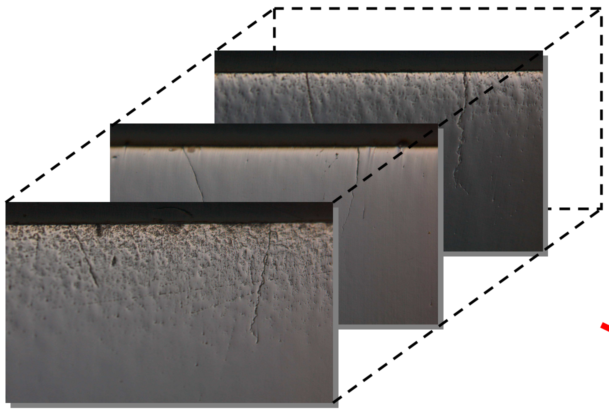
Sphere / plane experiment



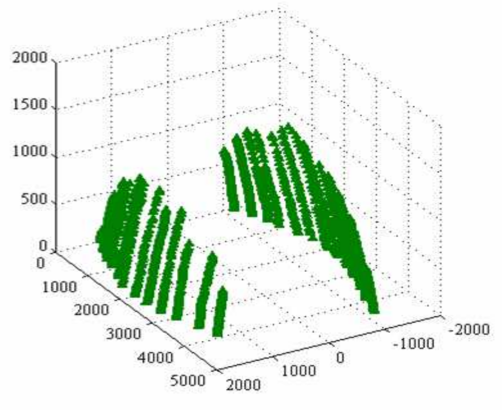
Experimental fretting crack



Transversal cut

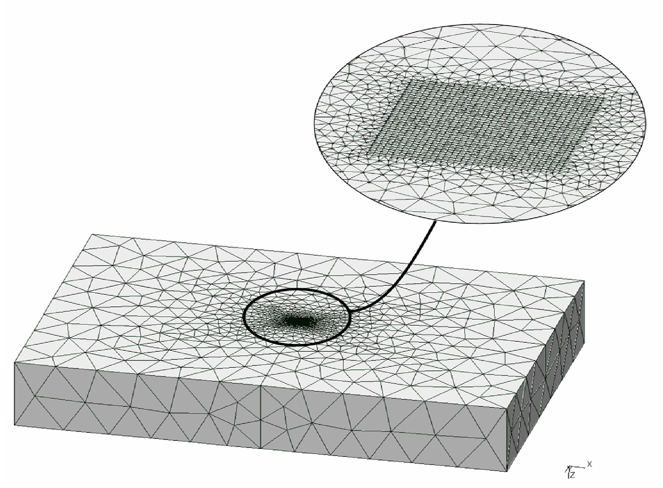


3D crack shape reconstruction

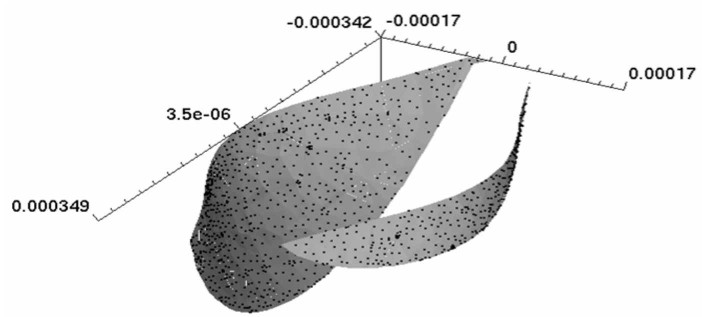


4 Global – Local X-FEM Simulation of a 3D fretting fatigue test

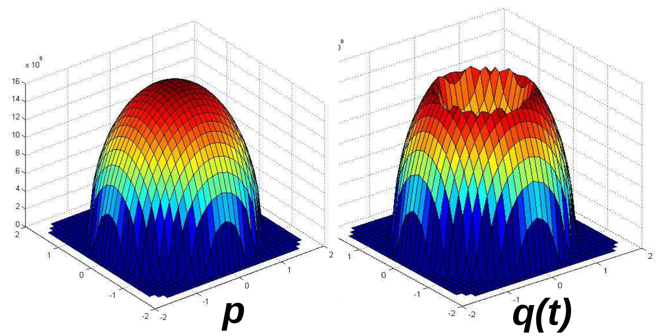
- <u>Specimen:</u>
  - 25mm × 16mm × 4mm
  - Steel : E = 210 GPa ; ν = 0.3
  - Mesh : 46266 tetraedra
  - local refinement close to the area of interest (contact zone)



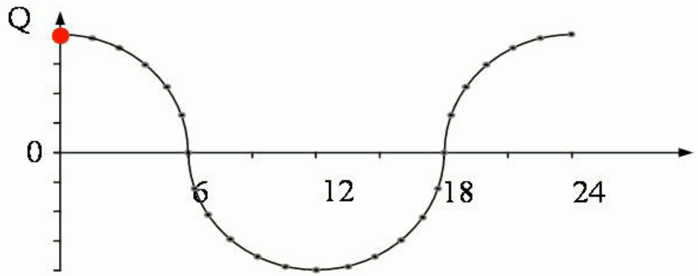
- <u>Crack:</u>
  - level sets
  - Discretization: 2574 interface elements
  - boundary crack length: ~600 µm
  - in the bulk: ~ 100 µm



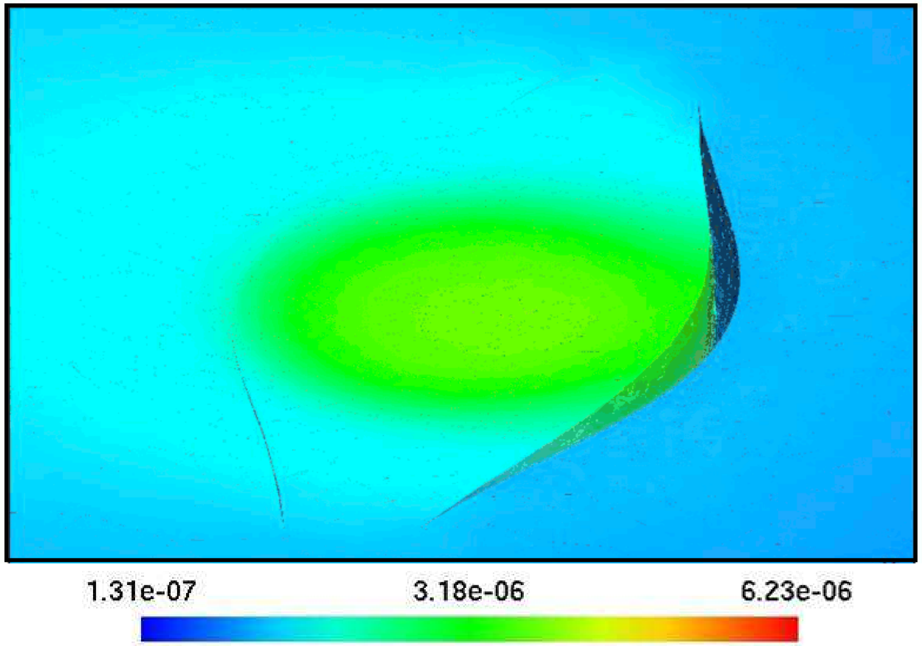
- <u>Loading:</u>
  - obtained from the two-body contact calculation
  - time discretization for 1 cycle: 25 time steps



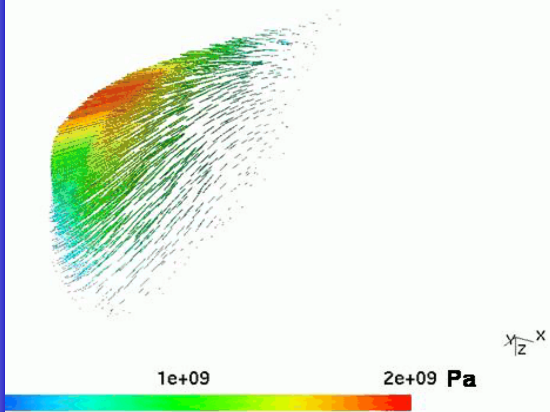
4 Global – Local X-FEM Simulation of a 3D fretting fatigue test



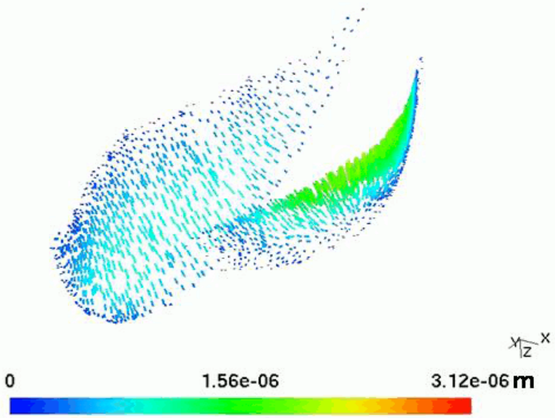
Fretting cycle



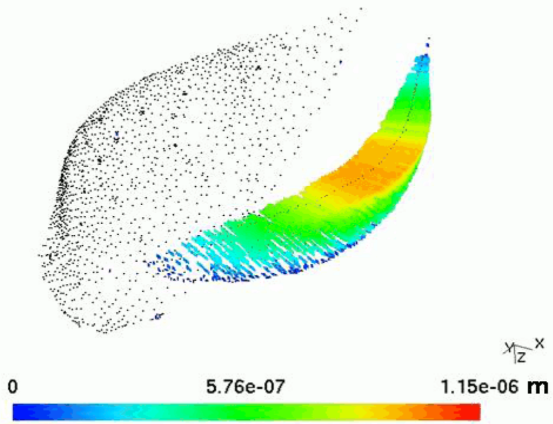
Global displacement field



contact load



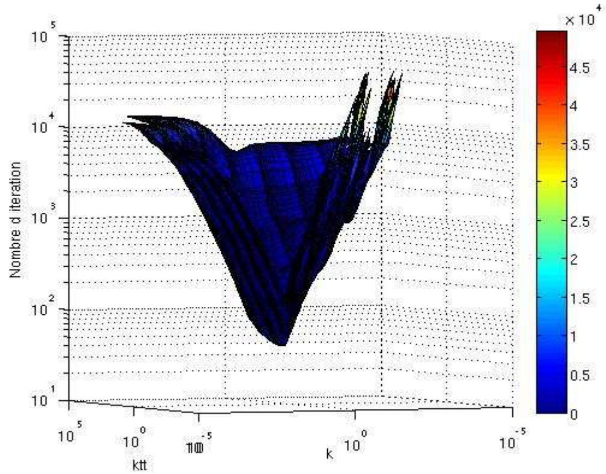
Local sliding



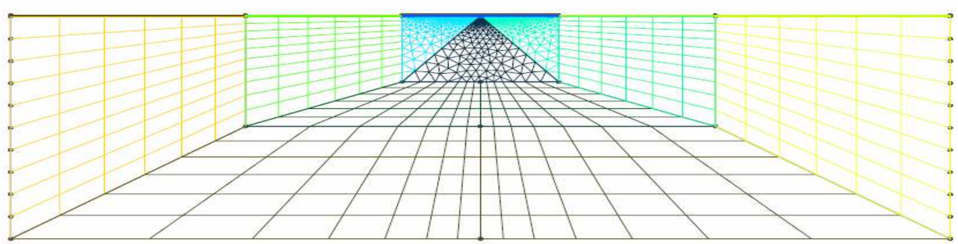
Local crack opening

4 TRAVAIL RÉALISÉ avec la SNCF: OPTIMISATION DES PERFORMANCES DU SOLVEUR X-FEM AVEC PRISE EN COMPTE DU CONTACT INTERFACIAL

- Optimisation des performances du solveur (2 paramètres: direction de recherche, terme de stabilisation)
  - Moins de 50 itérations NL pour un précision de 10-4
  - Indispensable pour des problèmes en 3 dimensions

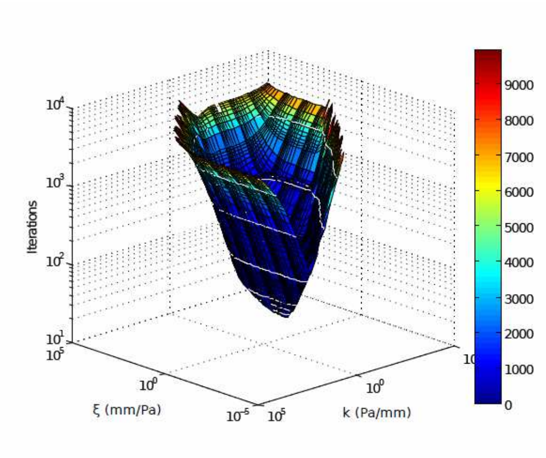


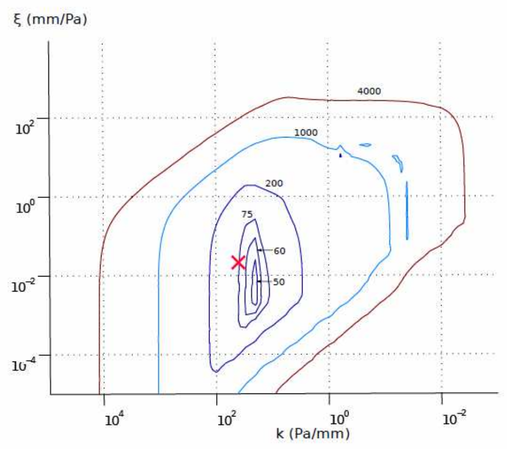
- Maillage multi-échelle paramétré



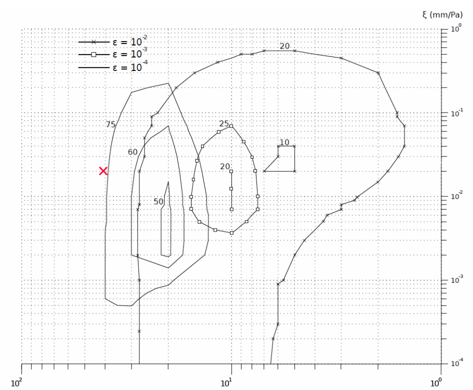
- ***k = E / l***   ***ξ = l / E***   ***E : module de Young du matériau***
  l : longueur de la fissure

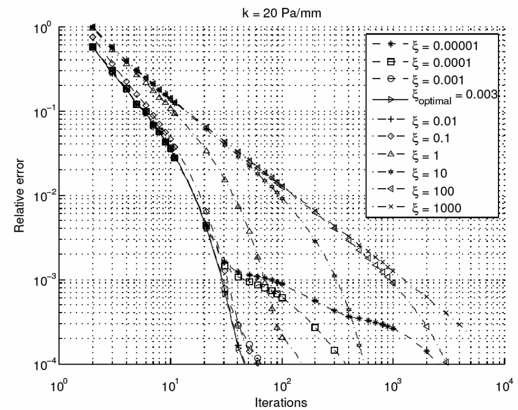
- ***Etude de l'influence des CL, géométrie, matériau, chargement, coefficient de frottement, fissure***





- ***Influence de la précision, Taux de convergence***





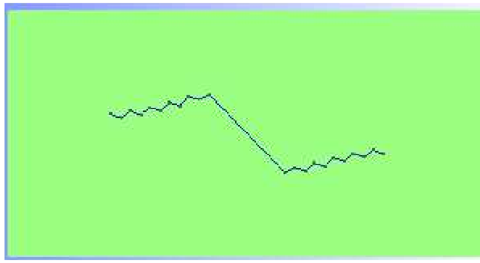
Convergence plus rapide vers une solution stabilisée
[B. Trollé, A. Gravouil, MC. Baietto, TML.Nguyen, FEAD, 2011, submitted]

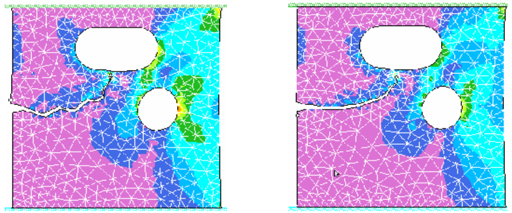
4

TRAVAIL RÉALISÉ : MAITRISE DES ARTEFACTS NUMERIQUES UTILISÉE POUR SIMULER LA PROPAGATION DES FISSURES

- Simulation de la propagation des fissures sensible à l'erreur de discrétisation et de résolution numérique

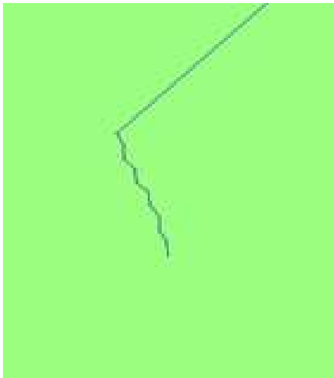
- Peu étudié sur un grand nombre de cycle

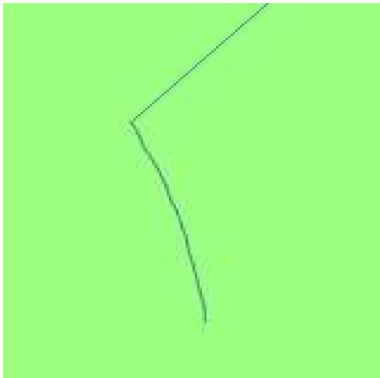




P. O. Bouchard,CONTRIBUTION A LA MODELISATION NUMERIQUE EN MECANIQUE DE LA RUPTURE ET STRUCTURES MULTIMATERIAUX, thèse école des Mines de Paris, 2000

- Algorithme adaptatif





4

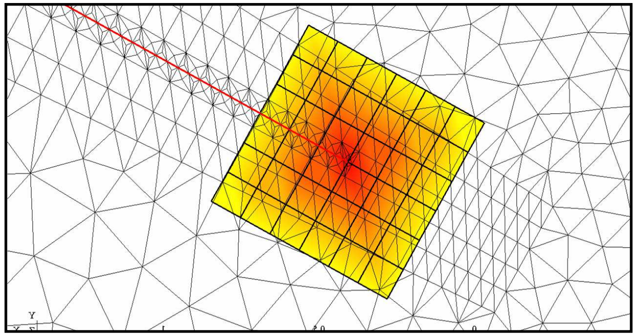
TRAVAIL RÉALISÉ : MAITRISE DES ARTEFACTS NUMERIQUES UTILISÉE POUR SIMULER LA PROPAGATION DES FISSURES

- **Stress intensity factors calculation**

  2D interaction integral

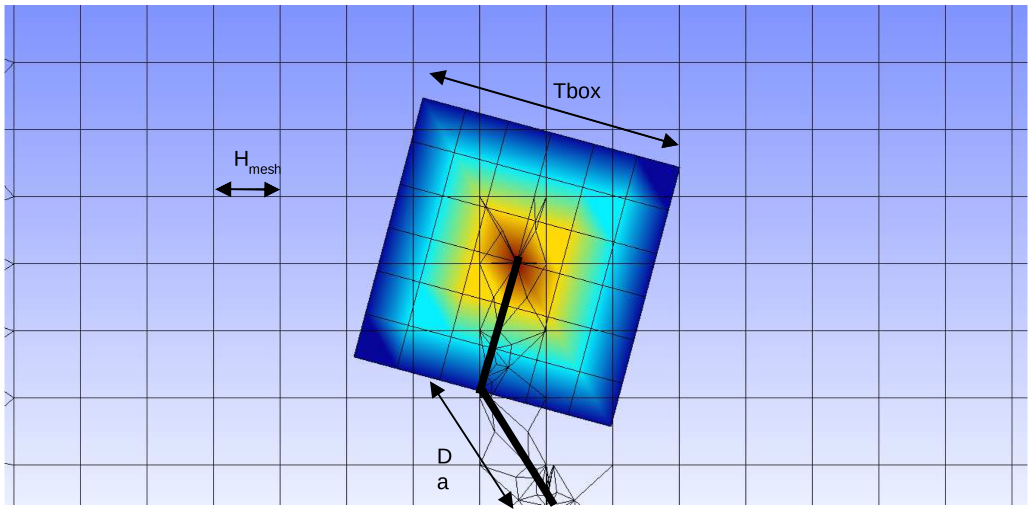
$$I^{\Re,aux} = \int_C \left(W_I^{\Re,aux}\delta_{1j} - \sigma_{ij}^{\Re}\frac{\partial u_i^{aux}}{\partial x_1} - \sigma_{ij}^{aux}\frac{\partial u_i^{\Re}}{\partial x_1} \right) n_j \, ds + \sigma_{12}^{\Re}(A)\,[u_1^{aux}(A)]$$

$$I^{\Re,aux} = \frac{2(1-\nu^2)}{E}\left(K_I^{\Re}K_I^{aux} + K_{II}^{\Re}K_{II}^{aux}\right)$$



Integration domain close to the crack t

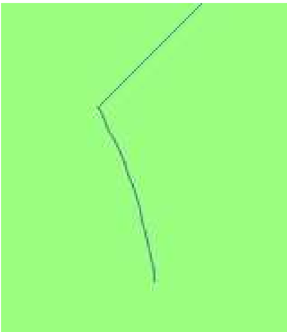
- **3 parameters for the adaptative algorithm**



4 TRAVAIL RÉALISÉ : MAITRISE DES ARTEFACTS NUMERIQUES UTILISÉE POUR SIMULER LA PROPAGATION DES FISSURES

- **Algorithme 1:** Algorithme de propagation adaptatif

si $\theta_N\theta_{N-1} < 0$ *et* $(\theta_N > \theta_{N-1}$ *ou* $\theta_N > \theta_{coin})$ *et* $\theta_N > \theta_{bruit}$ **alors**
  $\Delta a_N = k_{\Delta a}\ \Delta a_{N-1}$
fin
si $\theta_{N-1} > \theta_{bruit}$ **alors**
  si $(N - Ncoin - 1)\Delta a_{N-1} > 3\frac{H_{mesh}}{2}$ **alors**
    $Tox = 3H_{mesh}$ ou entrées utilisateurs
  fin
sinon
  $Tbox = 2\Delta a_{N-1}$
  si $(\theta_{N-1} > \theta_{coin})$ **alors**
    $N_{coin} = N$
  fin
fin
si $(\frac{Tbox}{2} < H_{mesh})$ **alors**
  $Tbox = H_{mesh}$
fin



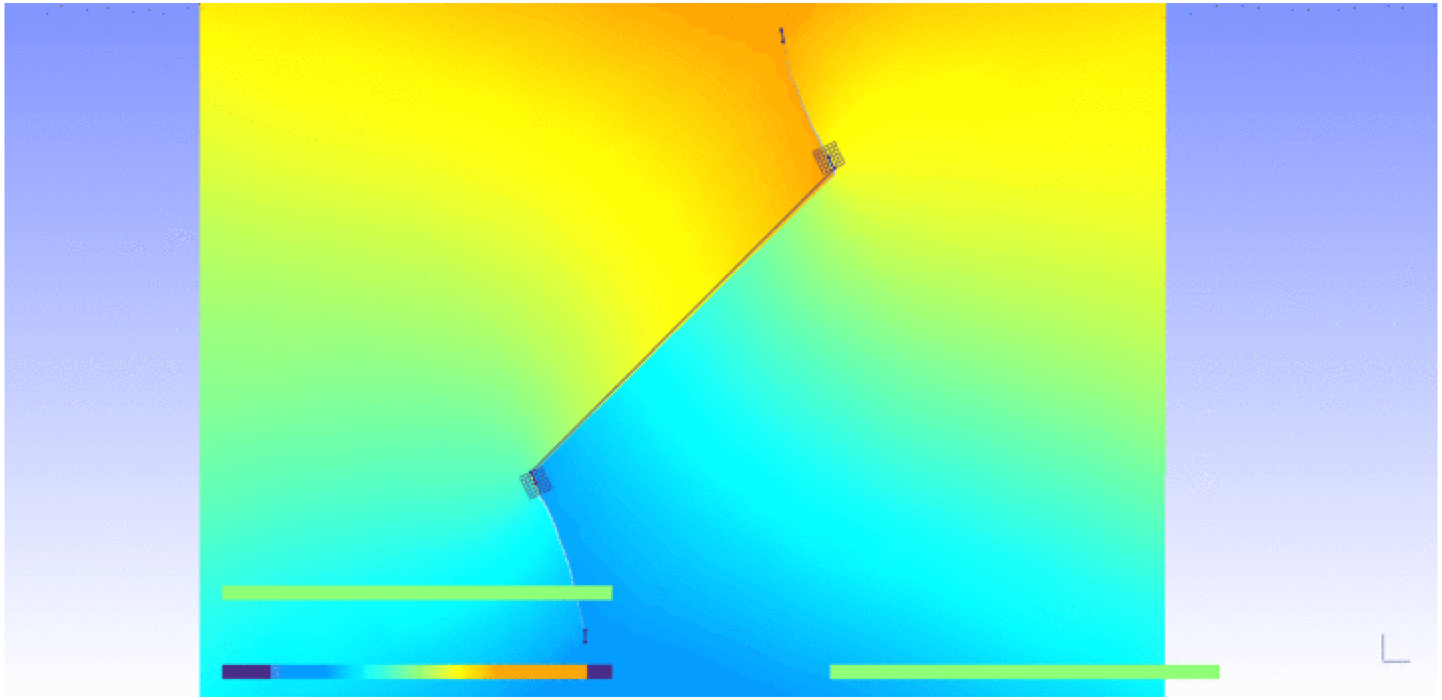
- A erreur de discrétisation fixée, contrôle des paramètres minimisant l'erreur numérique:

  Δa variable   Tbox s'adapte au Δa

→ Propagation robuste avec une maîtrise de l'erreur induite par les artefacts numériques

4 TRAVAIL RÉALISÉ : MAITRISE DES ARTEFACTS NUMERIQUES UTILISÉE POUR SIMULER LA PROPAGATION DES FISSURES

- *Adaptation des paramètres numériques en fonction des pas de propagation précédents*



4 LOI DE PROPAGATION EN MODE MIXTE DEDIEE À LA FATIGUE DE ROULEMENT

- Loi du projet ICON à partir d'essai sur machine à galet

$$\frac{da}{dn} = 2.10^{-9}\left(\Delta K_{eq}^{2}\right)^{1.665} \qquad \Delta K_{eq}^{2} = \Delta K_{I}^{2} + 0{,}772\,\Delta K_{II}^{2}$$

- Autre loi disponible:

$$\frac{da}{dn} = 0{,}000507\left(\Delta K_{eq}^{3{,}74} + \Delta K_{seuil}^{3{,}74}\right) \qquad \Delta K_{eq} = \sqrt{\Delta K_{I}^{2} + \left[\left(\frac{614}{307}\right)\Delta K_{II}^{3{,}21}\right]^{\frac{2}{374}}}$$

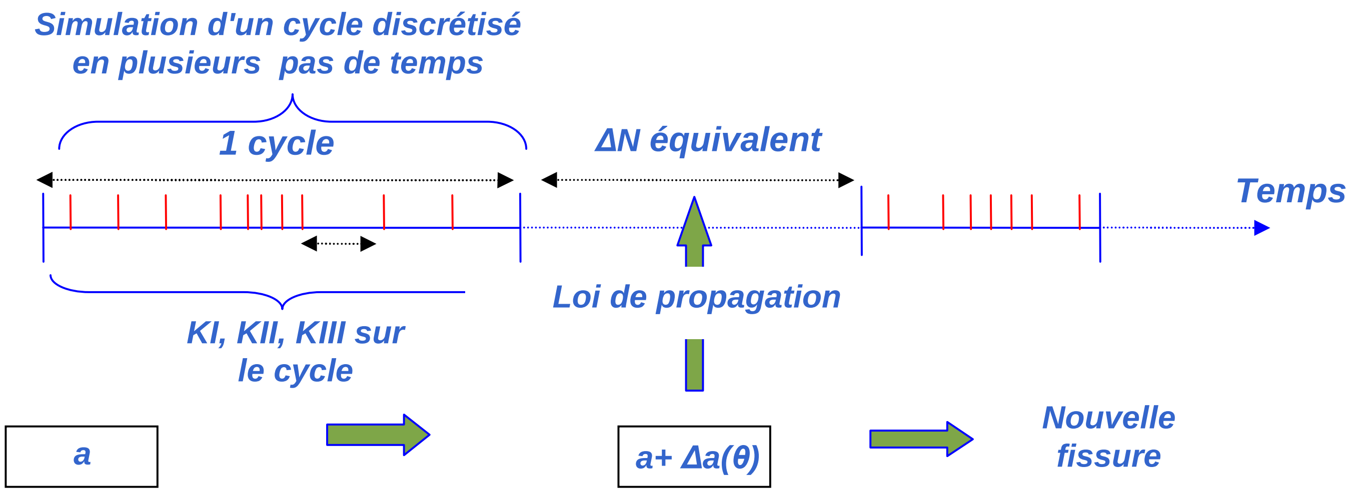
[Bold, P.; Brown, M. & Allen, R. Shear mode crack growth and rolling contact fatigue Wear, 1991, 144, 307-317]

- Réflexion sur la mise en place d'essais de fatigue de roulement sur machine à galet LaMCoS

4 Stratégie multi-échelle pour la simulation de la propagation des fissures

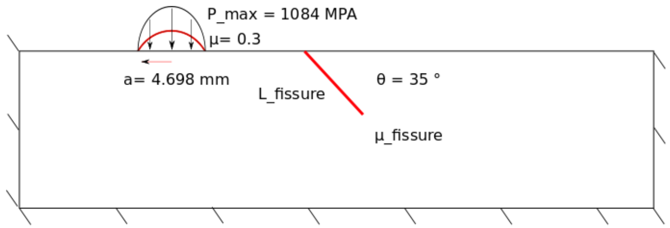
- Etape suivante : Propagation des fissures sur un grand nombre de cycle

- 2 échelles de temps :
  → Pas de temps au cours d'un cycle pour le calcul des FICs
  → Cycle entier pour la propagation

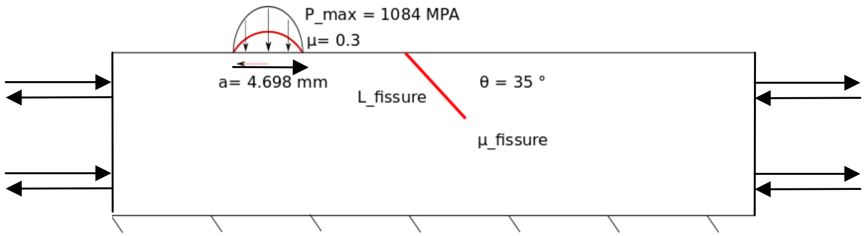
- 

4 Application à la fatigue de roulement

- **Première étude paramétrique**
  - **Longueur de la fissure**

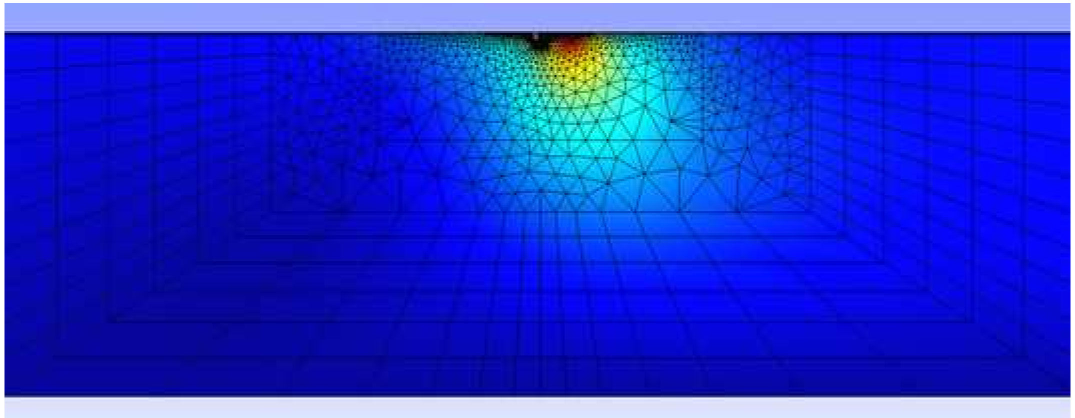


  - **Contraintes résiduelles**



Contraintes « résiduelles » : 125 MPa

-



4 DEVELOPPEMENT DE LA STRATEGIE SOUS CAST3M

- Stratégie de simulation multi-échelles maîtrisée (Logiciel dédié ELFE_3D / LaMCoS)

- En collaboration avec l'équipe CAST3M du CEA Saclay
  - Assemblage de la matrice globale du système linéaire
  - Loi de Coulomb pour l'interface
  - Résolution avec la méthode LATIN
  - Indicateur spécifique au problème d'une fissure en contact frottant

- Points durs dans l'implémentation:
  - formulation mixte à 3 champs (solveur de l'étape globale linéaire)
  - raccord faible entre le maillage et la fissure (similaire opérateurs de maillages incompatibles)
  - XFEM avec contact + solveur LATIN

- Remarque: la méthode explicite/implicite de représentation des fissures (triangulation+levelsets) développée par B P.rabel est parfaitement adaptée à une extension aux fissures frottantes dans

5 General Conclusions

- *Recovering with X-FEM a fully independence of the interface mesh from the structure mesh for contact and / or friction problems*
- *Describing accurately possible complex contact / friction states along the crack faces*
- *The following improvements are introduced:*
  - *The crack interface is considered as an autonomous entity with its own primal and dual variables (w,t) discretization, and constitutive law (unilateral frictional coulomb's law).*
  - *An automatic refinement of the interface discretization is performed according to size and shape criteria to get a contact solution at a prescribed accuracy. It leads to a crack discretization independent from the finite element structural mesh, further adapted to the scale of interest.*
  - *An innovative three-field weak formulation coupling bulk and crack variables (u,w,t) is adopted.*

5 Contributions récentes - perspectives

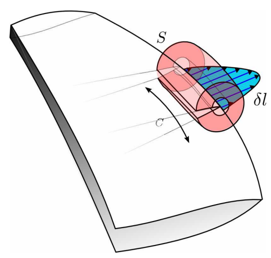
- Optimisation du solveur X-FEM Non-linéaire avec contact / frottement
- Optimisation de la direction de recherche du solveur LATIN et du terme de stabilisation (moins de 50 itérations pour une précision de 10-4)
  - Jusqu'à un facteur 20 de gain CPU par rapport à un calcul non-optimisé (indispensable dans la perspective du 3D)
- Algorithme de propagation adaptatif permet une propagation où l'erreur numérique est maîtrisée (liée au maillage, au pas et à l'angle de l'incrément de propagation)
- Implémentation de la méthode dans CAST3M (CEA)
- Implémentation d'une loi de propagation en fatigue de roulement (essais de validation éventuels)

~

Stress Intensity Factors calculation

- *issipated energy for a virtual crack extension*

$$\int \quad (\;) \quad (\;) \qquad \int \int \qquad \qquad (\qquad \qquad)$$

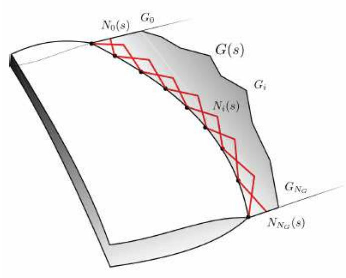


- *G(s) is discretized along the front with curvilinear shape functions:*

$$(\;) \; \sum \quad (\;) \qquad \qquad \int \quad (\;) \quad (\;)$$

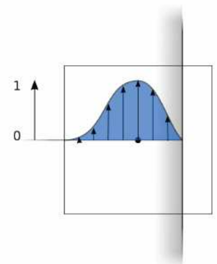
$$- \qquad \qquad - \quad \int$$

[Parks et al., 2000]



- *Diagonal matrix:* $\sum \quad \int \sum \; (\;) \; (\;) \; = \int \; (\;)$

- *Irwin formula* $-[\quad (\;) \quad (\;)] \; - \quad (\;)$



- *Interaction integral* $\int \quad [(\qquad) \quad (\qquad \qquad)]$

- *Contact, friction, plasticity, dynamics* $\int (\qquad \qquad) \qquad \int \qquad \left(- \quad - \right)$

[Combescure, Suo, 1986] [Moës N., Gravouil A., Belytschko T., IJNME 2002]

[Gosz & al. 1997,2002, Béchet 2005, Réthoré 2005, Elguedj 2006, Ribeaucourt 2006]

Augmented Lagrangian Iterative Solver

- *Iterative strategy similar to LATIN method*
- *Regularization of the 3 field weak formulation by penalty terms (influence on the convergence rate)*

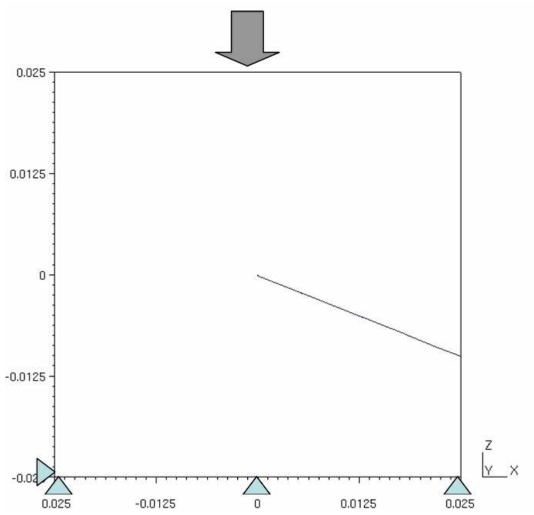
$$
\begin{aligned}
0 \;=\; & -\int_{\Omega} \boldsymbol{\sigma}_{i+1} : \boldsymbol{\epsilon}(\mathbf{u}^*) d\Omega + \int_{\Gamma^t} \mathbf{f}_t \cdot \mathbf{u}^* dS + \int_{\Gamma_C} \boldsymbol{\lambda}_{i+1} \cdot \mathbf{u}^* dS \\
& + \int_{\Gamma_C} (\mathbf{t}_i + \boldsymbol{\alpha} \mathbf{w}_i) \cdot \mathbf{w}^* dS - \int_{\Gamma_C} (\boldsymbol{\lambda}_{i+1} + \boldsymbol{\alpha} \mathbf{w}_{i+1}) \cdot \mathbf{w}^* dS \\
& + \int_{\Gamma_C} (\mathbf{u}_{i+1} - \mathbf{w}_{i+1}) \cdot \boldsymbol{\lambda}^* dS \;\; \forall \mathbf{u}^* \in U_0^*, \; \forall \mathbf{w}^* \in W^* \; and \; \forall \boldsymbol{\lambda}^* \in \Lambda^*
\end{aligned}
$$

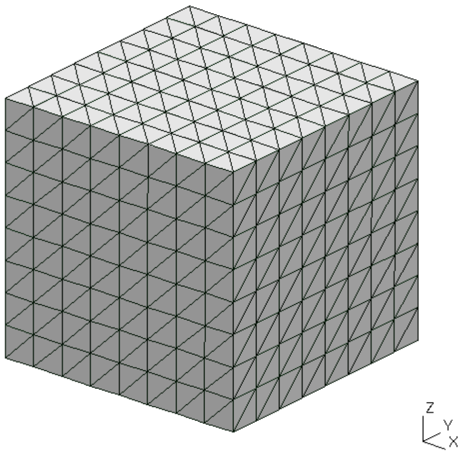
- *Discretized formulation:*

$$
\begin{bmatrix} \mathbf{K} & 0 & -\mathbf{K}_{u\lambda} \\ 0 & \mathbf{K}_{ww} & \mathbf{K}_{w\lambda} \\ -\mathbf{K}_{u\lambda}^T & \mathbf{K}_{w\lambda}^T & 0 \end{bmatrix}
\begin{pmatrix} \Delta \mathbf{U}_{i+1} \\ \Delta \mathbf{W}_{i+1} \\ \Delta \boldsymbol{\Lambda}_{i+1} \end{pmatrix}
=
\begin{pmatrix} \mathbf{F}_t + \mathbf{K}_{u\lambda} \cdot \boldsymbol{\Lambda}_i \\ \mathbf{K}_{w\lambda} \cdot (\mathbf{T}_i - \boldsymbol{\Lambda}_i) \\ \mathbf{K}_{u\lambda}^T \cdot \mathbf{U}_i - \mathbf{K}_{w\lambda}^T \cdot \mathbf{W}_i \end{pmatrix}
$$

$$
\left\{
\begin{aligned}
\mathbf{u}_{i+1} &= \Delta \mathbf{u}_{i+1} + \mathbf{u}_i \\
\mathbf{w}_{i+1} &= \Delta \mathbf{w}_{i+1} + \mathbf{w}_i \\
\boldsymbol{\Lambda}_{i+1} &= \Delta \boldsymbol{\Lambda}_{i+1} + \boldsymbol{\Lambda}_i
\end{aligned}
\right.
$$

Validation of the model: Crack submitted to frictional contact





Bulk mesh:

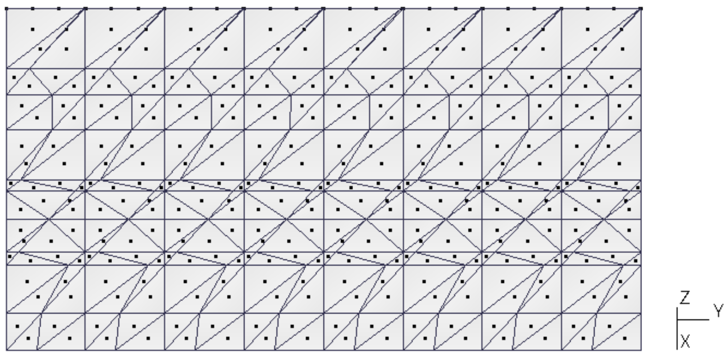
3072 tetrahedra

Geometry: (50mm; 50mm; 50mm)

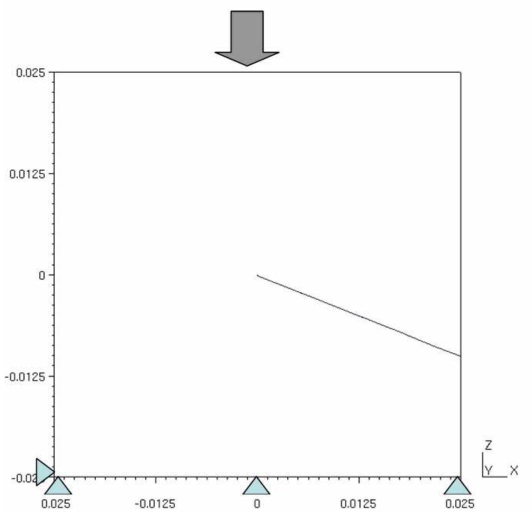
Material: E = 200 GPa, υ = 0,3

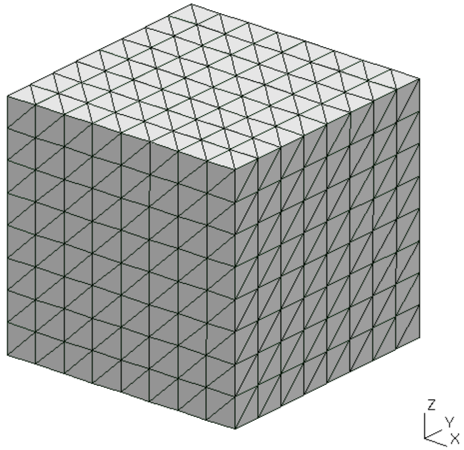
Compressive pressure: 50 MPa

Standard interface: 360 interface elements



Validation of the model: Crack submitted to frictional contact

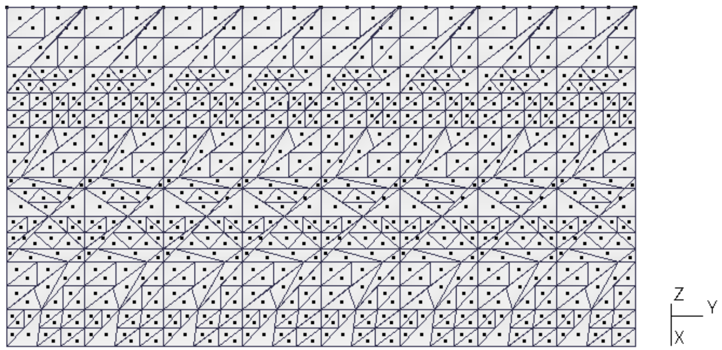




Bulk mesh:

3072 tetrahedra

Refined interface: 832 interface elements



Geometry: (50mm; 50mm; 50mm)

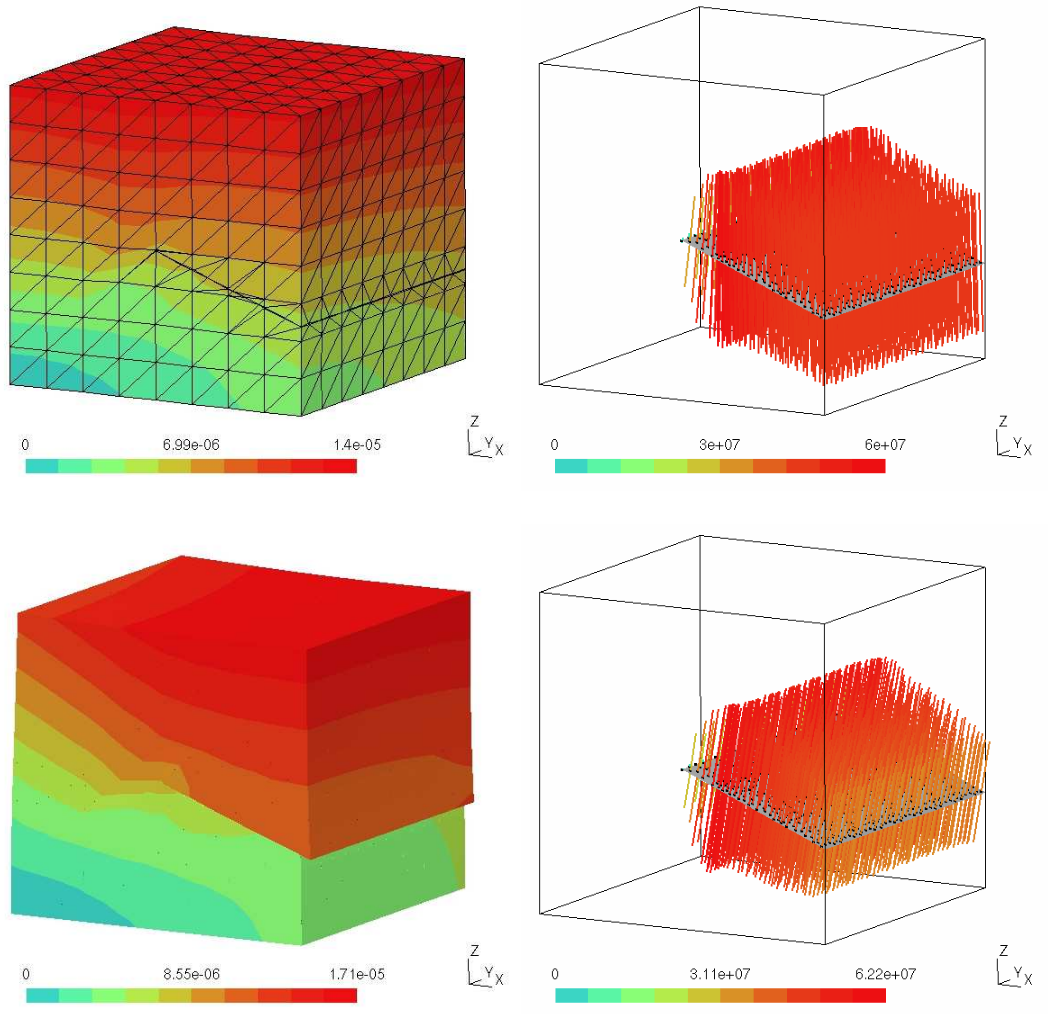
Material: E = 200 GPa, υ = 0,3

Compressive pressure: 50 MPa

Validation of the model: Crack submitted to frictional contact

Sticking case: μ = 1
Solution very close to uncracked body submitted to the same loading

Sliding case: μ = 0,2
Good agreement with FEM (ANSYS)

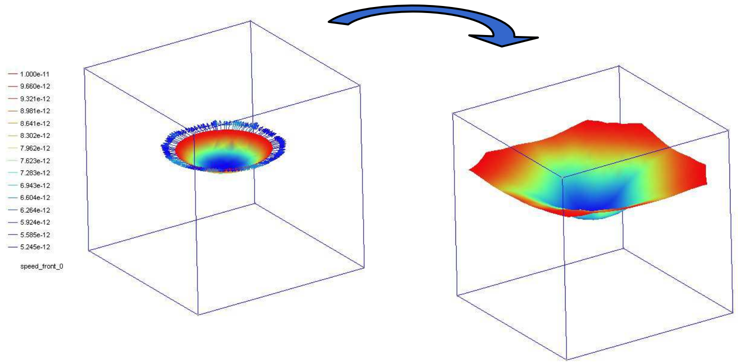


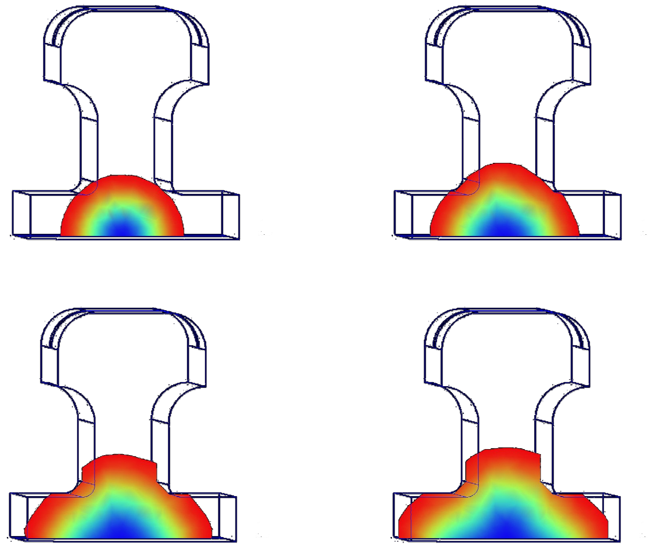
X-FEM + Level sets

- *eXtended Finite Element Method + level sets*

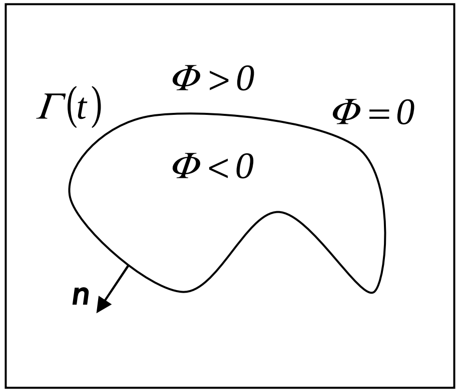
- *Advantages:*
  - *Similar to FEM*
  - *No remeshing*
  - *No field interpolation*
  - *Good topologic properties*
  - *Flexibility in the initialization of level sets*

- *Drawbacks:*
  - *Specific numerical integration and preconditioning*
  - *Post-treatment*
  - *Specific strategies of enrichment for time-dependent problems*





Level sets



Signed distance:

$$\begin{cases} \Phi(\mathbf{x},t) < 0 & in\ \Omega^-(t) \\ \Phi(\mathbf{x},t) = 0 & on\ \Gamma(t) \\ \Phi(\mathbf{x},t) > 0 & in\ \Omega^+(t) \end{cases}$$

→ *Allows to model implicitly moving interfaces*

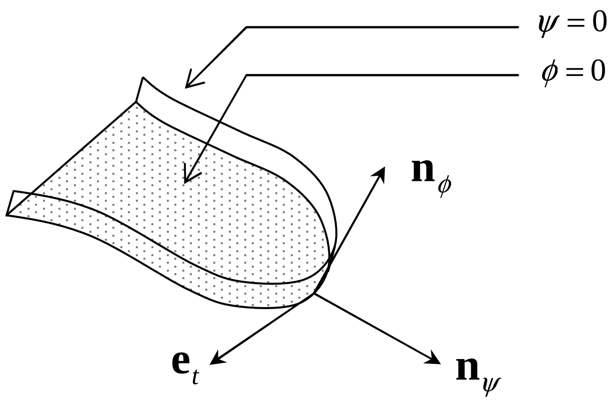
- **Governing equation:** $\frac{\partial \Phi}{\partial t} + V|\nabla \Phi| = 0$
- **Normal vector:** $\mathbf{n} = \frac{\nabla \Phi}{|\nabla \Phi|}$ **curvature:** $\kappa = \nabla . \frac{\nabla \Phi}{|\nabla \Phi|}$
- **Hausdorff measure:** $|\Gamma(t)| = \int \delta(\Phi)|\nabla \Phi| dx$

[Osher and Sethian 1988]

- **Lebesgue measure:** $|\Omega^-(t)| = \int H(-\Phi) dx$

Non-planar crack modeling

- Local basis linked to the level sets



$$\mathbf{n}_\psi = \nabla \psi$$

$$\mathbf{n}_\phi = \nabla \phi$$

$$\mathbf{e}_t = \mathbf{n}_\psi \times \mathbf{n}_\phi$$

- Component velocity on the local basis of the crack front V_ϕ V_ψ and

$$\mathbf{V} = V_\psi \mathbf{n}_\psi + V_\phi \mathbf{n}_\phi$$

Non-planar crack modeling

- **Time and space discretization for structured meshes**

$$\phi_{ij}^{n+1} = \phi_{ij}^{n} - \Delta t \left\{ \begin{array}{l} \left(s_{ij} n_{ij}^{x}\right)^{+} \dfrac{\phi_{ij} - \phi_{i-1j}}{\Delta x} + \left(s_{ij} n_{ij}^{x}\right)^{-} \dfrac{\phi_{i+1j} - \phi_{ij}}{\Delta x} \\ + \left(s_{ij} n_{ij}^{y}\right)^{+} \dfrac{\phi_{ij} - \phi_{ij-1}}{\Delta y} + \left(s_{ij} n_{ij}^{y}\right)^{-} \dfrac{\phi_{ij+1} - \phi_{ij}}{\Delta y} \end{array} \right\}$$

$$\begin{cases} \tilde{\Phi}^{n+1} = \Phi^{n} - \Delta t H\left(\Phi^{n}\right) \\ \Phi^{n+1} = \frac{\left(\Phi^{n} + \tilde{\Phi}^{n+1}\right)}{2} - \frac{\Delta t}{2} H\left(\tilde{\Phi}^{n+1}\right) \end{cases}$$

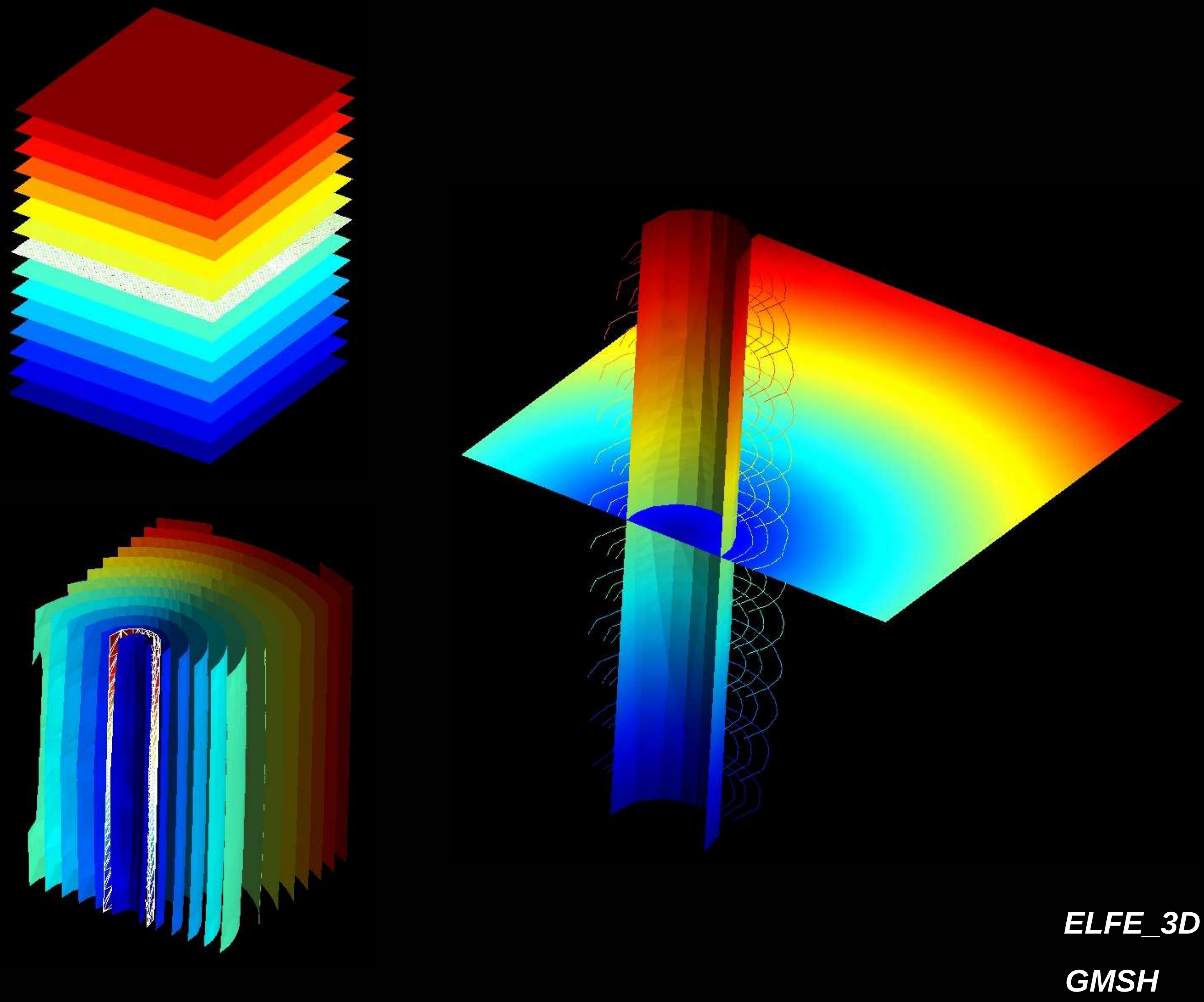
$$(x)^{+} = \max(x,0) \qquad (x)^{-} = \min(x,0) \qquad s_{ij} = \frac{\Phi_{ij}}{\sqrt{\Phi_{ij}^{2} + \Delta x^{2}}}$$

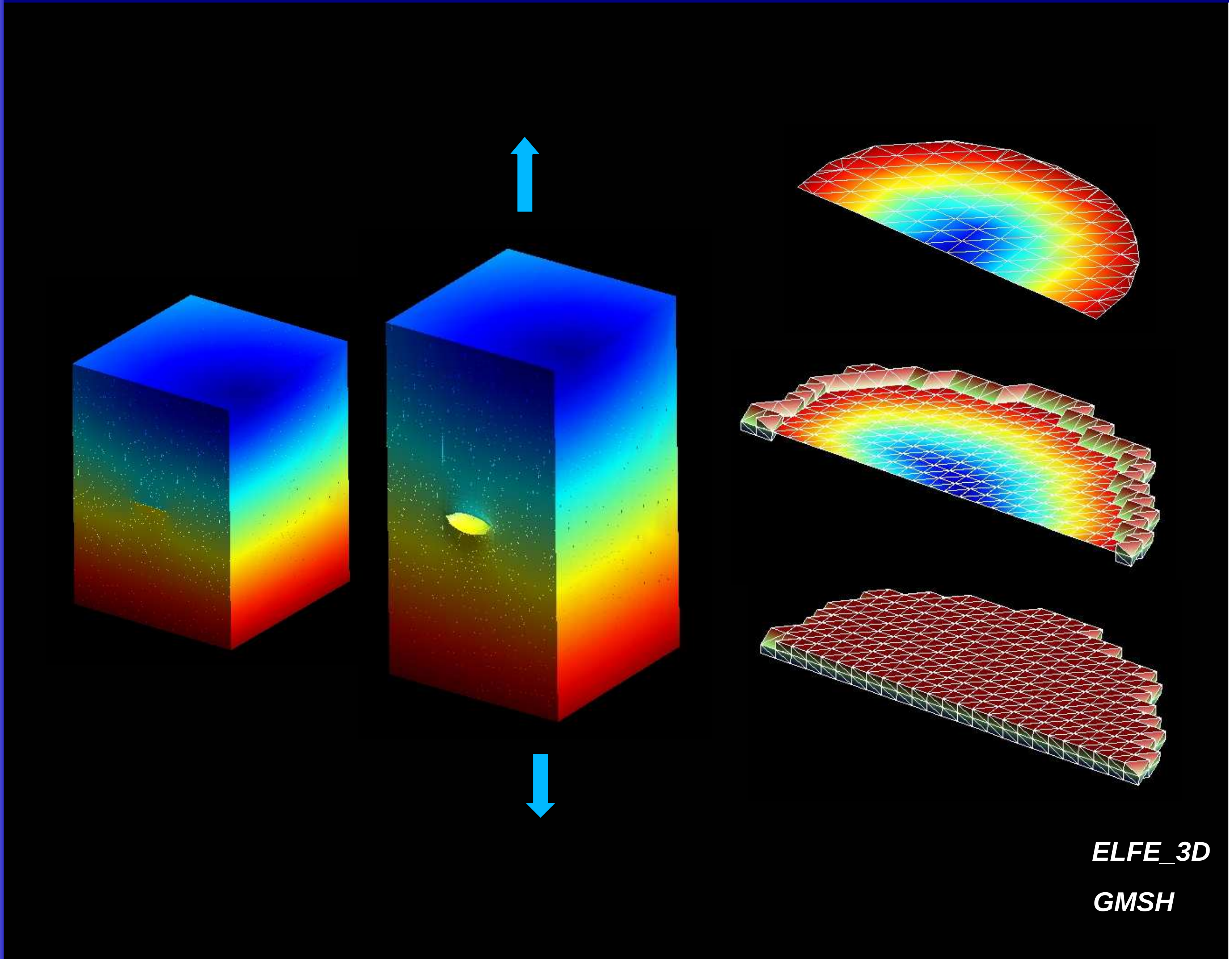
- **Time and space discretization for non-structured meshes**

$$\begin{cases} \dfrac{\partial \phi}{\partial t} + \mathrm{H}\left(\nabla \phi, \mathbf{x}, t\right) = 0 \\ \phi(\mathbf{x},0) = \phi_0(\mathbf{x}) \end{cases}$$

Space: [Barth and Sethian 1998]
Time: Runge Kutta

→ *Numerical schemes stable, accurate and convergent. However, finite difference approaches are more accurate for an equivalent size element mesh*


ELFE_3D
GMSH

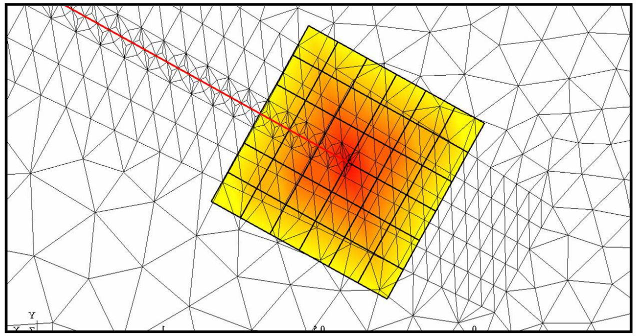

ELFE_3D
GMSH

Stress intensity factors calculation

2D interaction integral

$$I^{\Re,aux} = \int_C \left(W_l^{\Re,aux}\delta_{1j} - \sigma_{ij}^{\Re}\frac{\partial u_i^{aux}}{\partial x_1} - \sigma_{ij}^{aux}\frac{\partial u_i^{\Re}}{\partial x_1} \right) n_j \, ds + \sigma_{12}^{\Re}(A)\left[u_1^{aux}(A)\right]$$

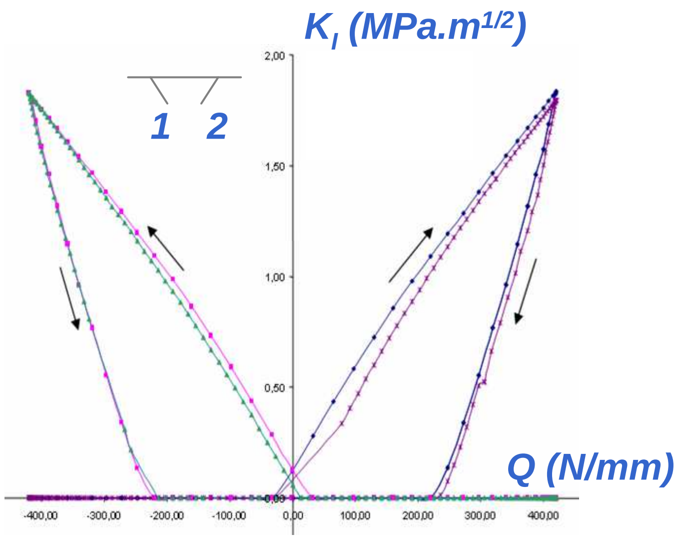
$$I^{\Re,aux} = \frac{2(1-\nu^2)}{E}\left(K_I^{\Re}K_I^{aux} + K_{II}^{\Re}K_{II}^{aux}\right)$$

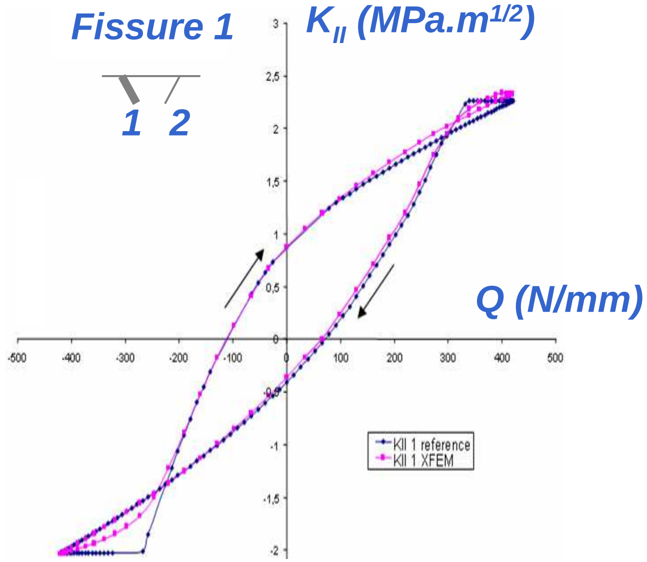


Integration domain close to the crack tip

Validation of the SIFs calculation:

Comparison with a semi-analytical model: [Dubourg et al, ASME J. Trib. 1992]





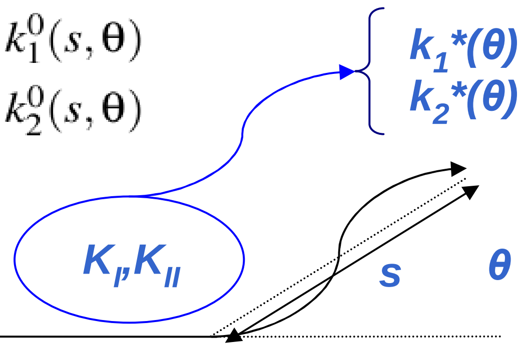
[M.C. Baietto, E. Pierres, A. Gravouil., IJSS 2010]

Critères prédisant la direction de propagation

Calcul des contraintes des contraintes à l'extrémité infinitésimale d'une fissure (Amestoy et Leblond 1979) :

$$\begin{pmatrix} k_1^0(s,\theta) \\ k_2^0(s,\theta) \end{pmatrix} = \begin{bmatrix} K_{11}(\theta) & K_{12}(\theta) \\ K_{21}(\theta) & K_{22}(\theta) \end{bmatrix} \begin{pmatrix} K_I \\ K_{II} \end{pmatrix}$$

$$\begin{aligned} k_1^*(\theta) &= \lim_{s\to 0} k_1^0(s,\theta) \\ k_2^*(\theta) &= \lim_{s\to 0} k_2^0(s,\theta) \end{aligned}$$



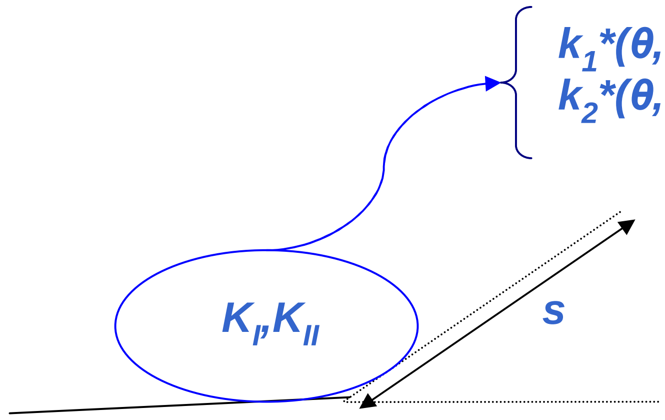
→Max $k_1^*(\theta)$
→$k_2^*(\theta)$

Critère basé sur les maximums des quantités calculés

Applicable à des chargements multi-axiaux proportionnels

Critères prédisant la direction de propagation

Extension des développements d'Amestoy par Pineau et Hourlier (1982)



→ max $k_1^*(\theta,t)$,
→ max $\Delta k_1^*(\theta,t)$
→ max da/dN (θ,t)

3 critères basés sur les maximums en espace et en temps sur un cycle entier

Critère applicable à des chargements multi-axiaux non proportionnels